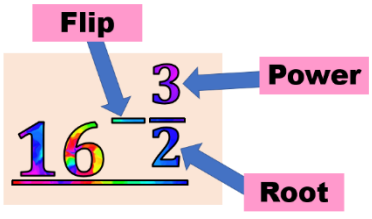
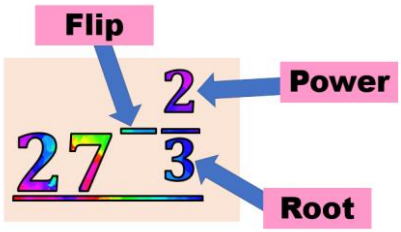
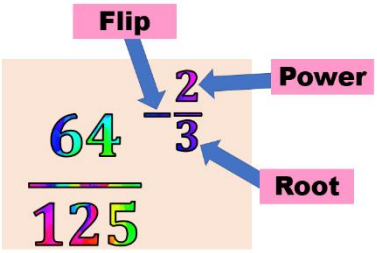
Maths Revision Pack

Year 10 Higher

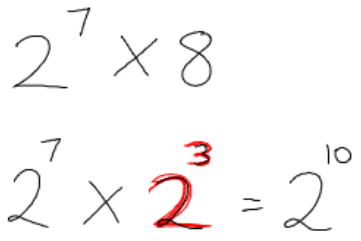
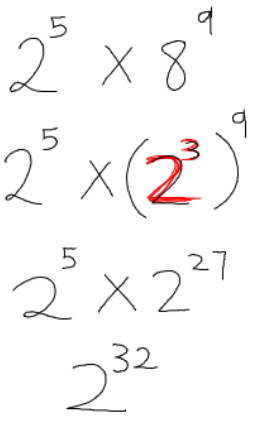
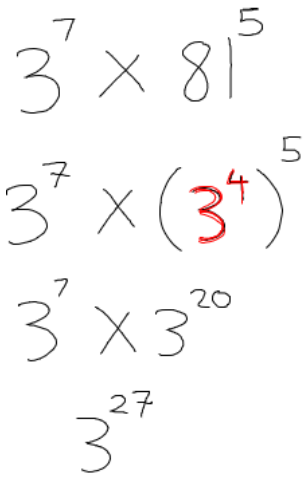
Assessment Week 1

Name: _____

Example 1	Example 2	Example 3
		
Step 1 : $\sqrt{16} = 4$ Step 2 : $4^3 = 64$ Step 3 : $\frac{1}{64}$	Step 1 : $\sqrt[3]{27} = 3$ Step 2 : $3^2 = 9$ Step 3 : $\frac{1}{9}$	Step 1 : $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$ Step 2 : $(\frac{3}{5})^2 = \frac{9}{25}$ Step 3 : $\frac{25}{9} = 2\frac{7}{9}$

Questions

2^3	3^{-3}	$25^{\frac{1}{2}}$
$25^{-\frac{1}{2}}$	$36^{\frac{1}{2}}$	$27^{\frac{1}{3}}$
$27^{-\frac{1}{3}}$	$64^{\frac{3}{2}}$	$64^{\frac{2}{3}}$
$125^{-\frac{2}{3}}$	$(\frac{16}{81})^{\frac{1}{2}}$	$(\frac{8}{27})^{-\frac{1}{3}}$
$(\frac{49}{121})^{\frac{3}{2}}$	$(\frac{9}{144})^{-\frac{3}{2}}$	$(\frac{1}{125})^{-\frac{2}{3}}$

Example 1	**Example 2**	**Example 3**
Write down $2^7 X\ 8$ *as a single power of* 2. 	Write down $2^5 X\ 8^9$ *as a single power of* 2. 	Write down $3^7 X\ 81^5$ *as a single power of* 3. 

Questions

Write these as single a single power

$2^4 X\ 16$	$2^9 X\ 32$	$3^5 X\ 27$
$2^5 X\ 8^7$	$2^5 X\ 16^3$	$3^2 X\ 9^5$
$2^4 X\ 8^3$	$4^5 X\ 16^9$	$4^2 X\ 64^6$
$5^2 X\ 25^2$	$5^6 X\ 125^4$	$16^3 X\ 64^6$
$25^2 X\ 125^2$	$9^5 X\ 27^2$	$27^2 X\ 81^6$

Example 1	Example 2
$(2x^5y^7)^3$ $2x^5y^7 \times 2x^5y^7 \times 2x^5y^7$ $8x^{15}y^{21}$	$(5a^4b^8)^3$ $5a^4b^8 \times 5a^4b^8 \times 5a^4b^8$

Questions

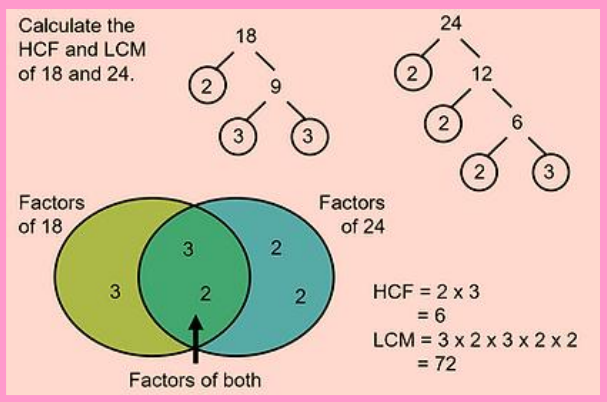
$(6x^4y^7)^2$	$(5x^8y^3)^2$
$(3a^3b^5)^2$	$(7x^3y^9)^2$
$(3x^4y^6)^3$	$(4a^5b^2)^3$

Example 1	Example 2
$9^0 = 1$ $7^2 = 49$ $9^0 \times 7^2$ 1 x 49 = 49	$b^0 = 1$ $8^1 = 8$ $b^0 + 8^1$ 1 + 8 = 9

