Topic 1 - Factors and Multiples

Definition

A **factor** is an integer that divides exactly into a whole number leaving no remainder.

The highest common factor (HCF) is the highest factors shared by 2 numbers.

Examples

1. List the factors of 18

1 x 18
2 x 9
3 x 6

The factors of 18 are 1, 2, 3, 6, 9, 18

2. List the factors of 24

1 x 24
2 x 12
3 x 8
4 x 6

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

3. What is the HCF of 18 and 24?

Factors of 18: 1, 2, 3, 6, 9, 18

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

The HCF of 18 and 24 is 6

Definition

A **multiple** is the result of multiplying a number by an integer.

The lowest common multiple (LCM) is the lowest multiple shared by 2 numbers.

Examples

1. List the first five multiples of 3

3, 6, 9, 12, 15

2. List the first five multiples of 4

4, 8, 12, 16, 20

3. What is the LCM of 3 and 4?

Multiples if 3: 3, 6, 9, 12, 15...

Multiples of 4: 4, 8, 12, 16, 20...

The LCM of 3 and 4 is 12

Questions

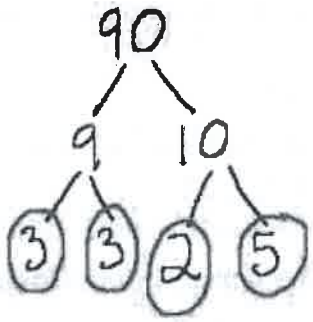
List the factors of 16	List the factors of 36	What is the HCF of 16 and 36
List the factors of 32	List the factors of 48	What is the HCF of 32 and 48
List the first six multiples of 5	List the first six multiples of 6	What is the LCM of 5 and 6
List the first six multiples of 7	List the first six multiples of 8	What is the LCM of 7 and 8

Prime Numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

A **factor Tree** is a method used to write a number as a product of prime factors

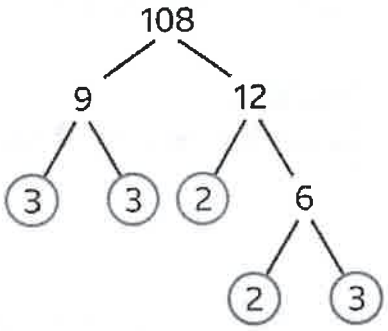
Examples

1. Write 90 as a product of prime factors in index form



Answer = $2 \times 3^2 \times 5$

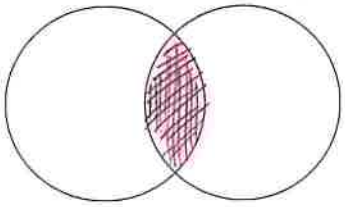
2. Write 108 as a product of prime factors in index form



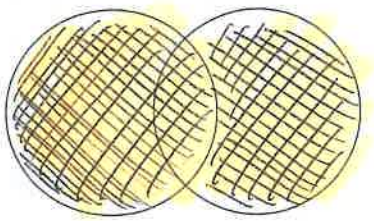
Answer = $2^2 \times 3^3$

We can use a **venn diagram** to find the HCF and LCM of a pair of numbers once we have wrote them as a product of prime factors

HCF – multiple the numbers in the middle



LCM – multiple ALL of the numbers

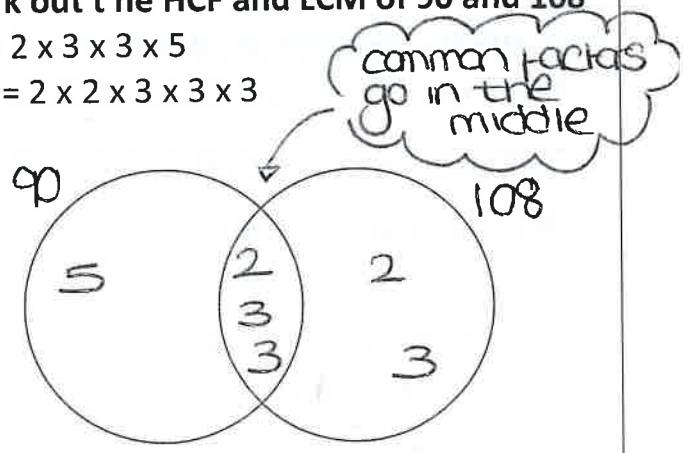


Example

Work out t he HCF and LCM of 90 and 108

90 = 2 x 3 x 3 x 5

108 = 2 x 2 x 3 x 3 x 3



HCF = 2 x 3 x 3 = 18

LCM = 2 x 2 x 3 x 3 x 3 x 5 = 540

Questions (use examples to help you)

List the first 10 prime numbers	Write 60 as a product of prime factors in index form	Write 72 as a product of prime factors in index form
Work out the HCF and LCM of 60 and 72	Work out the HCF and LCM of 60 and 108	Work out the HCF and LCM of 72 and 108

Topic 2- Powers and Roots

Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

The **power** of a number tells you how many times to multiply the number by itself.

Examples

1. $4^2 = 4 \times 4 = 16$
2. $5^3 = 5 \times 5 \times 5 = 125$
3. $3^2 + 6^3 = 9 + 216 = 225$

Square root is the inverse of power of 2
Cube root is the inverse if power of 3

Examples

1. $\sqrt{25} = 5$
2. $\sqrt[3]{512} = 8$
3. $\sqrt{16} + \sqrt[3]{125} = 4 + 5 = 9$

Questions

$\mathbf{11^2}$	$\mathbf{9^3}$	$\mathbf{5^2 + 3^2}$	$\mathbf{7^3 - 5^3}$
$\mathbf{12^2 + 3^3}$	$\mathbf{\sqrt{36} + \sqrt{25}}$	$\mathbf{\sqrt[3]{729} - \sqrt{81}}$	$\mathbf{5^3 + \sqrt{144}}$

Topic 3 – Rounding and Estimation

Rounding to the nearest integer

Rounding line goes after the unit/ones column

Examples

Round to the nearest integer

a) 15.3 $\approx$ 15 (decider)

b) 140.73 $\approx$ 141 (decider)

c) 300.5 $\approx$ 301 (decider)

Rounding to the nearest 10, 100, 1000

Rounding line goes **after** the place value

Examples

9847

1. Round to the nearest 10

9847 $\approx$ 9850 (decider)

2. Round to the nearest 100

9847 $\approx$ 9800 (decider)

3. Round to the nearest 1000

9847 $\approx$ 10000 (decider)

Questions

Round to the nearest integer:	Round to the nearest 10:	Round to the nearest 100:	Round to the nearest 1000:
a) 21.8	a) 89	a) 289	a) 5891
b) 84.51	b) 375	b) 8915	b) 9751
c) 125.29	c) 2841	c) 2973	c) 12583

Round to decimal places

Decimal places are the digits after the decimal point

1 decimal place = nearest tenth
2 decimal places = nearest hundredth
3 decimal places = nearest thousandth

Examples

153.2843

1. Round to 1 decimal place

2. Round to 2 decimal place

3. Round to 3 decimal places

Round to significant figures

We start counting significant from the left from the first non-zero digit.

