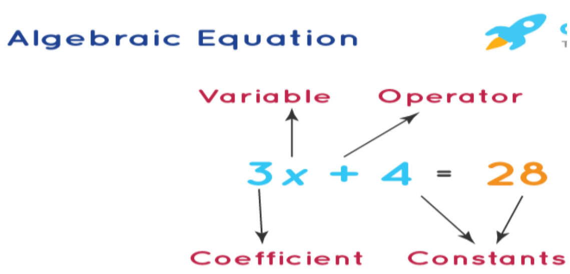


MATHS

HOMEWORK

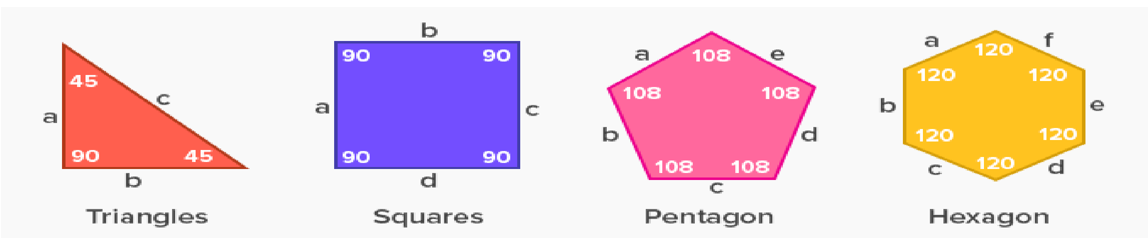
SUPPORT

BOOKLET

FOR

Parents

KS 3



Content Page

- Place Value And Rounding 3 - 7
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- Fractions 14 - 18
- Probability 19
- Ratio 20 - 21
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Place Value and Rounding

Booklet 1: Place Value and Rounding		
1	Place value	The value of a digit depending on its place in a number.
2	Number	A word, symbol, or figure, that represents a particular quantity. Used in counting and making calculations e.g. Two hundred and thirty four, 234, 45.7, $\sqrt{2}$.
3	Integer	A whole positive or negative number
4	Decimal	A number with digits after a decimal point
5	Decimal place	The number of digits after the decimal point
6	Round	Change a number to one which is easier to use
7	Estimate (verb)	To give a rough idea of an answer. "Estimate the answer to 4.6 x 19.2"
8	Estimate (noun)	The rough answer. "My estimate to 4.6 to 19.2 is 100".
Inequalities		
9	$=$	Equal to – the left is equal to the right
10	$\neq$	Not equal to e.g. $4 + 3 \neq 6$
11	$<$	Less than e.g. $3 < 4$
12	$>$	Greater than e.g. $4 > 3$
13	$\leq$	Less than or equal to
14	$\geq$	Greater than or equal to
15	$\approx$	Approximately $4.8 \approx 5$

Place Value

Value of the Digit

The column the digit is in tells us its **value**.

Example			
Thousands	**Hundreds**	**Tens**	**Units**
3	5	0	2

The value of the 5 is 500 because it is in the hundreds column (5 x 100 = 500).

The value of the 3 is 3000 because it is in the thousands column (3 x 1000 = 3000)

Example:

48	6542	80321
Is made up of 4 tens 8 units	Is made up of 6 thousands 5 hundreds 4 tens 2 units	Is made up of 8 ten thousands 0 thousands 3 hundreds 2 tens 1 unit

Place Value Table

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths

Rounding whole Numbers

We have a rhyme when rounding Under Line, draw a line/say the rhyme **5 or more add 1 more, 4 or less let it rest**

Round to nearest 10

6/7 ↖ say the rhyme = 70

10/2 ↖ say the rhyme = 100

Round to nearest 100

1/6 ↖ 7 say the rhyme = 200

1/3 ↖ 3 say the rhyme = 100

Rounding decimals

Rounding means making a number simpler but keeping its value close to what it was.

Examples:

Round 59.9261 to the following place value…		
a) to 1 decimal place/nearest tenth **S**	**b) to 2 decimal places/nearest hundredth** **U** 59 . 9261	**c) to 3 decimal places/nearest thousandth** **S**

59 . 9261 decider The decider is 2, so we keep it the same. 59.9261 ≈ 59.9	1 decider 5 9 . 93 The decider is 6, so we round up the 2 by adding 1. 59.9261 ≈ 59.93	59 . 9261 decider The decider is 1, so we keep it the same. 59.9261 ≈ 59.926

Dealing with 9s

We have looked at rounding decimals to different place values. What we haven't looked at yet is what do you do when you have to round up a '9'? In our base ten system, '9' is the biggest single digit number there is, so when rounding a '9' up you make the '9' a '0' and carry over a '1' to the next biggest place value. The easiest way to do this is using a column addition layout. This is especially useful when there are lots of 9s!