Questions

$8^1 + 7^0$	$3^2 \times a^0$
$13^0 + 6^2$	$15^1 - 35^0$

Example

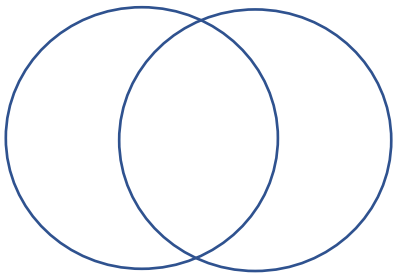


$18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

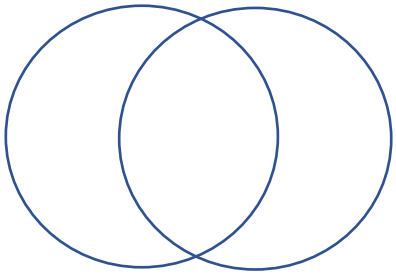
Questions

Find the HCF and LCM of 54 and 36.



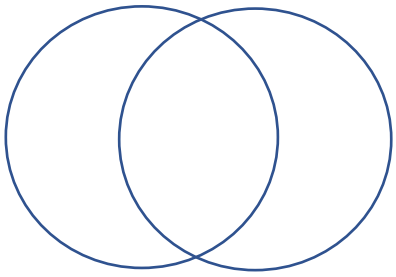
HCF =
LCM =

Find the HCF and LCM of 120 and 220.

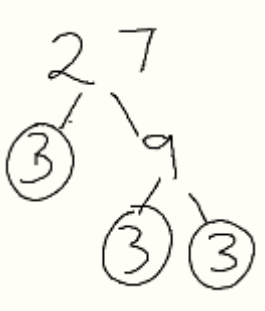


HCF =
LCM =

Find the HCF and LCM of 240 and 180.



HCF =
LCM =

Example 1	**Example 2**	**Example 3**
$N=3^4 \times 5^3 \times 7^5$ What is 3N? $3 \times 3^4 \times 5^3 \times 7^5$ $= 3^5 \times 5^3 \times 7^5$	$N=2^3 \times 3^3 \times 5^2$ What is $3N^2$? $3 \times 2^3 \times 3^3 \times 5^2 \times 2^3 \times 3^3 \times 5^2$ $2^6 \times 3^7 \times 5^4$	$N=2^3 \times 3^8 \times 5^2$ What is $27N^2$?  $27 = 3^3$ $3^3 \times 2^3 \times 3^8 \times 5^2 \times 2^3 \times 3^8 \times 5^2$ $2^6 \times 3^{19} \times 5^4$
Questions		
$N=2^4 \times 5^3 \times 7^5$ What is 5N?	$N=2^3 \times 3^3 \times 5^2$ What is $2N^2$?	$N=2^3 \times 3^8 \times 5^2$ What is $24N^2$?
$N=3^4 \times 5^3 \times 7^5$ What is 2N?	$N=2^9 \times 3^3 \times 7^2$ What is $3N^2$?	$N=2^3 \times 3^7 \times 5^2$ What is $18N^2$?
$N=2^7 \times 3^8 \times 5^3 \times 7^5$ What is 7N?	$N=2^5 \times 3^8 \times 5^3 \times 7^5$ What is $5N^2$?	$N=2^4 \times 3^8 \times 5^2$ What is $125N^2$?

Example 1

SOLVE FOR X:

$15 + 6x = 45 + 8x$

$15 + 6x = 45 + 8x$

$-6x \quad -6x$

$15 = 45 + 2x$

$-45 \quad -45$

$\frac{-30}{2} = \frac{2x}{2}$

$-15 = x$

Check your answer:

$15 + 6(-15) \stackrel{?}{=} 45 + 8(-15)$

$15 + (-90) \stackrel{?}{=} 45 + (-120)$

$-75 = -75$ ✓

Example 2

$\frac{2x}{5} = 3$ ← Add a one underneath to make it a fraction

$\frac{2x}{5} = \frac{3}{1}$ ← Fraction=fraction Use scissors

$2x(1) = 3(5)$

$\frac{2x}{2} = \frac{15}{2}$

$x = 15/2 = 7\frac{1}{2}$

Example 3

$\frac{3w+4}{5} = \frac{2}{7}$ ← Fraction=fraction Use scissors

$7(3w+4) = 5(2)$

$21w + 28 = 10$

$(-28) \quad (-28)$

$\frac{21w}{21} = \frac{-18}{21}$

$w = -18/21$

Example 4

Fractions Both Sides Equation

Solve the Equation : $\frac{8h + 7}{2} = 3h - 4$

1a. Cross Multiply : $8h + 7$ (2) = $3h - 4$ (1)