Examples

4981

1. Round to 1 significant figure

2. Round to 2 significant figures

3. Round to 3 significant figures

Examples

0.5681

1. Round to 1 significant figure

2. Round to 2 significant figures

3. Round to 3 significant figures

Questions

Round to 1 decimal place:	Round to 2 decimal places:	Round to 3 decimal places:
a) 51.97	a) 0.4956	a) 183.1697
b) 108.542	b) 138.1571	b) 48.2438
c) 0.8681	c) 57.1346	c) 0.05931
Round to 1 significant figure:	**Round to 2 significant figures:**	**Round to 3 significant figures:**
a) 4210	a) 123	a) 3450
b) 0.891	b) 2.493	b) 0.86152
c) 12.861	c) 0.1056	c) 25.8961

Estimation/Approximation

When we estimate we round each number to 1 significant figure first, then calculate.

Example

Estimate the answer to

48.7×61.2

Step 1 - Round each number to 1 significant figure

$48.7 \approx 50$ (decider)

$61.2 \approx 60$ (decider)

Step 2 – Rewrite and calculate

$48.7 \times 61.2 \approx 50 \times 60 = 300$

Answer = 300

Example

Estimate the answer to $\dfrac{8.3 \times 18.7}{0.52}$

Step 1 - Round each number to 1 significant figure

$8.3 \approx 8$ (decider)

$18.7 \approx 20$ (decider)

$0.52 \approx 0.5$ (decider)

Step 2 – Rewrite and calculate

$$\frac{8.3 \times 18.7}{0.52} \approx \frac{8 \times 20}{0.5} = \frac{160}{0.5} = \frac{1600}{5} = 320$$

Answer = 320

When estimating with a square root, we round to the nearest square number first

Examples

1. $\sqrt{19} \approx \sqrt{16} = 4$
2. $\sqrt{38} \approx \sqrt{36} = 6$
3. $\sqrt{98} \approx \sqrt{100} = 10$

Example

Estimate the answer to $\dfrac{2.3^2 \times \sqrt{99}}{4.962}$

Step 1 - Round each number

$2.3 \approx 2$ (decider)

$4.962 \approx 5$ (decider)

$99 \approx 100$ (nearest square number)

Step 2 – Rewrite and calculate

$$\frac{2.3^2 \times \sqrt{99}}{4.962} \approx \frac{2^2 \times \sqrt{100}}{5} = \frac{4 \times 10}{5} = \frac{40}{5} = 8$$

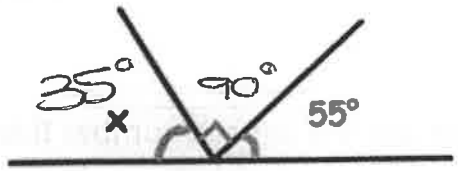
Answer = 8

Questions		
Estimate $\frac{28.4 \times 21.05}{5.9}$	Approximate $\frac{41.2 \times 19.8}{0.49}$	Estimate $\frac{21.75 + \sqrt{98.1}}{0.192}$
Estimate 5.9^2	Estimate $\sqrt{120}$	Approximate $\sqrt{83}$
Approximate $9.04 + 19.85 \times 2.99 - 5.03$	Estimate $\frac{431.1}{14.3 + 3.8^2}$	Estimate $\frac{804 \times 2.86}{0.513}$

Topic 4 – Basic Angles and Parallel Lines

Angles on a straight line at a point sum to 180°

Find x:



$x + 90 + 55 = 180$

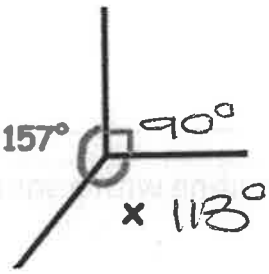
$x + 145 = 180$

$-145 \quad -145$

$x = 35°$

Angles around a point sum to 360°

Find x:



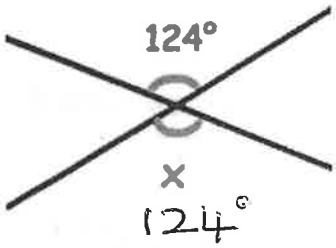
$x + 90 + 157 = 360$

$x + 247 = 360$

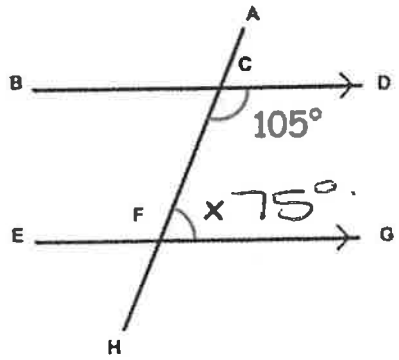
$-247 \quad -247°$

$x = 113°$

Vertically opposite angles are equal



Co-interior angles sum to 180°

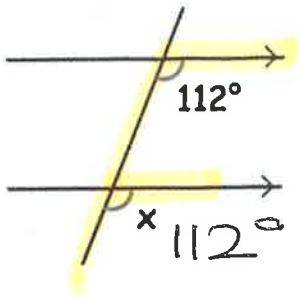


$x + 105 = 180$

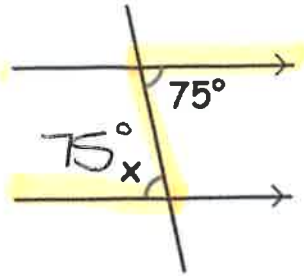
$-105 \quad -105$

$x = 75°$

Corresponding angles are equal



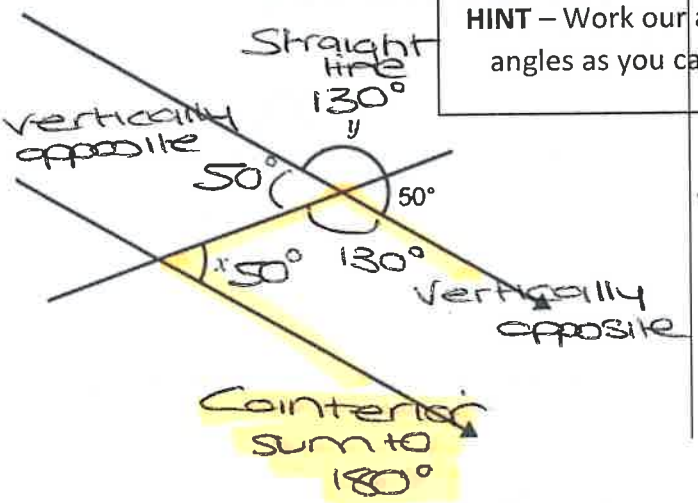
Alternate angles are equal



Examples

Calculate the size of angles x and y

HINT – Work our as many angles as you can first

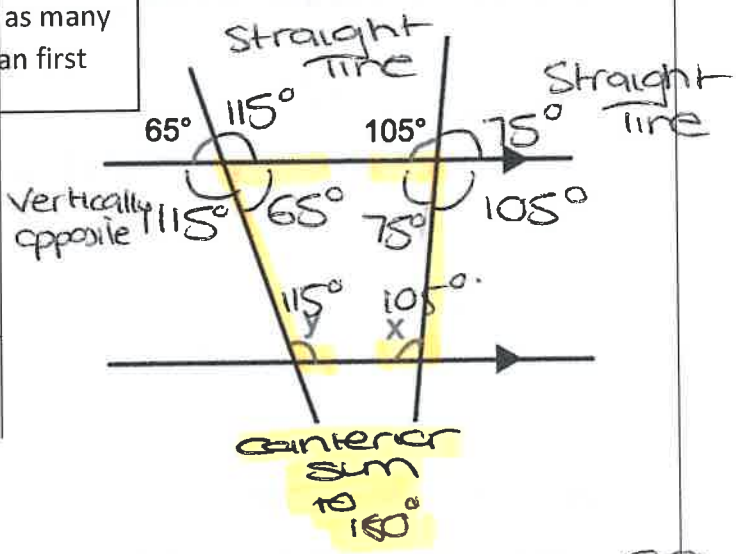


$x + 130 = 180$

$-130 \quad -130$

$x = 50°$

Calculate the sizes of angles x and y



$y + 65 = 180$

$-65 \quad -65$

$y = 115°$

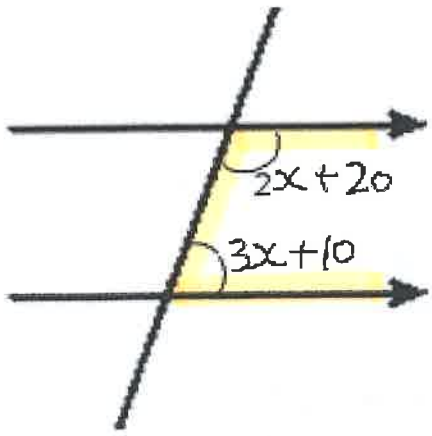
$x + 75 = 180$

$-75 \quad -75$

$x = 105°$

Work out the value of x

1. Form an equation



$2x + 20 + 3x + 10 = 180$

$5x + 30 = 180$

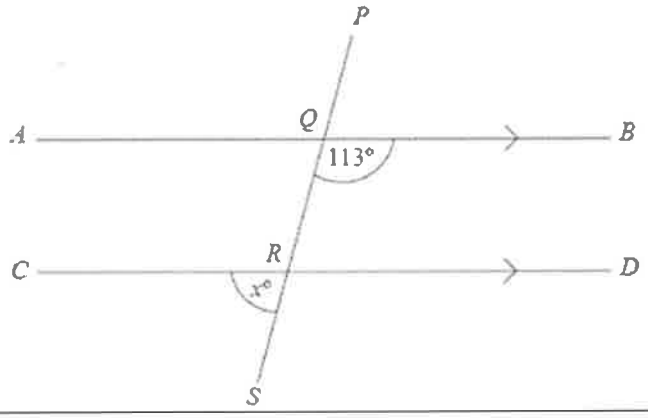
$-30 \quad -30$

$\frac{5x}{5} = \frac{150}{5}$

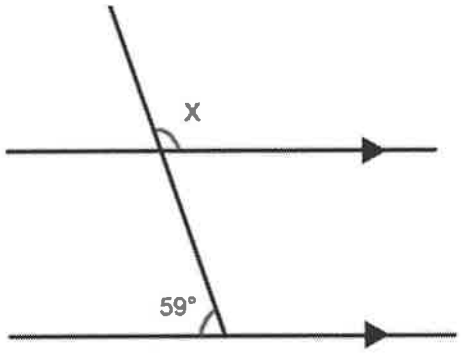
$x = 30.$

Questions

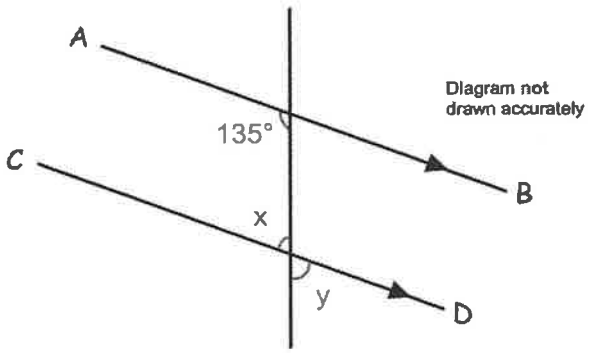
Work out the size of angles x



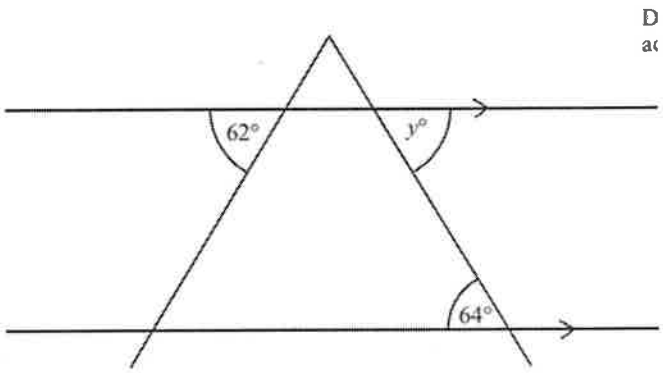
Work out the size of angle x



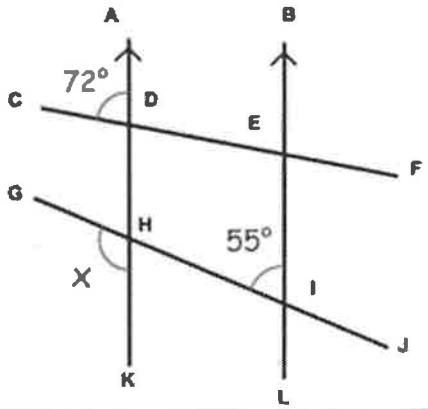
Work out the size of angles x and y



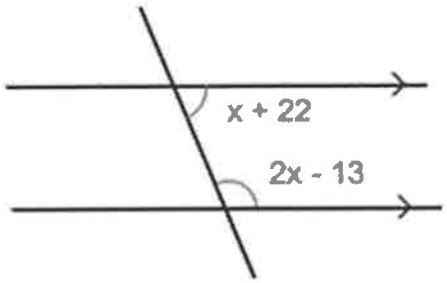
Work out the size of angle y



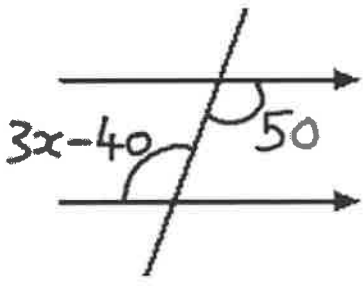
Work out the size of angle x



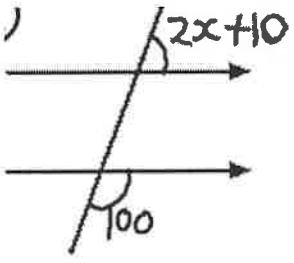
Work out x



Work out x



Work out x

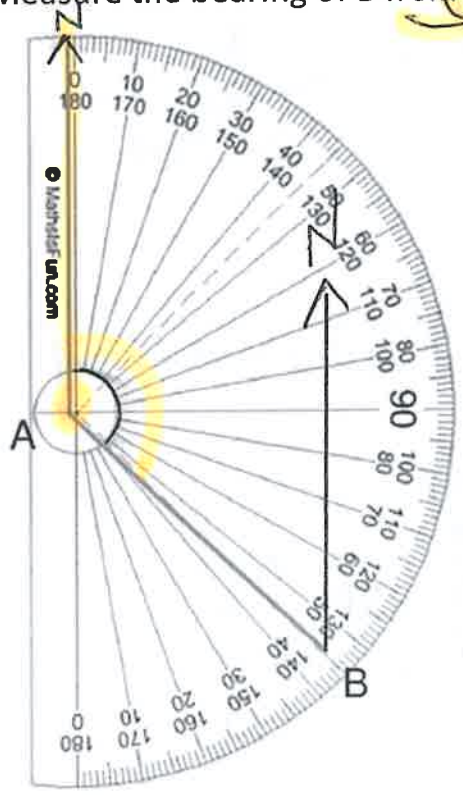


Bearings follow 3 rules:

1. Measure from North
2. Measure clockwise
3. Must be 3 figures

Example

Measure the bearing of B from A

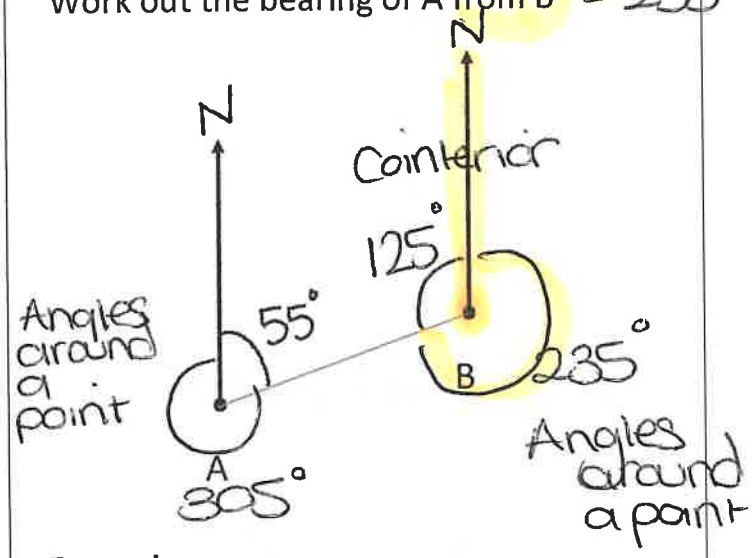


Answer = 135°

HINT – Work our as many angles as you can first

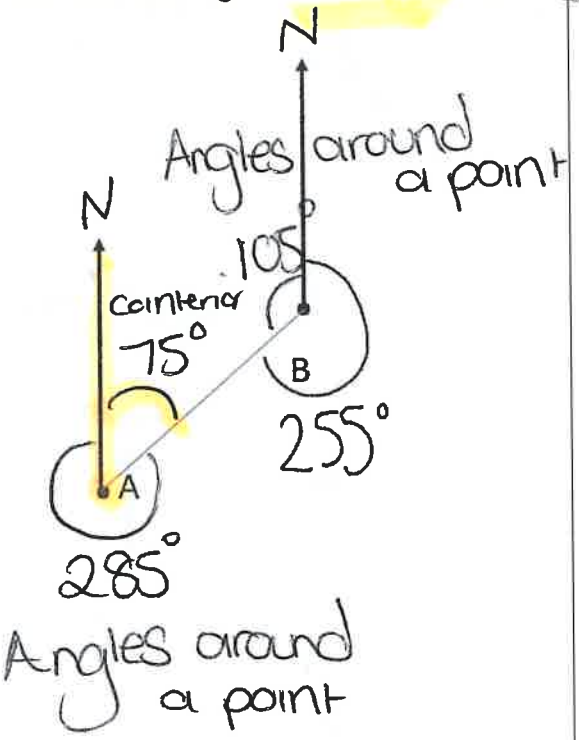
Example

Work out the bearing of A from B = 235°



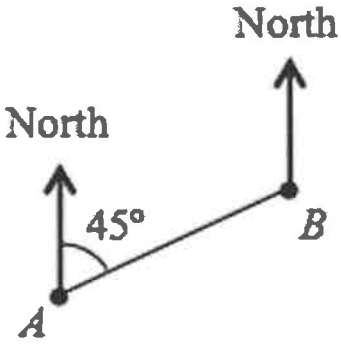
Example

Work out the bearing of B from A = 075°

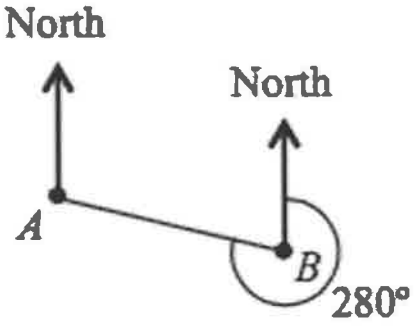


Questions

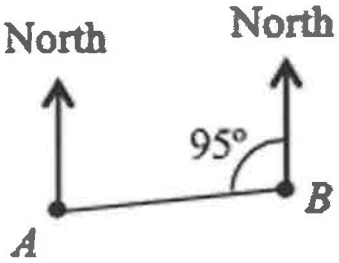
Find the bearing of *B* from *A*



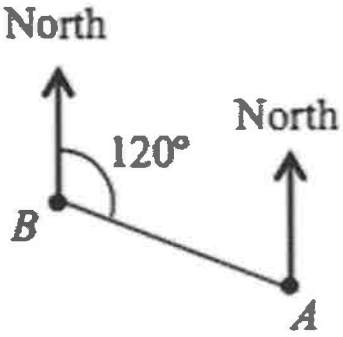
Find the bearing of *A* from *B*



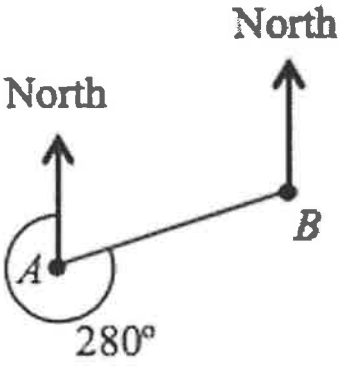
Find the bearing of *B* from *A*



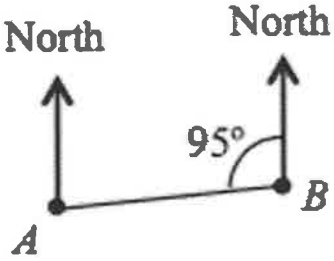
Find the bearing of *B* from *A*



Find the bearing of *A* from *B*



Find the bearing of *A* from *B*



A **scale** is the ratio that defines the relation between the actual figure and its model

Example

A map has a scale of 1cm : 3 miles.

a) On the map, the distance between two towns is 7cm. What is the actual distance between the two towns?

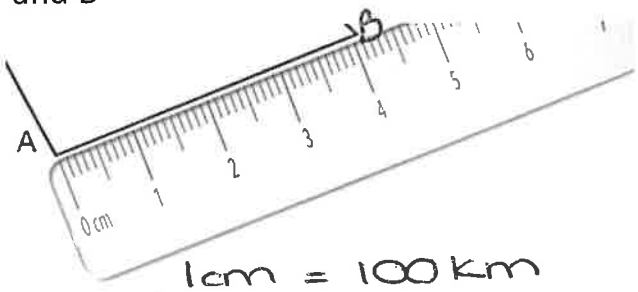
$\frac{7}{1} = 7$ ×7 (1cm : 3 miles) ×7
7cm : 21 miles

b) The actual distance between another two towns is 90 miles. What is the distance between the towns on the map?