Examples:

Rounding		
a) to 1 decimal place/nearest tenth U 14 . 9684 decider	**b) to 2 decimal places/nearest hundredth** U 14 . 9982 decider	**c) to 3 decimal places/nearest thousandth** U 99. 9998 decider

. 1 15 . 0 1	. 1 15 . 00 1 1	. 1 100. 000 1 1 1 1

___________	___________	___________ ___________
The decider is a '6', so we round up. The easiest way is to "add 1" using column addition. We carry '1' to the ones. The '4' turns into a '5'. **14.9694 ≈ 15.0** There is one digit after the decimal point. It is rounded to 1 decimal place.	The decider is a '8', so we round up. The easiest way is to "add 1" using column addition, carrying each time. **14.9982 ≈ 15.00** There are two digits after the decimal point. It is rounded to 2 decimal places.	The decider is an '8', so we round up. The easiest way is to "add 1" using column addition, carrying each time. **99.9988 ≈ 100.000** There are three digits after the decimal point. It is rounded to 3 decimal places.

Factors and Multiples

Factors and Multiples

A **divisibility rule** is a **short, easy** way of determining whether an integer is divisible by a whole number, without performing the division itself. It is also a very quick way of finding a **factor** of a number.

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
9	If the sum of the digits is divisible by 9
10	If the last digit is 0

If a number is divisible by 2, then the number 2 as one of its factors. The number itself is a multiple of 2.

Example

- 982 is divisible by 2.
- 2 is a factor of 982.
- 982 is a multiple of 2.

Non-Example

- 835 is **not** divisible by 2.

- 2 is **not** a factor of 835
- 835 is **not** a multiple of 2.

Digit Sums: Divisibility rules for 3s and 9s.

- If a number is in the 3 times table then the **sum of its digits** is divisible by 3.
- If a number is in the 9 times table then the **sum of its digits** is divisible by 9.

Divisibility rule for 6

- All numbers in the 6 times table have 3 and 2 as a factor.
- If a number is in **both** the 3 times table **and** 2 times table then the number is divisible by 6.
- You can remember this as "Test for 2 and 3… if it has two ticks then it divides by six!"

Multiples

- Multiples of a number are the numbers which are in its times table.

Multiples of a number are the '**same or more**'.

Common Multiples

Common multiples are numbers which are in the times table of two or more numbers.

Lowest Common Multiple

The smallest number that is a common multiple of two or more numbers is called the **lowest common multiple (LCM).** The LCM is very useful when adding or subtracting fractions.

Example:

What is the lowest common multiple of 3 and 4 3, 6, 9, **12**, 15, 18, 21, 24, 27, 30, 33, 36… 4, 8, **12**, 16, 20, 24, 28, 32, 36 … 12, 24 and 36 (and others not shown here) are common multiples of 3 and 4. The **lowest common multiple** (LCM) is 12.

Factors

A **factor pair** of a number are two numbers that you multiply together that results in that given number. If the number is repeated (for example 4 x 4), we only list each factor once. For this reason, we can say that numbers have either an **even** an **odd** number of **unique factors**.

Example

Write all the factor pairs for 100.

1 x 100
2 x 50
4 x 25
5 x 20
10 x 10

List the factors of 100

1, 2, 4, 5, 10, 20, 25, 50, 100.

It has 9 unique factors.

Prime Factors

A prime factor is a factor of a number which is also prime. We will discover just how important these prime factors are!

Prime factors only have <u>TWO factors 1 and itself.</u>

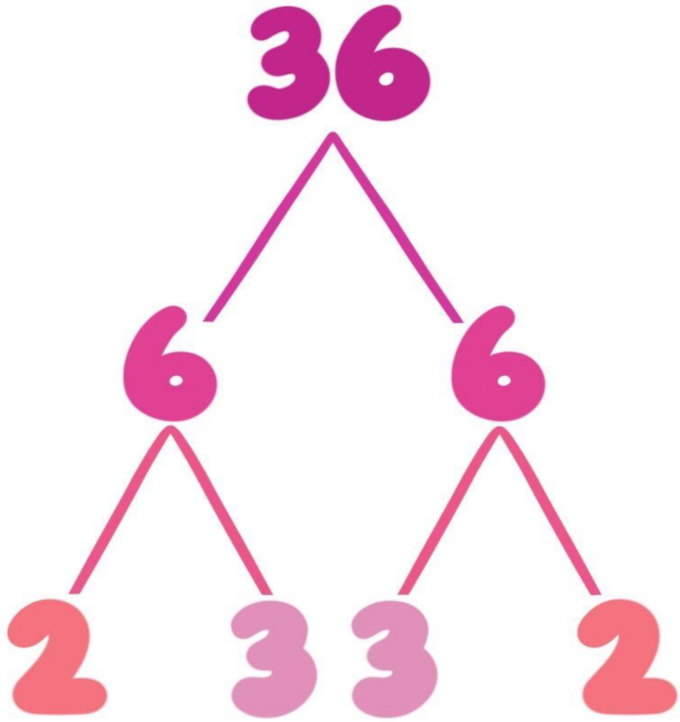
The first 10 prime numbers: 2,3,5,7,11,13,17,19,23,29

For example, 2 is prime number which is a factor of 6.