1b. Use Brackets : $1\,(8h + 7) = 2\,(3h - 4)$

1c. Expand Out : $8h + 7 = 6h - 8$

Now solve as letter both sides

Questions

Solve these equations

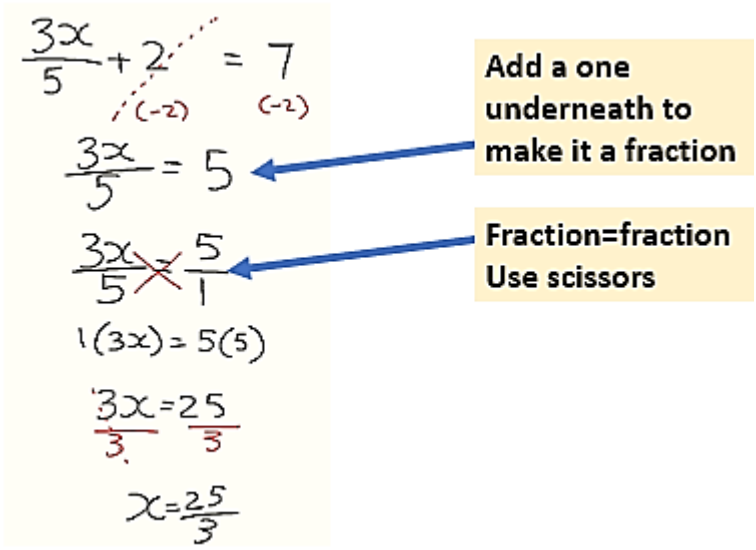
$\frac{x}{4} = 8$

$\frac{2a}{3} = 8$

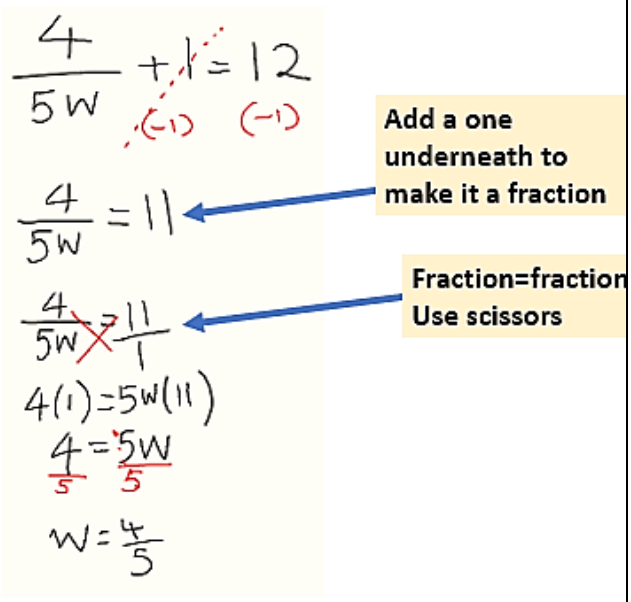
$\frac{10x - 1}{7} = 8$

$\frac{7c + 4}{8} = 11$	$\frac{53 - 2x}{5} = 7$	$\frac{11 - w}{5} = 3 + w$
$\frac{9(4x - 1)}{2x} = 15$	$\frac{x}{2x - 1} = 5$	$\frac{2(3x-6)}{5}=\frac{3x}{2}$
$\frac{x+15}{3} = 2(x + 10)$	$\frac{5x}{5}=3x - 8$	$2(5 - 6x)=\frac{3x}{2}$

Example 1



Example 2



Solve these equations

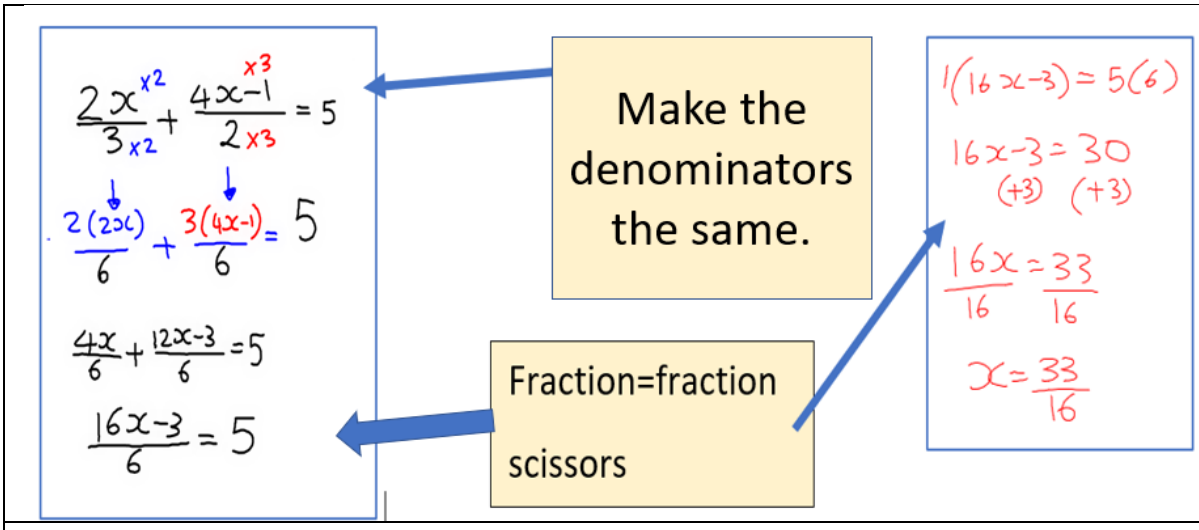
$\frac{x}{2} + 1 = 3$

$\frac{2r}{5} - 3 = 2$

$\frac{3}{c} + 8 = 2$

$\frac{4}{p} + 3 = 5$

Example 1



Solve these equations

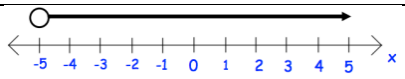
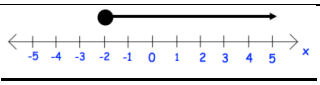
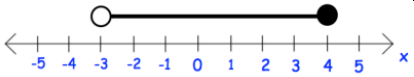
$\frac{x+5}{3} + \frac{x+1}{2} = 8$

$\frac{2x-1}{4} + \frac{x-3}{5} = 3$

$\frac{x+5}{10} - \frac{x+4}{2} = 1$

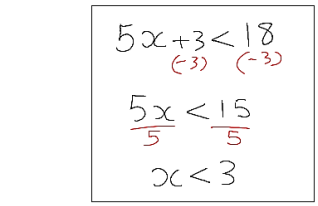
$\frac{x-3}{4} - \frac{x-8}{3} = 4$

Examples		

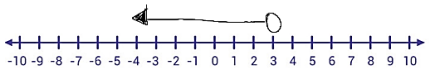
● Includes the number ○ Does not Include the	 x *is more than* -5	 x *is more than or equal to* -2
< or > ○ **≤ or ≥** ●	 x *is more than* -3 *but less than or equal to* 4	 x *is less than* -3