$\frac{30}{1} = 30$ ×30 (1cm : 3 miles) ×30
30cm : 90 miles

Example

The diagram shows a scale drawing.
Scale: 1cm = 100km
Calculate the actual distance between A and B



1cm = 100 km
×4·1 ( ) ×4·1
4·1cm = 410 km

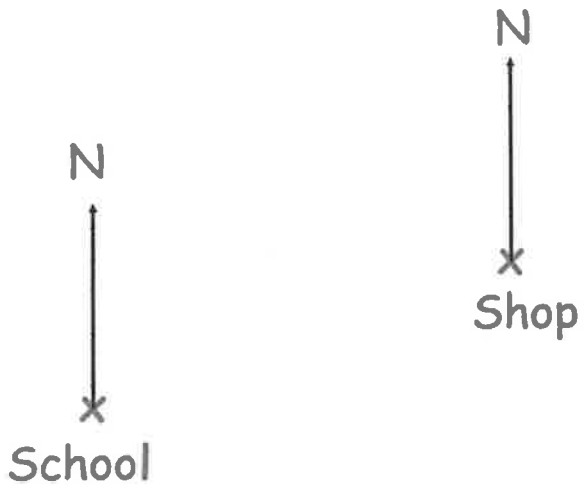
$\frac{4 \cdot 1}{1} = 4 \cdot 1$

Questions

A map has a scale of 1cm : 4 kilometres. The actual distance between two cities is 52 kilometres. What is the distance between the cities on the map?

A map has a scale of 1cm : 2.5 miles. On the map, the distance between two cities is 8cm. What is the actual distance between the two cities?

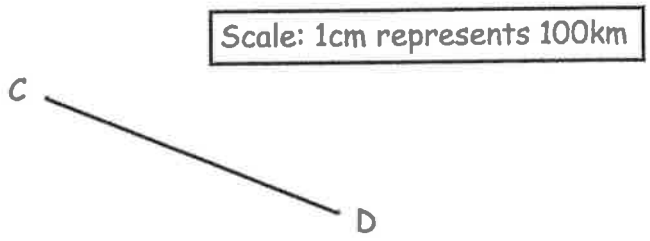
The diagram shows part of a map.
It shows the position of a school and a shop.



The scale of the map is 1cm = 100 metres.

Work out the real distance between the school and the shop.
Give your answer in metres.

The diagram shows a scale drawing.



(a) Use the diagram to calculate the actual distance from C to D.

Topic 6 - Indices

The rules below can only be used if the base is the same

$3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5
3 is the base
5 is the power
We read this as "3 to the power of 5"

Anything to the power of zero is 1

Examples

$3^0 = 1$ $\quad 5^0 = 1$

$a^0 = 1$ $\quad x^0 = 1$

Anything to the power of 1 is itself

Examples

$3^1 = 3$ $\quad 5^1 = 5$

$a^1 = a$ $\quad x^1 = x$

When we multiply we add the powers

Examples

- $3^2 \times 3^5 = 3^{2+5} = 3^7$
- $x^6 \times x^2 = x^{6+2} = x^8$
- $n^5 \times n^{-2} = n^{5+-2} = n^{5-2} = n^3$
- $a^3b^4 \times a^{-5}b^2 = a^{3+-5}b^{4+2} = a^{-2}b^6$

When there are coefficients, we multiply them first and then work out the power

- $4a^3 \times 6a^5 = 24a^{3+5} = 24a^8$

When we divide we subtract the powers

Examples

- $3^7 \div 3^3 = 3^{7-3} = 3^4$
- $\frac{x^3}{x^7} = x^{3-7} = x^{-4}$
- $\frac{n^4}{n^{-5}} = n^{4--5} = n^9$
- $\frac{x^5y^2}{x^2y^6} = x^{5-2}y^{2-6} = x^3y^{-4}$

When there are coefficients, we divide them first and then work out the power

- $\frac{30a^7}{5a^4} = 6a^{7-4} = 6a^3$

When we have brackets we multiply the powers

Examples

- $(3^2)^5 = 3^{2\times5} = 3^{10}$
- $(x^4)^3 = x^{4\times3} = x^{12}$
- $(n^{-3})^6 = n^{-3\times6} = n^{-18}$

When there are coefficients, we also need to put the coefficient to the power

- $(2x^4)^3 = 2x^4 \times 2x^4 \times 2x^4 = 8x^{12}$

(We can do this quickly by doing 2^3)

Sometimes one questions may contain several rules

Examples

$$\frac{a^3 \times a^4}{a^5} = \frac{a^7}{a^5} = a^2$$

$$\frac{(b^2)^4 \times b^3}{b^2 \times b^5} = \frac{b^8 \times b^3}{b^7} = \frac{b^{11}}{b^7} = b^4$$

$$\frac{6x^2y \times 5x^3y^2}{3xy^5} = \frac{30x^5y^3}{3xy^5} = 10x^4y^{-2}$$

Questions				
$7^2 \times 7^6$	$10^5 \div 10^3$	$(4^5)^4$	$a^4 \times a^{-9}$	$x^8 \div x^3$
$(b^3)^{-5}$	$2x^3 \times 8x^5$	$\frac{m^6}{m^{-2}}$	$(5a^6)^2$	$2x^4$
$m^3n^2 \times m^{-4}n^5$	$\frac{a^3b^5}{a^7b^2}$	$5x^4y^2 \times 8x^{-3}y^2$		
$\frac{24a^3b^5}{6ab^9}$	$\frac{x^6 \times x^{-2}}{x^4}$	$\frac{(p^4)^{-3} \times p^5}{p^2 \times p^3}$		
$\frac{4x^3y \times 9x^{-2}y^3}{2x^2y^6}$	$\frac{8a^5b^3 \times 5a^4b^9}{4ab^7}$			

Sometimes you are required to change the base.

"Write as a single power of 2" means that you want the base to be 2 ie 2

"Write as a single power of 3" means that you want the base to be 3 ie 3

Examples

- **Write 9 as a single power of 3**
  $9 = 3 \times 3 = 3^2$

- **Write 8^4 as a single power of 2**
  $8 = 2 \times 2 \times 2 = 2^3$
  We then replace 8 *with* 2^3
  $(2^3)^4 = 2^{12}$ (brackets = multiply)

- **Write 16 as a single power of 2**
  $16 = 2 \times 2 \times 2 \times 2 = 2^4$

- **Write 25^3 as a single power of 5**
  $25 = 5 \times 5 = 5^2$
  We then replace 25 *with* 5^2
  $(5^2)^3 = 5^6$ (brackets = multiply)

Questions	
Write 27 as a single power of 3	Write 4 as a single power of 2
Write 81 as a single power of 3	Write 125 as a single power of 5
Write 9^5 as a single power of 3	Write 16^3 as a single power of 2
Write 8^5 as a single power of 2	Write 32^4 as a single power of 2
Write 27^9 as a single power of 3	Write 25^7 as a single power of 5

Topic 7 – Expand and factorise single brackets

Expand means to multiply out the brackets. You multiply the term outside the brackets but each term inside the bracket. You should always check to see if you can simplify your answer.