Therefore 2 is a prime factor of 6.

Example
What are the **prime factors** of 12?
1) Factor pairs of 12
<u>12</u>
1 12
2 6
3 4
2) Circle the prime numbers
3) Prime factors: 2 and 3

Factor Trees: a way to find prime factorization



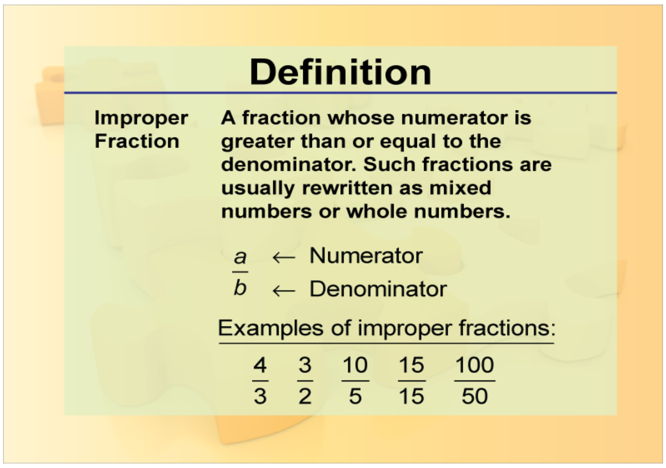
Index Form

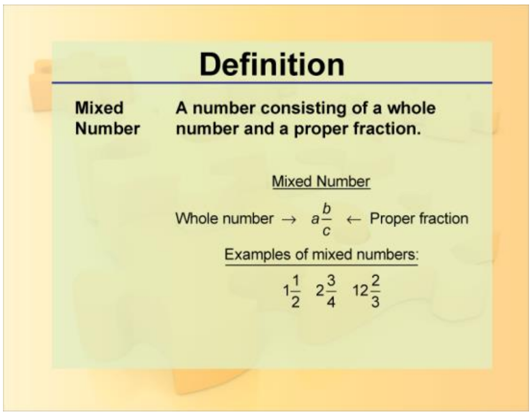
2x2x3x3

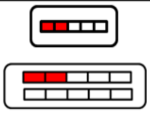
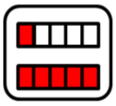
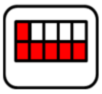
$= 2^2 \times 3^2$

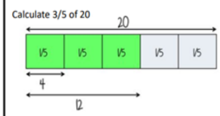
Fractions

Booklet 2: Working with Fractions			
16	Fraction	Part of a whole	
17	Equivalent fraction	Two or more fractions with the same value	$\frac{1}{2} = \frac{3}{6}$
18	Numerator	The top of a fraction	$\frac{Numerator}{Denominator}$
19	Denominator	The bottom of a fraction	
20	Unit Fraction	A fraction where the numerator is 1	½, 1/3, 1/6
21	Proper Fraction	Value is less than one. Numerator is smaller than the denominator	2/7 , 1/6, 100/365
22	Improper Fraction	Value is larger than 1. Numerator is larger than the denominator.	8/5, 13/2, 47/1
23	Mixed Number	A number written as an integer and a proper fraction.	$2\frac{1}{3}$, $12\frac{11}{30}$
24	Fraction in its *simplest form*	The numerator and denominator have no common factors larger than 1.	$\frac{3}{6}$ not in its simplest form as top and bottom ÷3 $\frac{2}{5}$ is in its simplest form.
25	Fraction represents an integer	Numerator is a multiple of the denominator	$\frac{12}{6} = 2$, $\frac{2a}{a} = 2$
26	Fraction is equal to 1	Numerator and denominator are equal	$\frac{6}{6} = 1$, $\frac{a}{a} = 1$
27	Integers as a fraction	Write as a fraction with denominator 1.	$6 = \frac{6}{1}$


Definition
Improper Fraction
A fraction whose numerator is greater than or equal to the denominator. Such fractions are usually rewritten as mixed numbers or whole numbers.
$\frac{a}{b}$ ← Numerator ← Denominator
Examples of improper fractions:
$\frac{4}{3}$ $\frac{3}{2}$ $\frac{10}{5}$ $\frac{15}{15}$ $\frac{100}{50}$


Definition
Mixed Number
A number consisting of a whole number and a proper fraction.
Mixed Number
Whole number → $a\frac{b}{c}$ ← Proper fraction
Examples of mixed numbers:
$1\frac{1}{2}$ $2\frac{3}{4}$ $12\frac{2}{3}$

Representing Fractions using pictures			
33	**The whole**	One shape in a fraction drawing.	
34	**Equal parts**	In a fraction drawing the whole is split into **equal** parts.	
35	**Fraction diagrams**	Proper Fraction	Improper Fraction
		Each whole is split into five equal parts two are shaded: $\frac{2}{5}$ 	Each whole is split into five equal parts, six are shaded. $\frac{6}{5}$ 
		Each whole is split into 10 equal parts, 6 are shaded: $\frac{6}{10}$ 	