Solve:

$$5x + 3 < 15$$



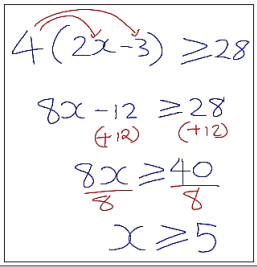
x *is less than* 3



2,1,0,-1,-2……

Solve:

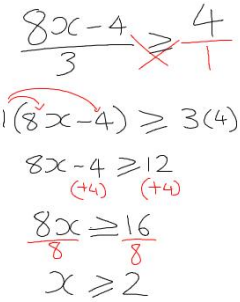
$$4(2x - 3) \geq 28$$

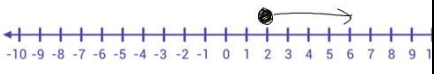


x *is more than or equal to than* 3

5,6,7,8,9,10,11……

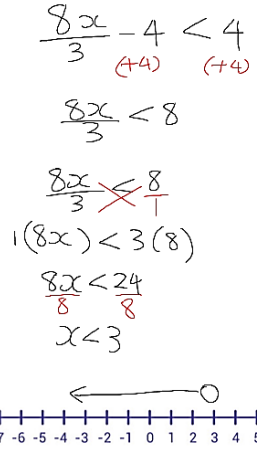
$$\frac{8x - 4}{3} \geq 4$$





2,3,4,5,6,7,8……

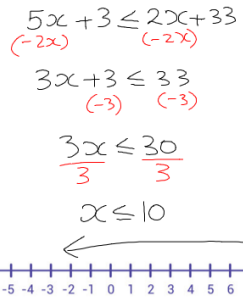
$$\frac{8x}{3} - 4 < 4$$

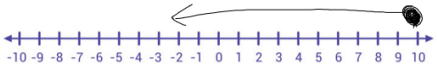


2,1,0,-1,-2……

Solve:

$$5x + 3 \leq 2x + 33$$

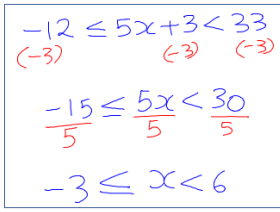




10,9,8,7,6,5,4……

Solve:

$$-12 \leq 5x + 3 < 33$$



x *is more than or equal to* -3 *but lesst than* 6

-3,-2,-1,0,1,2,3,4,5

Questions

Solve these inequalities, draw a number line and write down the integers that satisfy the inequalities.

$2x + 1 \leq 9$	$3x - 5 > 16$	$4(2x + 1) > 9$
$5x - 2 \geq 68$	$4x \leq x + 6$	$\frac{x}{9} - 6 > 4$
$\frac{x}{2} + 1 \leq 5$	$\frac{x+3}{2} \geq 5$	$\frac{x-5}{4} > 2$

$\frac{x+18}{4} \leq 5$	$4x + 6 \leq x + 21$
$9x + 4 > 7x + 15$	$4x - 4 \leq 7x - 19$
$5 < 2y < 12$	$4 \leq 2x \leq 8$
$-3 \leq x + 2 < 2$	$1 \leq 2y - 3 < 9$
$16 \leq 5x + 1 < 31$	$7 < 2y - 3 \leq 25$

Example1

$y = 2x + 1$

x	-1	0	1	2	3
y	-1	1	3	5	7

$2(3) + 1 = \mathbf{7}$

$2(2) + 1 = \mathbf{5}$

$2(1) + 1 = \mathbf{3}$

$2(0) + 1 = \mathbf{1}$

$2(-1) + 1 = \mathbf{-1}$

Example2

$x + 2y = -2$

x	-1	0	1	2	3
y	-0.5	-1	-1.5	-2	-2.5

$-1-\frac{1}{2}(3) = \mathbf{-2.5}$

$-1-\frac{1}{2}(2) = \mathbf{-2}$

$-1-\frac{1}{2}(1) = \mathbf{-1.5}$

$-1-\frac{1}{2}(0) = \mathbf{-1}$

$-1-\frac{1}{2}(-1) = \mathbf{-0.5}$

Re-arrange to make y the subject

$x + 2y = -2$

$(-x) \quad (-x)$

$\frac{2y}{2} = \frac{-2-x}{2}$

$y = -1 - \frac{1}{2}x$

Questions

$y = 3x - 1$

x	-1	0	1	2	3
y	-4			5	

$y = 4x - 2$

x	-2	-1	0	1	2	3
y						

$y = \frac{1}{2}x + 1$

x	-2	-1	0	1	2	3	4
y							

$2x + y = 4$

x	-1	0	1	2	3
y					

Example1

Gradients from two points

Find the gradient of the line passing through (2,5) and (6,13)

TSD
- Two coordinates
- Subtract
- Divide

Find the gradient of the line passing through (2,5) and (6,13)

6, 13
− 2, 5
4, 8

$\frac{y}{x} = \frac{8}{4} = 2$

Gradient = 2

Example2

Gradients from two points

Find the gradient of the line passing through (-2,9) and (-4,7)

TSD
- Two coordinates
- Subtract
- Divide

Find the gradient of the line passing through (-2,9) and (-4,7)