Remember:

$- \times - = +$

$+ \times - = -$

Examples

- Expand $3(x-4) \equiv 3x - 12$
- Expand $-(x-5) \equiv -x + 5$ (invisible 1)
- Expand $-2(4x+3) \equiv -8x - 6$
- Expand $4a(5b+8c) \equiv 20ab + 32ac$
- Expand $x(3-2x) \equiv 3x - 2x^2$
- Expand $4x^2(3x-10) \equiv 12x^3 - 40x^2$
- Expand and simplify $5x + 2(3x-4) \equiv 5x + 6x - 8 = \mathbf{11x - 8}$
- Expand and simplify $2x(3x+5) - 7x \equiv 6x^2 + 10x - 7x \equiv \mathbf{6x^2 + 3x}$
- Expand and simplify $4(2x+5) - 2(3x-9) \equiv 8x + 20 - 6x + 18 \equiv \mathbf{2x + 38}$

Questions

A1 Expand: $7(m+3)$	**A2** Expand: $4(m-4)$	**B1** Expand: $2(3p+7)$	**B2** Expand: $3(p+2q)$
C1 Expand: $8(rs-7)$	**C2** Expand: $7(2rs+3)$	**D1** Expand and simplify: $3w+4(w+5)$	**D2** Expand and simplify: $8w+2(3w-7)$
E1 Expand and simplify: $4(x+1)+7$	**E2** Expand and simplify: $6(x+3)+2x$	**D3** Expand: $3x^2(6x-5)$	**D4** Expand: $4ax(3x+2a)$
E3 Expand: $3a^2(5a+4b-2ab)$	**E4** Expand: $5a^2b(2a-ab+3b)$	**B4** Expand: $-5(-3x+y-4)$	**C4** Expand: $-x(6-y)$
E2 Expand: $8x-4(7-3x)$	**E3** Expand: $9x-x(x+5)$	**D1** Expand and simplify $2(3b-1)+(b+9)$	**D2** Expand and simplify $5(k-3)-4(3k+2)$

Factorising is the opposite of expand. You put the highest common factor of all terms outside the bracket.

Examples

- Factorise $5x + 10 \equiv 5(x + 2)$
- Factorise $12x - 18 \equiv 6(2x - 3)$
- Factorise $24 - 9x \equiv 3(8 - 3x)$
- Factorise $4ab + 5ac \equiv a(4b + 5c)$
- Factorise $x^2 + 6x \equiv x(x + 6)$
- Factorise $6x^2 - 15x \equiv 3x(2x - 5)$
- Factorise $12x^2y + 8xy^2 \equiv 4xy(3x + 2y)$

Questions

$12x + 16$	$x^2 + 5x$	$4xy + 8y$	**A4** Factorise fully: $9x^3 - 36x^2$
$16x - 12$	$x^2 - x$	$10xy - 5x^2y$	**B4** Factorise fully: $9x^3 - 12x^4$
$16y - 12x$	$4a - 7a^2$	$x^2y + 4xy^2$	**C4** Factorise fully: $6w^2xy + 2wx^2y$
$10x + 5y - 30$	$3y + xy$	$9x^3y^2 - 6x^2y^5$	**D4** Factorise fully: $7x^2y^3 + x^4y$
	$5x^2 + xy - 3x$	$12x^5y - 6x^4y^2 + 9x^3y^3$	

Topic 8 – FDP and Operations with fractions

Fraction	Decimal	Percentage
$\frac{0.6}{1} \xrightarrow{\times 10} \frac{6}{10} \xrightarrow{\div 2} \frac{3}{5}$ 1. put over 1 2. multiply 3. simplify	0.6	$\xrightarrow{\times 100}$ $0.6 \times 100 = 60\%$
$\frac{2}{5}$ 1. write over 100	0.4	$\frac{2}{5} \xrightarrow{\times 20} \frac{40}{100} = 40\%$
$\frac{1.42}{1} \xrightarrow{\times 100} \frac{142}{100} \xrightarrow{\div 2} \frac{71}{50}$	$142 \div 100 = 1.42$ $\xleftarrow{\div 100}$	142%

Questions

1. Convert these decimals into percentages

(a) 0.25 (b) 0.75 (c) 0.13 (d) 0.88

(e) 0.075 (f) 0.021 (g) 0.1425 (h) 0.9682

(a) 1.63 (b) 1.25 (c) 1.8 (d) 1.01

2. Convert these percentages into decimal

(a) 53% (b) 19% (c) 25% (d) 74%

(e) 1.7% (f) 6.8% (g) 8.15% (h) 0.5%

(a) 162% (b) 190% (c) 115% (d) 144%

3. Convert these fractions into percentages

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{3}{20}$ (e) $\frac{3}{25}$ (f) $\frac{4}{5}$ (c) $\frac{5}{8}$

4. Convert these percentages into fractions

(i) 15% (j) 29% (k) 16% (l) 44%

(m) 99% (n) 85% (o) 52% (p) 80%

(a) 111%	(b) 130%	(c) 150%	(d) 110%
(a) 12.5%	(b) 0.2%	(c) 1.8%	(d) 15.2%

5. Convert these decimals into fractions

(e) 0.8	**(f) 0.2**	(g) 0.9	(h) 0.6
(i) 0.13	(j) 0.22	(k) 0.31	(l) 0.12
(a) 0.123	(b) 0.402	(c) 0.676	(d) 0.888
(e) 2.5	(f) 3.9	(g) 8.5	(h) 1.12

You may also be asked to order fractions, decimals and percentages. To do this you must convert them all into the same form, usually decimals or percentages.

Example

Write these numbers in order of size, starting with the smallest.

73% $\frac{2}{3}$ 0.9 $\frac{3}{4}$ 0.87

$73 \div 100 = 0.73$ $\frac{2}{3} = 0.\dot{6}$ $\frac{3}{4} = \frac{75}{100} = 0.75$

$\frac{2}{3}$, 73% , $\frac{3}{4}$, 0.87 , 0.9

Smallest Largest

Questions

(a) $\frac{1}{4}$ 0.19 0.3 26% $\frac{1}{5}$

(b) 0.9 $\frac{17}{20}$ $\frac{4}{5}$ 88% 0.79

(c) 11% 0.2 13% $\frac{3}{20}$ $\frac{1}{8}$

(d) $\frac{2}{3}$ 65% 0.68 $\frac{7}{10}$ $\frac{5}{8}$

(e) 101% $\frac{11}{10}$ 1.2 $\frac{19}{20}$ 0.9

(f) 1.5 $\frac{5}{3}$ 82% $\frac{7}{4}$ $\frac{37}{40}$

Topic 8 – Four Rules with Fractions

Addition and Subtraction

In order to add fractions then denominator must be the same. We can make it the same by using the lowest common multiple.

Example

- $\frac{4}{5} + \frac{3}{4}$ The LCM of 4 and 5 is 20:

$$\frac{4}{5} + \frac{3}{4} = \frac{16}{20} + \frac{15}{20} = \frac{31}{20} = 1\frac{11}{20}$$

- $\frac{4}{7} - \frac{2}{5}$ The LCM of 5 and 7 is 35:

$$\frac{4}{7} - \frac{2}{5} = \frac{20}{35} - \frac{14}{35} = \frac{6}{35}$$

- $3\frac{2}{5} + 2\frac{1}{6}$ First we must convert the mixed numbers into improper fractions. The LCM of 5 and 6 is 30:

$$3\frac{2}{5} + 2\frac{1}{6} = \frac{17}{5} + \frac{13}{6} = \frac{102}{30} - \frac{65}{30} = \frac{37}{30} = 1\frac{7}{30}$$

Questions

(i) $\frac{7}{15} - \frac{1}{5}$ (j) $\frac{3}{4} - \frac{2}{5}$ (k) $\frac{3}{10} + \frac{3}{8}$ (l) $\frac{2}{5} + \frac{4}{7}$

(e) $\frac{19}{20} + \frac{4}{5}$ (f) $\frac{5}{9} + \frac{13}{18}$ (g) $\frac{5}{12} + \frac{9}{10}$ (h) $\frac{4}{7} + \frac{7}{8}$

(e) $2\frac{1}{2} + 1\frac{1}{3}$ (f) $2\frac{2}{9} - 1\frac{1}{3}$ (g) $2\frac{2}{9} + \frac{5}{6}$ (h) $1\frac{5}{12} + 1\frac{5}{8}$

Multiply

To multiply fractions we multiply the numerators together and multiply denominators together. We should check to see if we can cross cancel first (diaginals only). You should also simplify your answer where possible.