Operations with Fractions			
36	Add or subtract fractions	Must have a common denominator fist. Add numerators. Denominator stays the same.	$\frac{1}{3}+\frac{2}{5}$ becomes $\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$
37	Find fraction of an amount	• divide by bottom • Multiply by top. Can also solve using multiplying with fractions ($\frac{3}{5}$ x $\frac{20}{1}=\frac{60}{5}$ = 12)	
38	Multiply fractions	Does **not** need common denominator • Top x top • Bottom x bottom. • Simplify result where possible.	$\frac{1}{3}$ x $\frac{3}{5}=\frac{3}{15}=\frac{1}{5}$
39	"of"	Means multiply.	
40	Multiply fractions by an integer	All integers can be written as fractions with 1 as the denominator. Multiply as normal.	$\frac{1}{3}$ x 27 $\frac{1}{3}$ x $\frac{27}{1}=\frac{27}{3}=9$
41	Divide with fractions	Use "Keep, Flip, Change" Keep the first fraction same Flip the second fraction upside down Change the ÷ to x. Multiply as normal.	$\frac{1}{3}\div\frac{2}{5}$ K C F $\frac{1}{3}$ x $\frac{5}{2}=\frac{5}{6}$
42	Calculations with mixed numbers	Usually best to convert to improper fractions first. (see left)	$2\frac{1}{3}+\frac{2}{3}$ $\frac{7}{3}+\frac{2}{3}=\frac{9}{3}$ simplifies to 3

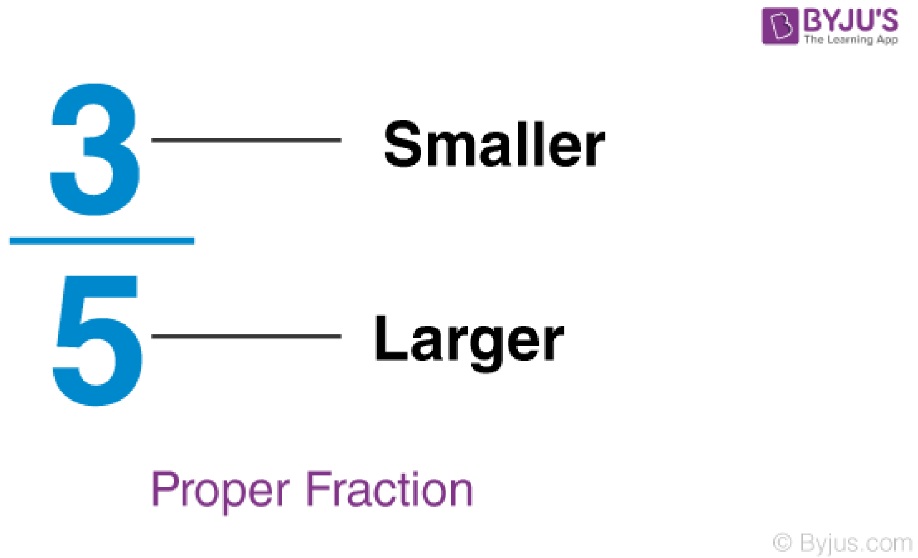
In Maths, there are three major types of fractions. They are **proper fractions, improper fractions and mixed fractions**. Fractions are those terms which have numerator and denominator. Based on these two terms we define its types.

Proper Fraction

A fraction where the numerator is less than the denominator, then it is known as a proper fraction.

i.e., Numerator < Denominator

For example,

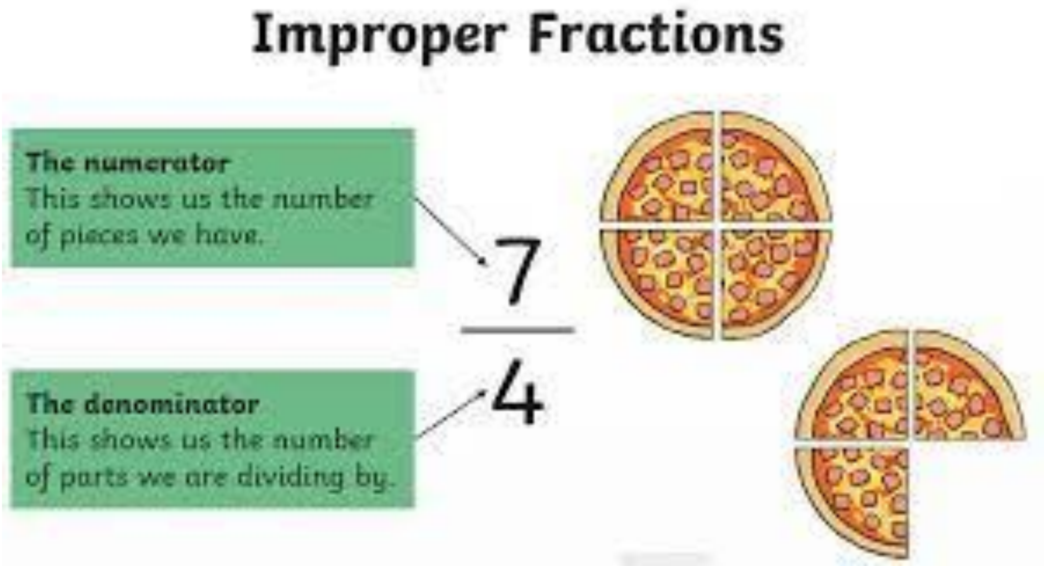


Note:

- The value of proper fraction after further simplification is always less than 1.