$-2 - -4 = 2$

−2, 9
−4, 7
2, 2

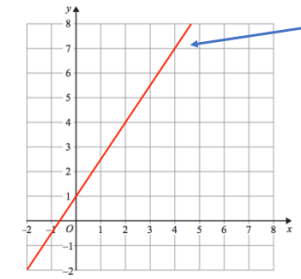
$\frac{y}{x} = \frac{2}{2} = 1$

Gradient = 1

Questions

(1, 4) and (3, 10)	(0, 0) and (3, 12)
(5, −2) and (9, 14)	(−5, −9) and (1, 3)
(−7, −2) and (1, −4)	(−2, 1) and (8, −7)

Example

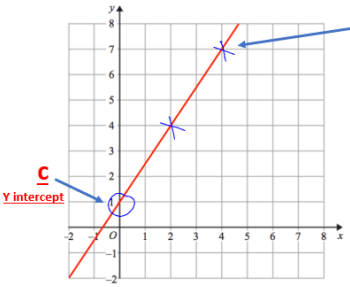


Straight line

Y = mx + c

Gradient

TSD: Two coordinates, Subtract, Divide



TSD: Two coordinates, Subtract, Divide

4,7 − 2,4 = 2,3

$\frac{y}{x} = \frac{3}{2}$

c

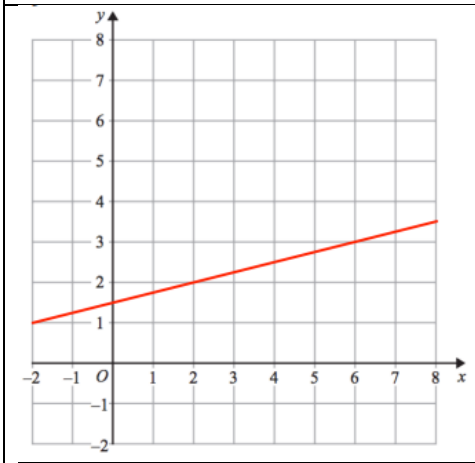
Y intercept

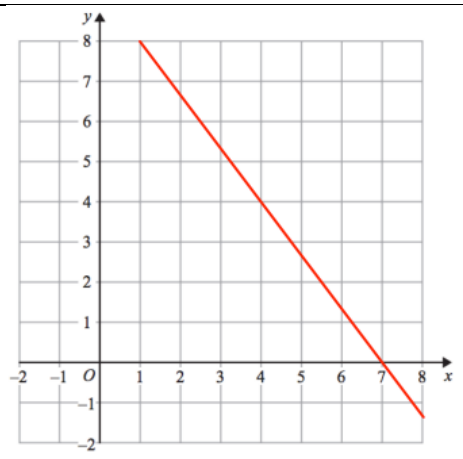
Gradient (m) = $\frac{3}{2}$

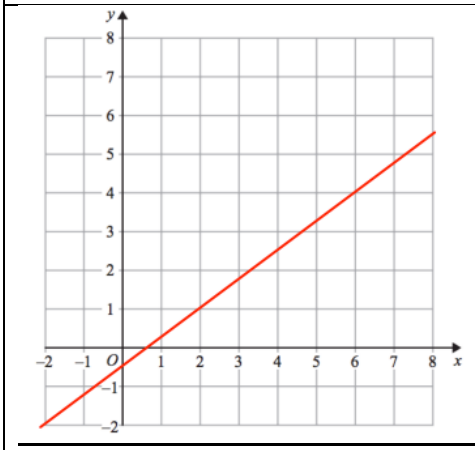
Y-intercept (c) = 1

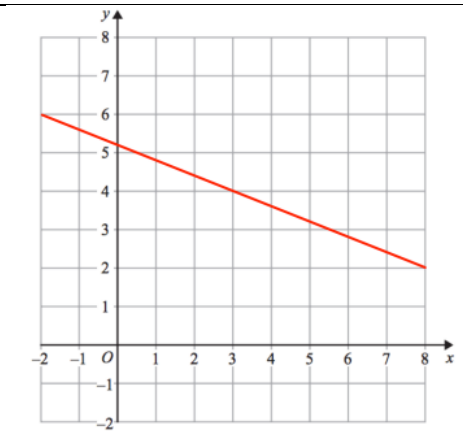
Equation: $y = \frac{3}{2}x + 1$

Questions: find the equations



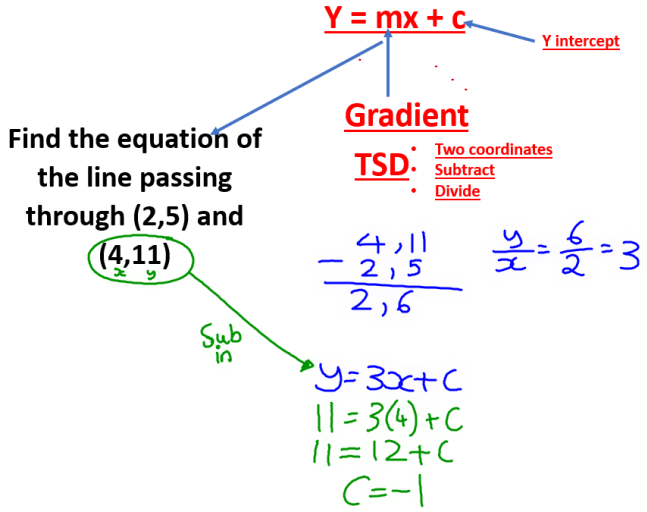






Example1

Find the equation of the line passing through (2,5) and (4,11)