Example

- We can cross cancel the 3 and 6 by dividing both by 3, we can also cross calcel the 4 and the 8 by dividing by 4:

$$\frac{3}{8} \times \frac{4}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

- We can cross cancel the 10 and 15 by dividing both by 5:

$$\frac{10}{12} \times \frac{7}{15} = \frac{2}{12} \times \frac{7}{3} = \frac{2 \times 7}{12 \times 3} = \frac{14}{36} = \frac{7}{18}$$

- Turn 3 into a fraction over 1

$$\frac{1}{8} \times 3 = \frac{1}{8} \times \frac{3}{1} = \frac{1 \times 3}{8 \times 1} = \frac{3}{8}$$

- Convert mixed numbers into improper fractions:

$$2\frac{3}{4} \times 1\frac{2}{7} = \frac{15}{4} \times \frac{9}{7} = \frac{15 \times 9}{4 \times 7} = \frac{135}{28} = 4\frac{23}{28}$$

Questions

(q) $\frac{6}{7} \times \frac{2}{9}$ (r) $\frac{3}{10} \times \frac{5}{6}$ (s) $\frac{6}{15} \times \frac{3}{4}$ (t) $\frac{3}{5} \times \frac{11}{15}$

(e) $8 \times \frac{3}{4}$ (f) $\frac{2}{3} \times 12$ (g) $5 \times \frac{1}{3}$ (h) $8 \times \frac{2}{5}$

(e) $\frac{1}{4} \times 3\frac{1}{3}$ (f) $1\frac{2}{3} \times 1\frac{1}{4}$ (g) $4\frac{3}{5} \times 1\frac{2}{3}$ (h) $1\frac{2}{11} \times \frac{8}{9}$

Divide

To divide fractions we use KFC; **K**eep the first fractions, **F**lip the second fraction, **C**hange the divide into a multiply. Then the multiply the fractions, checking to cross cancel first.

Example

- KFC:

$$\frac{1}{4} \div \frac{3}{5} = \frac{1}{4} \times \frac{5}{3} = \frac{1 \times 5}{4 \times 3} = \frac{5}{12}$$

- KFC. Cross cancel 2 and 8 by dividing both by 2:

$$\frac{2}{3} \div \frac{8}{10} = \frac{2 \div 2}{3} \times \frac{10}{8} = \frac{1}{3} \times \frac{10}{4} = \frac{1 \times 10}{3 \times 4} = \frac{10}{12} = \frac{5}{6}$$

÷2

- Write 4 over 1. KFC.

$$\frac{5}{6} \div 4 = \frac{5}{6} \div \frac{4}{1} = \frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \frac{5}{24}$$

- Convert into improper fractions. KFC.

$$2\frac{3}{4} \div 1\frac{1}{3} = \frac{11}{4} \div \frac{4}{3} = \frac{11}{4} \times \frac{3}{4} = \frac{11 \times 3}{4 \times 4} = \frac{33}{16} = 2\frac{1}{16}$$

Questions

(i) $\frac{3}{5} \div \frac{1}{2}$ (j) $\frac{7}{9} \div \frac{2}{3}$ (k) $\frac{8}{15} \div \frac{7}{10}$ (l) $\frac{9}{10} \div \frac{1}{3}$

(a) $\frac{3}{4} \div 2$ (b) $\frac{4}{7} \div 8$ (c) $\frac{11}{20} \div 3$ (d) $\frac{9}{40} \div 5$

(a) $\frac{2}{3} \div 1\frac{4}{5}$ (b) $1\frac{1}{2} \div 1\frac{9}{10}$ (c) $2\frac{3}{7} \div \frac{1}{2}$ (d) $2\frac{1}{3} \div 5\frac{1}{2}$

Example

Find $\frac{2}{5}$ of 35

$\frac{2}{5} \times \frac{35}{1}$ (÷5 above 35, ÷5 below 5) $= \frac{2}{1} \times \frac{7}{1}$

$= \frac{14}{1}$

$= 14$ ✓

Example

$\frac{3}{7}$ *of a number is* 9. What is the number?

$\square \times \frac{3}{7} = 9$

÷ $\frac{3}{7}$ ÷ $\frac{3}{7}$

$\square = \frac{9}{1} \div \frac{3}{7} = \frac{9}{1} \times \frac{7}{3}$ (÷3 above 9, ÷3 below 3)

$= \frac{3}{1} \times \frac{7}{1} = 21$ ✓

Questions

(a) $\frac{1}{2}$ of 10 (b) $\frac{1}{3}$ of 18 (c) $\frac{1}{5}$ of 20 (d) $\frac{1}{4}$ of 24

(a) $\frac{2}{3}$ of 15 (b) $\frac{7}{10}$ of 20 (c) $\frac{2}{5}$ of 30 (d) $\frac{3}{4}$ of 32

(e) $\frac{3}{5}$ of 45 (f) $\frac{2}{7}$ of 28 (g) $\frac{3}{8}$ of 88 (h) $\frac{3}{10}$ of 120

(a) $\frac{1}{2}$ of a number is 7, what is the number? (b) $\frac{1}{3}$ of a number is 4, what is the number?

(c) $\frac{1}{4}$ of a number is 8, what is the number? (d) $\frac{1}{5}$ of a number is 9, what is the number?

(a) $\frac{2}{3}$ of a number is 12, what is the number? (b) $\frac{2}{5}$ of a number is 10, what is the number?

(c) $\frac{2}{7}$ of a number is 6, what is the number? (d) $\frac{3}{10}$ of a number is 60, what is the number?