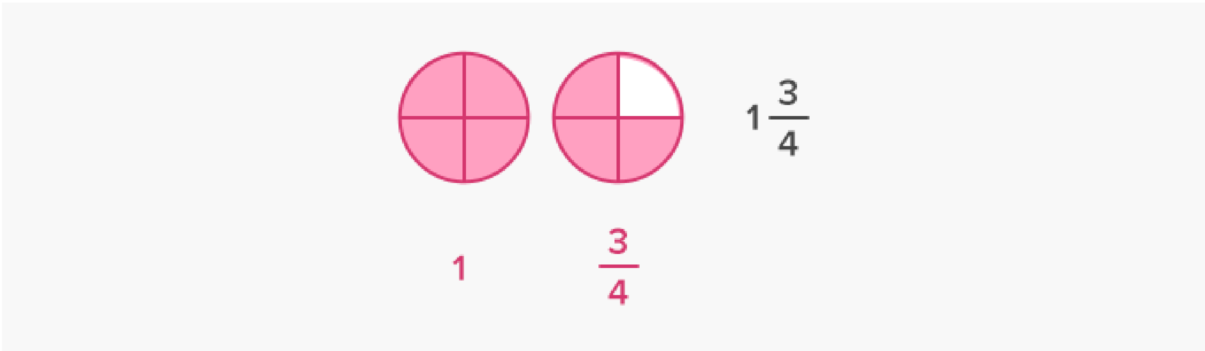
<u>Improper Fraction</u>

An improper fraction has a numerator greater than the denominator. For example, **3/2** is an improper fraction, but 2/3 is a proper fraction, whose denominator is greater than the numerator.



<u>Mixed Number</u>

A mixed number, or mixed fraction, is **a number that contains both an integer (whole number) and a proper fraction (a fraction whose numerator is less than its denominator)**.

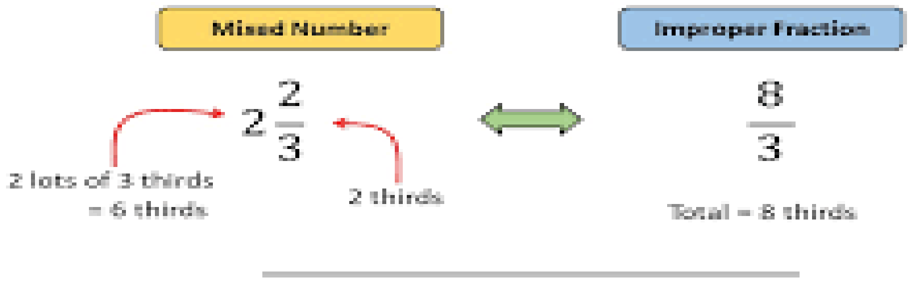


<u>Converting Fractions</u>

In order to convert a mixed number to an improper fraction:

1. Multiply the whole number by the denominator.
2. Add on the numerator.

3. Write the improper fraction by using the calculated value as the numerator over the original denominator.



Improper Fraction to Whole Number

To change an **improper fraction** to a **mixed number**, you must divide the numerator by the denominator. This will give you how many whole numbers the improper ...

Equivalent Fractions

When two or more fractions have the same result after simplification for which they represent the same portion of the whole, then such fractions are equal to each other and are called equivalent fractions.

For example, 1/2 and 2/4 are equivalent.

1/3 and 3/9 are equivalent.

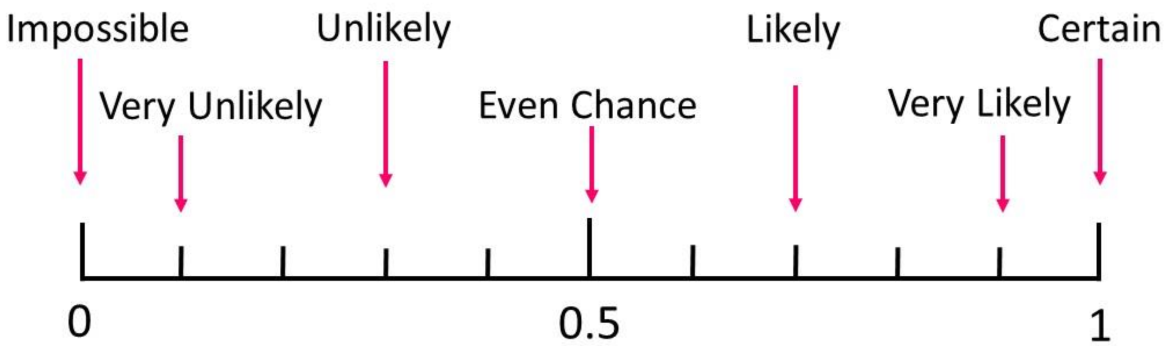
What is probability?