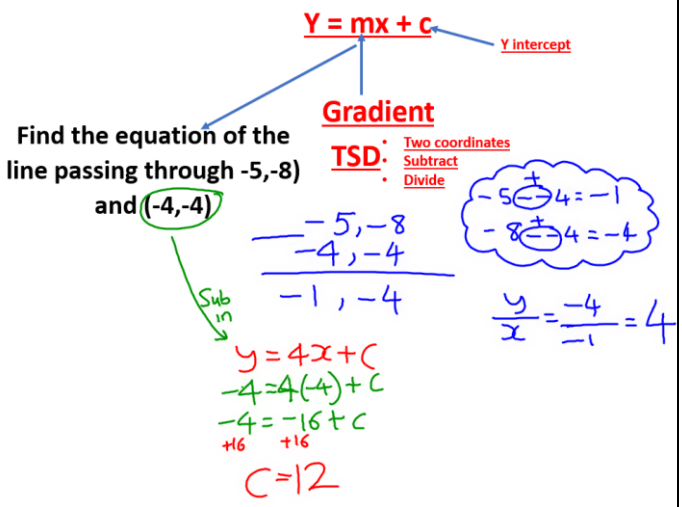
Gradient (m) = 3

Y-intercept (c) = -1

Equation: y = 3x - 1

Example2

Find the equation of the line passing through (-5,-8) and (-4,-4)



Gradient (m) = 4

Y-intercept (c) = 12

Equation: y = 4x + 12

Questions: find the equations

(1, 6) and (5, 4)	(3, 5) and (4, 1)
(−4, 2) and (1, 7)	(−5, 4) and (5, 2)
(−6, −4) and (−3, 2)	(−10, −5) and (−7, 4)

Examples

The equation of line L_1 is $y = 5x + 1$

The equation of line L_2 is $2y - 10x + 3 = 0$

Show that these two lines are parallel.

Parallel → Same gradient (m)

Re-arrange $y = mx + c$

L_1 $y = 5x + 1$ $m = 5$

L_2 $2y - 10x + 3 = 0$ (+10x) (+10x)

$2y + 3 = 10x$ (−3) (−3)

$\frac{2y}{2} = \frac{10x - 3}{2}$

$y = 5x - \frac{3}{2}$ $m = 5$

Same Gradient

Parallel to $y = 6x + 3$ and passing through $(10, 5)$

Parallel m is also 6. $m = 6$

Same gradient

$y = 6x + c$

$5 = 6(10) + c$

$5 = 60 + c$

(−60) (−60)

$-55 = c$

$y = 6x - 55$

Find the equation of the straight line passing through the point (0, 1) which is perpendicular to the line

$y = -2x + 2$

Perpendicular → Change sign & Flip

gradient

$y = -2x + 2$ $m = -2$

* Perpendicular gradient $= +\frac{1}{2}$

$(0, 1)$ x y

$y = \frac{1}{2}x + c$ ← Sub in

$1 = \frac{1}{2}(0) + c$

$1 = 0 + c$

$c = 1$

$y = \frac{1}{2}x + 1$

Questions

Find the equation of the line that is **parallel** to $y = \frac{1}{2}x - 3$ and passes through the point (6,-2).

Parallel to $y = 5x - 4$ and passing through $(2, 9)$

Parallel to $y = 4x$ and passing through $(-1, 3)$

Perpendicular to $y = -3x + 4$ and passing through $(6, 1)$

Examples

a:b = 2:5
What fraction is b?

a : b Total
2 : 5 7

$b = \frac{5}{7}$

$\frac{5}{9}$ of the apples are green. The rest are red.
Write down the ratio of red : green

Write down the ratio of green : red

$\frac{5}{9}$ = Green (5: Green; 9: Total)

R : G Total
4 : 5 9

G : R Total
5 : 4 9

Amy and Ben share some money. Amy gets four times as much as Ben.
Write down the ratio of Amy : Ben

Amy : Ben
4 : 1
×4

Write 4:7 in the form of n:1

4 : 7
÷7 ↓ ↓ ÷7
4/7 : 1

Write 4:7 in the form of 1:n

4 : 7
÷4 ↓ ↓ ÷4
1 : 7/4

a:b = 2:5
a + b = 21. what is a – b?

a : b Total
2 : 5 7
×3 ×3 ×3
6 : 15 21

a + b = 21 (Total)

21 ÷ 7 = 3

a:b = 2:5
b – a = 21, what is b + a?

a : b Total
2 : 5 7
×7 (3) ×7 ×7
14 21 35 49

b − a = 21 (difference in parts)

Amy and Ben share some money in the ratio 4:9.
Amy gets £35 less than Ben. How much did they share?

A : B Total
4 : 9 13
×7 (5) ×7 ×7
28 35 63 91

£35 less than means B − A (difference in parts)

$a : b = 7 : 4$

$b : c = 2 : 5$

Work out $a : c$

Give your answer in its simplest form.

a : b b : c
7 : 4 2 : 5
×1 ×1 ×2 ×2
7 4 4 10

LCM of 4 & 2 is 4

a : c
7 : 10

a : b is 4:9. Write down an equation

3 : 9 simplifies to 1 : 3

3 × 3 = 1 × 9
9 = 9

a : b
4 : 9

9a = 4b

8x = 3y Write down the ratio of x:y

8x = 3y
x : y
3 : 8

Jamie and Alastair share some sweets in the ratio 7 : 5. Jamie gives 2 sweets to Alastair. The ratio of sweets is now 13 : 11.
How many sweets did each initially have?

J : A
before 7x : 5x
Now 7x-2 : 5x+2
Now 13 : 11

$11(7x-2)=13(5x+2)$
$77x-22=65x+26$
$+22 \quad +22$
$77x=65x+48$
$-65x \quad -65x$
$\frac{12x}{12}=\frac{48}{12}$
$x=4$

The ratio of green to yellow beads is 4 : 5, and there are initially 27 beads. I add some yellow beads and the ratio is now 1 : 3. How many yellow beads did I add?

G : Y Total
4 : 5 9
x3 x3 x3
before 12 : 15 27
Now 12 : 15+x
Now 1 : 3

$\frac{27}{9}=3$

$3(12)=1(15+x)$
$36=15+x$
$-15 \quad -15$
$21=x$

Questions

a:b = 3:5
What fraction is b?