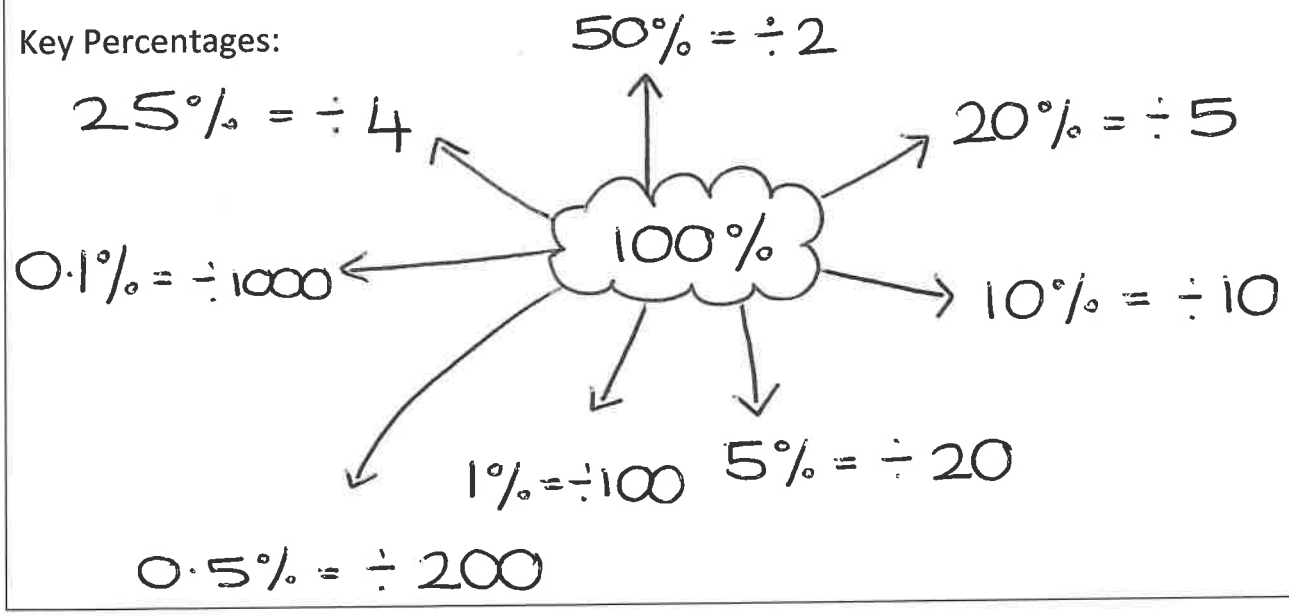
(e) $\frac{4}{9}$ of a number is 12, what is the number? (f) $\frac{2}{3}$ of a number is 3, what is the number?

Topic 9- Percentages

NON-CALCULATOR

To calculate a percentage without a calculator we use the build-up method. This is where we use key percentages and add them together.

Key Percentages:



Questions

(a) 10% of 70m	(b) 25% of 16 seconds	(c) 10% of 400kg	(d) 50% of 26g
(e) 75% of 40ml	(f) 1% of £300	(g) 25% of 36 days	(h) 50% of 9 days
(i) 75% of 24p	(j) 25% of £18	(k) 1% of $6300	(l) 10% of £7
(m) 1% of 60m	(n) 75% of 8 miles	(o) 1% of 80kg	(p) 50% of 1.6km

Example

Find 43% of £420.

1. Find key percentages

100% = £420

÷10 ↓ ↓ ÷10

10% = £42

÷10 ↓ ↓ ÷10

1% = £4.20

2. Build up

10 % = £42

×4 ↓ ↓ ×4

40% = £168

1 % = £4.20

×3 ↓ ↓ ×3

3% = £12.60

3. Add percentages together

40 % = £168.00

+ 3% = £12.60

43% = £180.60

Example

Find 17.5% of £160.

1. Find key percentages

100% = £160

÷10 ↓ ↓ ÷10

10% = £16

÷10 ↓ ↓ ÷10

1% = £1.60

÷2 ↓ ↓ ÷2

0.5% = £0.80

2. Build up

1 % = £1.60

×7 ↓ ↓ ×7

7% = £11.20

3. Add percentages together

10 % = £160.00

7% = £11.20

+ 0.5% = £0.80

17.5% = £172.00

Questions

(a) 35% of £800	(b) 6% of 160g	(c) 23% of 330cm	(d) 52% of 910m
(e) 61% of 1400	(f) 7% of 640GB	(g) 45% of 350g	(h) 80% of 450 people
(i) 90% of 1250ml	(j) 76% of £80,000	(k) 85% of 90 hours	(l) 12% of £6

Percentage Increase

Percentage increase means that we **add** the percent onto the original amount

Example

Increase £280 by 22%.

1. Find key percentages

100% = £280

÷10 ↓ ÷10

10% = £28

÷10 ↓ ÷10

1% = £2.80

2. Build up together

10% = £28 | 1% = £2.80

×2 ↓ ×2

20% = £56 | **2% = £5.60**

3. Add percentages together

20 % = £56.00

\+ 2% = £5.60

22% = £61.60

4. **Add** onto the original amount

100 % = £280.00

\+ 22% = £61.60

122% = £341.60

Percentage Decrease

Percentage decrease means that we **subtract** the percent from the original amount

Example

Decrease £130 by 31%.

1. Find key percentages

100% = £130

÷10 ↓ ÷10

10% = £13

÷10 ↓ ÷10

1% = £1.30

2. Build up together

10% = £13 | **1% = £1.30**

×3 ↓ ×3

30% = £39

3. Add percentages together

30 % = £39.00

\+ 1% = £1.30

33% = £40.30

4. **Subtract** from the original amount

100 % = £280.00

\- 33% = £40.30

67% = £239.70

Questions

(a) Increase 20 by 50%

(b) Increase 60p by 10%

(c) Increase 12g by 25%

(d) Increase 400 litres by 20%

(e) Increase 32ml by 75%

(f) Increase 70m by 40%

(a) Decrease 40 by 10%

(b) Decrease 30 hours by 50%

(c) Decrease 8kg by 25%

(d) Decrease 55cm by 40%

(e) Decrease 64 by 75%

(f) Decrease £3 by 10%

CALCULATOR

To calculate a percentage with a calculator we use the multiplier method. To find the multiplier you divide the percentage by 100.

Examples – Find the multiplier

- $45\% = \frac{45}{100} = 0.45$
- $63\% = \frac{63}{100} = 0.63$
- $192\% = \frac{192}{100} = 1.92$

Questions

(a) 50%	(b) 80%	(c) 10%	(d) 25%
(e) 45%	(f) 95%	(g) 5%	(h) 3%
(i) 7%	(j) 36%	(k) 71%	(l) 44%
(m) 0%	(n) 175%	(o) 104%	(p) 160%

Example

Find 12% of 132kg.

Amount: 132

Rate: 12%

Multiplier: $\frac{12}{100} = 0.12$

Sum: $132 \times 0.12 = 15.84$kg

Find 154% of 14

Amount: 14

Rate: 154%

Multiplier: $\frac{154}{100} = 1.54$

Sum: $14 \times 1.54 = 21.56$

Find 9.7% of 40m.

Amount: 40

Rate =9.7%

Multiplier: $\frac{9.7}{100} = 0.097$

Sum: $40 \times 0.097 = 3.88$m

Questions

(a) 15% of 80ml (b) 9% of 205kg (c) 45% of £135 (d) 17% of 540km

(e) 53% of 700g (f) 14% of 12 hours (g) 31% of 280kg (h) 6% of 4GB

(a) 2.5% of 60cm (b) 7.2% of 104ml (c) 24.5% of 30m (d) 47.9% of £3200

(a) 109% of 30m (b) 124% of 38 seconds (c) 186% of £40 (d) 196% of 20 miles

Percentage increase

Percentage increase means that we **add** the percent onto 100%

Example

Increase £425 by 78%

Amount: £425

Rate: 100% + 78% = 178%

Multiplier: $\frac{178}{100} = 1.78$

Sum: $425 \times 1.78 = £756.50$

Percentage decrease

Percentage decrease means that we **subtract** the percent from 100%

Example

Decrease £390 by 27%

Amount: £390

Rate: 100% - 27% = 73%

Multiplier: $\frac{73}{100} = 0.73$

Sum: $390 \times 0.73 = £284.70$

Questions

(a) Increase 80ml by 9% (b) Increase 420g by 70% (c) Decrease 8 by 12%

(d) Decrease £1250 by 38% (e) Increase 6000km by 23% (f) Decrease 48GB by 6%

(g) Increase 204 by 98% (h) Decrease 149mm by 91% (i) Increase 88 by 185%