Probability

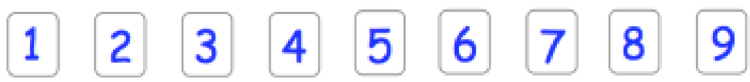
Probability is about estimating or calculating how likely something is to happen.

In maths, probabilities are always written as fractions, decimals or percentages with values between 0 and 1.

The Probability Scale



Question 3: Ralph has 9 cards, each with a number on it.



He picks a card at random.
Write down the probability that the chosen card is

(a) the number 8 (b) an even number (c) a number less than 7

(d) a multiple of 4 (e) a square number (f) a prime number

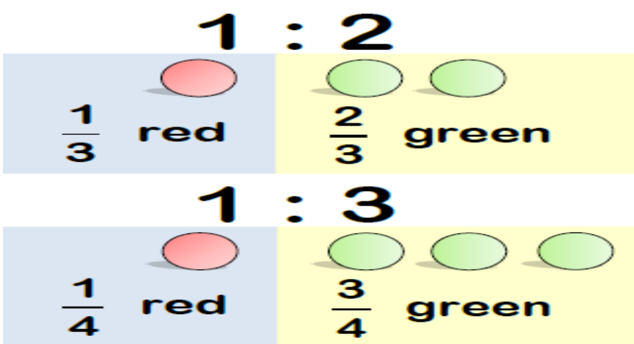
Ratio

A ratio **shows how much of one thing there is compared to another**. Ratios are usually written in the form a:b. If you are making orange squash and you mix one part orange to four parts water, then the ratio of orange to water will be 1:4 (1 to 4). The order in which a ratio is stated is important.

91	Ratio		The ratio of boys to girls is 2:3. For every 2 boys, there are 3 girls. **Boys : Girls** **2 : 3**
92	Proportion	The relationship between a part and a whole.	The ratio of boys to girls is 2:3

		Can be expressed as a fraction, decimal or percentage.	The proportion of the group which are boys is $\frac{2}{5}$
93	Express ratio as a fraction	Add the parts to find the denominator.	
94	Simplify Ratio	Divide both sides by a common factor	Squares : Diamonds is 9:12 3:4.

Example



ALGEBRA

Inverse operations

In maths, every operation has an inverse. When using the balance method to solve equations, we 'balance' or 'cancel' out an operation by using its inverse. Here are the inverse operations that you need to know:

Operation	Inverse
$\times$	$\div$
$\div$	$\times$
$+$	$-$
$-$	$+$
^{2} (square/power of 2)	$\sqrt{\ }$

Remember in algebra that

- $3a$ means $3 \times a$.
- $\frac{a}{3}$ means $a \div 3$

Examples:

Solve $3x = 12$ $\frac{\cancel{3}x}{\cancel{3}} = \frac{12}{3}$ $x = 4$ Check 3(4) = 12 □	Solve $20 = -4a$ $\frac{20}{-4} = \frac{\cancel{-4}a}{\cancel{-4}}$ $-5 = a$ Check -4(-5) = 20 □
Solve $\frac{x}{2} = 6$ $\frac{x}{2} = 6$ (×2) (×2) $x = 12$	Solve $3 = \frac{a}{7}$ $3 = \frac{a}{7}$ (×7) (×7) $21 = a$
Check $\frac{12}{2}$ = 6 □	Check 3 = $\frac{21}{7}$ □

<u>Collecting then solving</u>

Examples:

Solve $12 = 2x + 4$ $12 = 2x + 4$ $-4 \qquad -4$ $\frac{8}{2} = \frac{\cancel{2}x}{\cancel{2}}$ $4 = x$	Solve $7 = 3a - 5$ $7 = 3a - 5$ $+5 \qquad +5$ $\frac{12}{3} = \frac{\cancel{3}a}{\cancel{3}}$ $4 = a$
Check $2(4) + 4$ $12 = 8 + 4$ ✓	Check $3(4) - 5$ $7 = 12 - 5$ ✓
$30 = 10 + 5a$ $-10 \qquad -10$ $\frac{20}{5} = \frac{\cancel{5}a}{\cancel{5}}$ $4 = a$ Solve $30 = 10 + 5a$	$18 = -12 + 6x$ $+12 \qquad +12$ $\frac{30}{6} = \frac{\cancel{6}x}{\cancel{6}}$ $5 = x$ Solve $18 = -12 + 6x$
Check $10 + 5(4)$ $30 = 10 + 20$ ✓	Check $-12 + 6(5)$ $-12 + 30 = 18$ ✓

Examples:

<table>
<tr>
<td>$3x + 2x = 30$
$\frac{\cancel{5}x}{\cancel{5}} = \frac{30}{5}$
$x = 6$

Solve $3x + 2x = 30$

Check 3(6) + 2(6) = 30 ✓
18 + 12 = 30 ✓</td>
<td>Given that $x + 4 = 9$
a) What is the value of $2x$?
b) What is the value of $x - 3$?
Firstly, work out the value of x:
$x + 4 = 9$
$-4 \quad -4$
$x = 5$

Now substitute into find required expression:
a) $2x = 2 \times 5 = 10$
b) $x - 3 = 5 - 3 = 2$</td>
</tr>
</table>

Solving Two Step Equations

Two-step equations are equations that require us to do two balance steps to solve. It is really important that we do the balance steps in the correct order.