$\frac{8}{9}$ of the apples are green. The rest are red.
Write down the ratio of red : green

Write down the ratio of green : red

Amy and Ben share some money. Ben gets five times as much as Amy.
Write down the ratio of Amy : Ben

Write 3:8 in the form of n:1

Write 3:8 in the form of 1:n

a:b = 3:5
a + b = 32. what is a – b?

a:b = 3:5
b – a = 32, what is b + a?

Amy and Ben share some money in the ratio 5:8.
Amy gets £33 less than Ben. How much did they share?

a, *b* and *c* are positive integers.
$a : b = 3 : 8$ and $b : c = 6 : 11$
Work out the **smallest** possible value of $a + b + c$

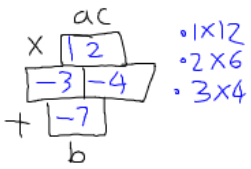
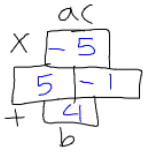
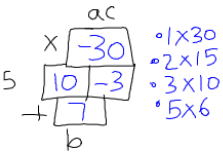
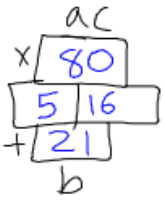
a : b is 4:9. Write down an equation

8x = 3y Write down the ratio of x:y

The ratio of the number of boys to girls at a party is 3 : 4
Six boys leave the party.
The ratio of the number of boys to girls at the party is now 5 : 8

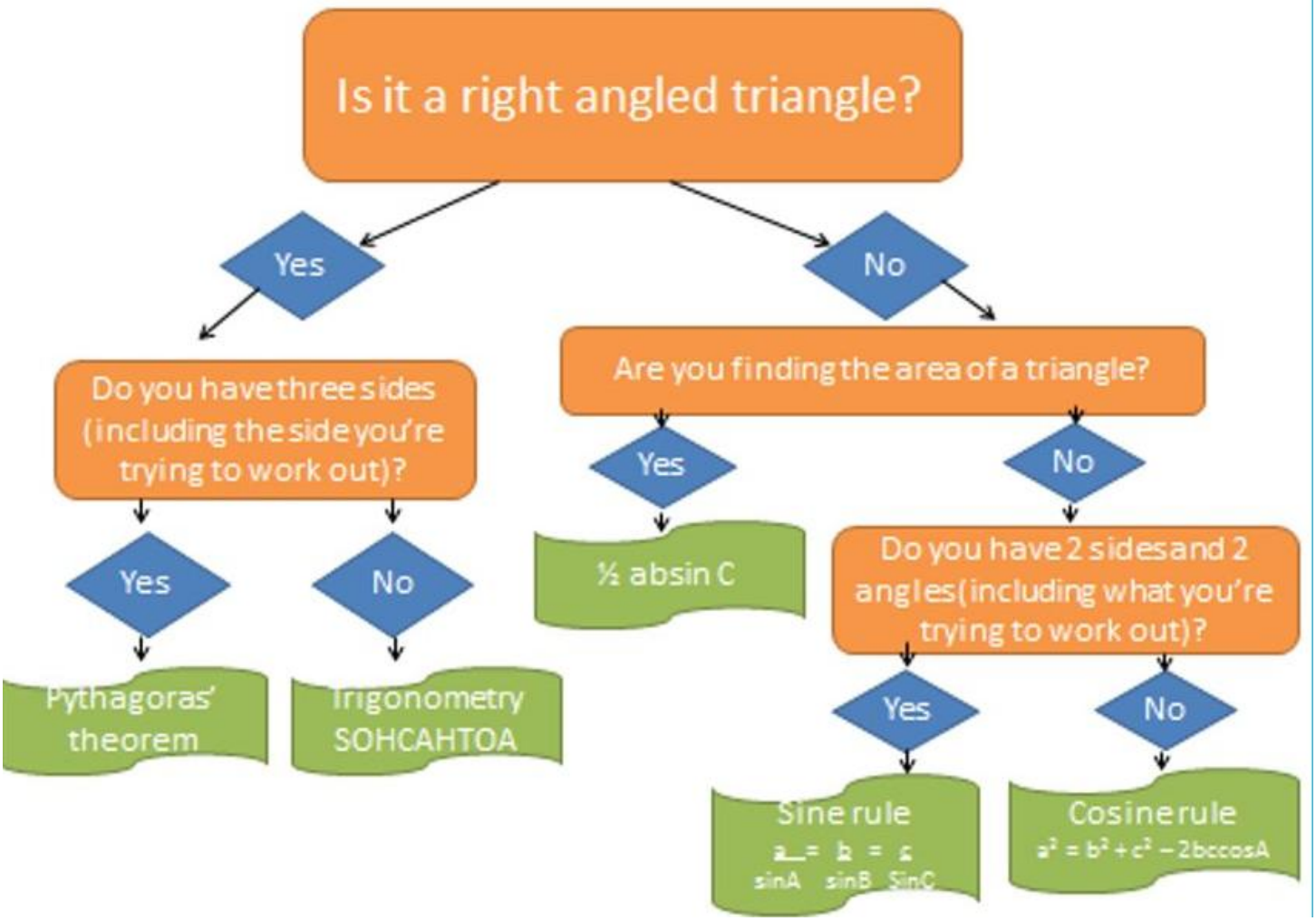
Work out the number of girls at the party.

The ratio of red to blue counters is 2 : 5 and there are 40 blue counters. When I remove some red counters the ratio is now 1 : 4. How many red counters did I remove?