Examples:

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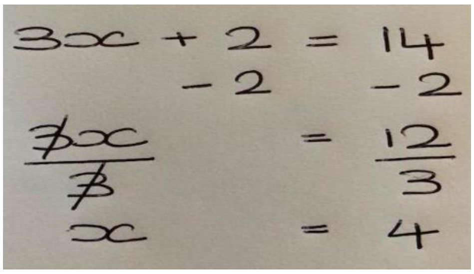
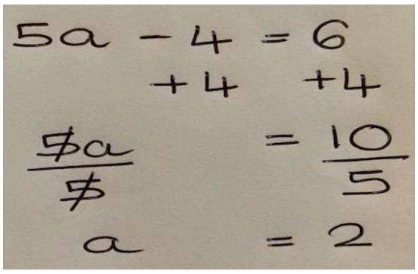
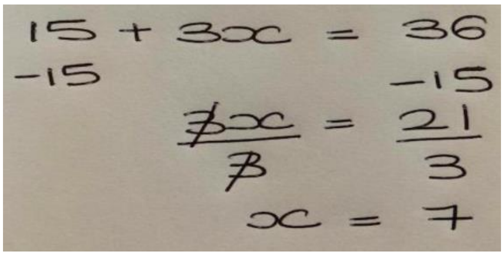
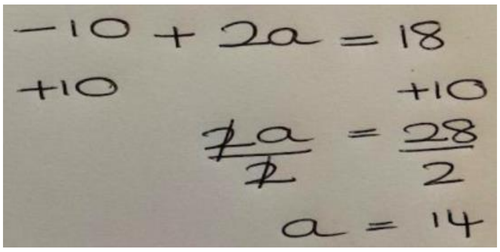
<td>

Solve $3x + 2 = 14$</td>
<td>

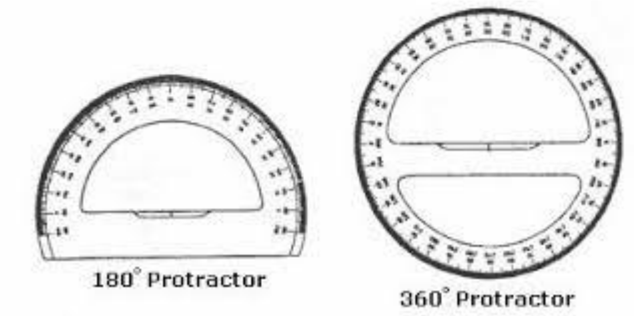
Solve $5a - 4 = 6$</td>
</tr>
<tr>
<td>Check 3(4) + 2
12 + 2 = 14 ✓</td>
<td>Check 5(2) - 4
10 - 4 = 6 ✓</td>
</tr>
<tr>
<td>

Solve $15 + 3x = 36$</td>
<td>

Solve $-10 + 2a = 18$</td>
</tr>
<tr>
<td>Check 15 + 3(7)
15 + 21 = 36 ✓</td>
<td>Check -10 + 2(14) = 18
-10 + 28 = 18 ✓</td>
</tr>
</table>

Angles

Protractor: Equipment used to measure angles.



Angle: A measure of turn

Acute Angle: Turn greater than 0°and less than 90°

Right Angle: 90°

Obtuse: Turn greater than 90° and less than 180°

Straight line: 180° turn

Reflex: Turn greater than 180° and less than 360°

Full Turn: 360° turn

Perpendicular line(s): Pair of lines that meet or cross at 90°

Polygon: 2D shape with straight lines

Interior Angles: The angles inside a polygon

Exterior Angles: Formed by extending a straight line next to interior angle.

Angles on a Straight line: Sum to 180°

Angles around a point: Sum to 360°
Vertical opposite angles: Are equal
Angles in Triangle's: Sum to 180 °
Angles in Quadrilaterals: Sum to 360°

Angle Facts: Straight Lines

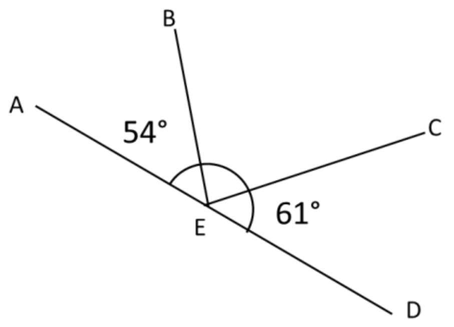
An angle is a measure of turn. The lengths of the lines do not matter. We measure angles in degrees. The larger the number, the greater the turn. A complete turn is 360°. Half a turn is 180°.

This is where we get our first angle fact

Adjacent angles on a straight line add up to 180°.

They must share a point.

Recap Example:	
The diagram below is not drawn accurately. Line AD is a straight line. Work out the size of angle B$\hat{E}C$.	State fact: Add up known angles: Subtract answer from known total:

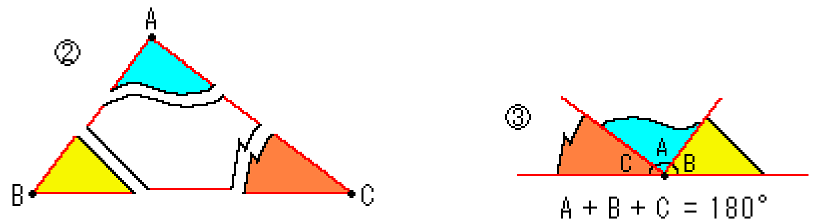
	

Angles in Triangles

We can use the angles on a straight line fact from lesson one to derive another fact.

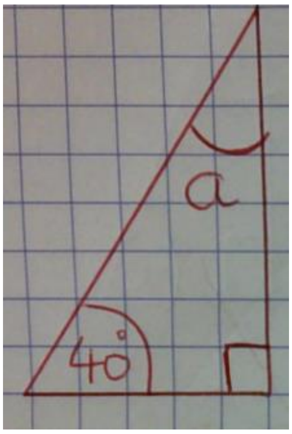
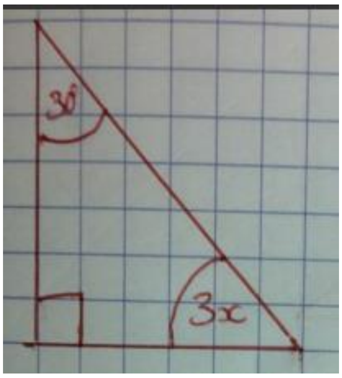
Angle Fact: Interior Angles of a Triangle add up to 180°

Demonstration: If you tear the three angles off a triangle and put them together they will always make a straight line.



*You can also see this if you fold a triangle inwards

Example with one missing angle	Example with algebra

 Add any information to diagram Work out missing angle	 Add any information to diagram Form equation Solve

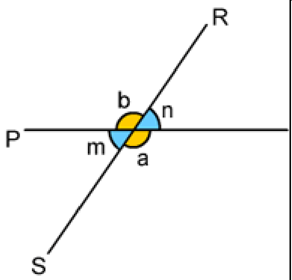
Vertically Opposite Angles

We have seen in the first lesson that adjacent angles on a straight line always add up to 180°.

We can use this fact to derive (work out) another angle fact.

Angle Fact: Vertically Opposite angles are equal

Demonstration



Fact: Adjacent angles on a straight line add up to 180°

180°	
b	n

180°	
a	n

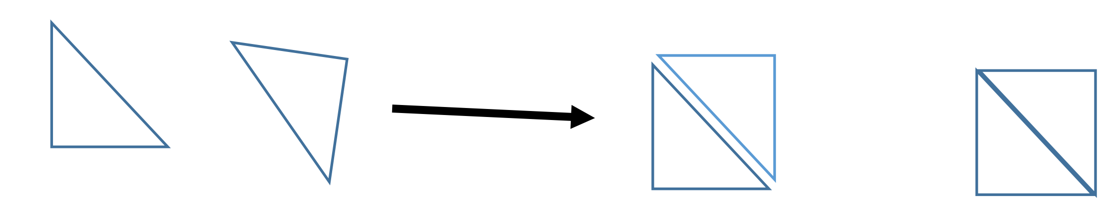
$b + n = 180°$ $\quad$ $a + n = 180°$

180°	
b	n
a	n

Angle a must be the same size as angle b

Angle Fact: The interior angles of a quadrilateral add up to 360°.

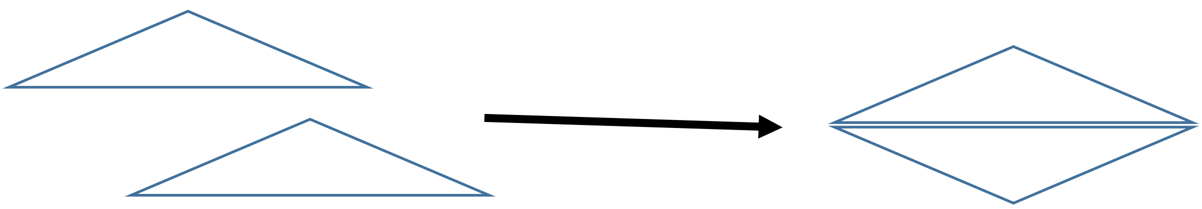
Demonstration



180°		
90°	40°	50°

180°		
90°	50°	40°

360°					
90°	40°	50°	50°	40°	90°



- Angles in a triangle add up to 180°
- 2 triangles make up a quadrilateral
- **Therefore, angles in a quadrilateral add up to 360°**