Examples	
$x^2 - 7x + 12$	ax^2+bx+c $x^2-7x+12$ $(x-3)(x-4)$ $a=1$ $b=-7$ $c=12$  •1×12 •2×6 •3×4
$x^2 + 4x - 5$	ax^2+bx+c x^2+4x-5 $(x+5)(x-1)$ $a=1$ $b=4$ $c=-5$  •1×5
$x^2 - 81$	x^2-81 ← difference of two squares. $(x+9)(x-9)$
$2x^2 + 7x - 15$	ax^2+bx+c $2x^2+7x-15$ $2x^2+10x-3x-15$ $2x(x+5) \mid -3(x+5)$ $(2x-3)(x+5)$ $a=2$ $b=7$ $c=-15$  •1×30 •2×15 •3×10 •5×6
$\frac{20x^2 + 21x + 4}{16x^2 - 1}$	ax^2+bx+c $20x^2+21x+4$ $20x^2+5x+16x+4$ $5x(4x+1) \mid 4(4x+1)$ $(5x+4)(4x+1)$ $a=20$ $b=21$ $c=4$  •1×80 •2×40 •4×20 •5×16 $16x^2-1$ ← difference of 2 squares $(4x+1)(4x-1)$ $\frac{(5x+4)(4x+1)}{(4x+1)(4x-1)} = \frac{5x+4}{4x-1}$

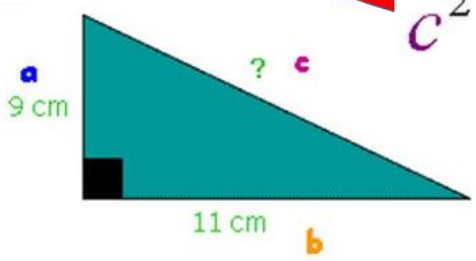
Factorise: $2w + 6$	$2w+6$ HCF = 2 $2(w+3)$
Factorise: $w^2 - 16$	difference of 2 squares w^2-16 $(w+4)(w-4)$
Factorise: $w^2 + 4w + 3$	w^2+4w+3 $a=1$ $b=4$ $c=3$ x | 3 | 1 | 3 | + | 4 | $(w+1)(w+3)$
Simplify $\frac{2x+8}{x^2-16}$	$2x+8 = 2(x+4)$ $x^2-16 = (x+4)(x-4)$ $\frac{2(x+4)}{(x+4)(x-4)} = \frac{2}{x-4}$
Simplify $\frac{x^2-8x+12}{x^2-36}$	
Simplify $\frac{2x^2+7x-9}{4x^2-81}$	

TRIGONOMETRY



PYTHAGORAS

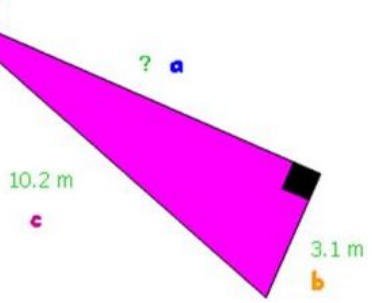
$$c^2 = a^2 + b^2$$



1. Label the sides
2. Use the formula: $c^2 = a^2 + b^2$
3. Put in the numbers:

$$c^2 = 9^2 + 11^2$$
$$c^2 = 81 + 121$$
$$c^2 = 202$$
$$c = \sqrt{202}$$
$$c = 14.2cm \quad (1dp)$$

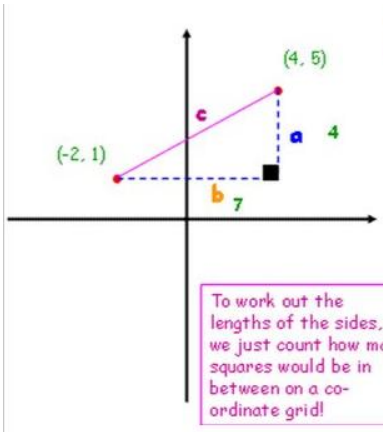
Square root both sides!



1. Label the sides
2. Use the formula: $a^2 = c^2 - b^2$
3. Put in the numbers:

$$a^2 = 10.2^2 - 3.1^2$$
$$a^2 = 104.04^2 - 9.61^2$$
$$a^2 = 94.43$$
$$a = \sqrt{94.43}$$
$$a = 9.72m \quad (2dp)$$

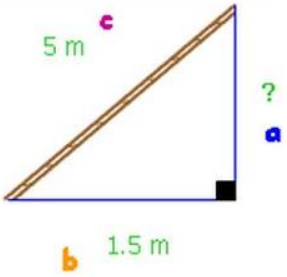
Square root both sides!



To work out the lengths of the sides, we just count how many squares would be in between on a co-ordinate grid!

PYTHAGORAS

A 5m ladder rests against the side of a house. The foot of the ladder is 1.5m away from the house. How far up the side of the house does the ladder reach?



1. Label the sides
2. Use the formula: $c^2 = a^2 + b^2$
3. Put in the numbers:

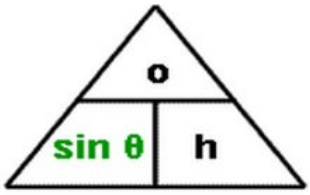
$$c^2 = 4^2 + 7^2$$
$$c^2 = 16 + 49$$
$$c^2 = 65$$
$$c = \sqrt{65}$$
$$c = 8.1 \quad (1dp)$$

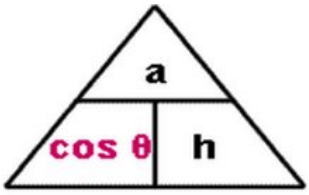
Square root both sides!

1. Label the sides
2. Use the formula: $a^2 = c^2 - b^2$
3. Put in the numbers:

$$a^2 = 5^2 - 1.5^2$$
$$a^2 = 25^2 - 2.25^2$$
$$a^2 = 22.75$$
$$a = \sqrt{22.75}$$
$$a = 4.77m \quad (2dp)$$

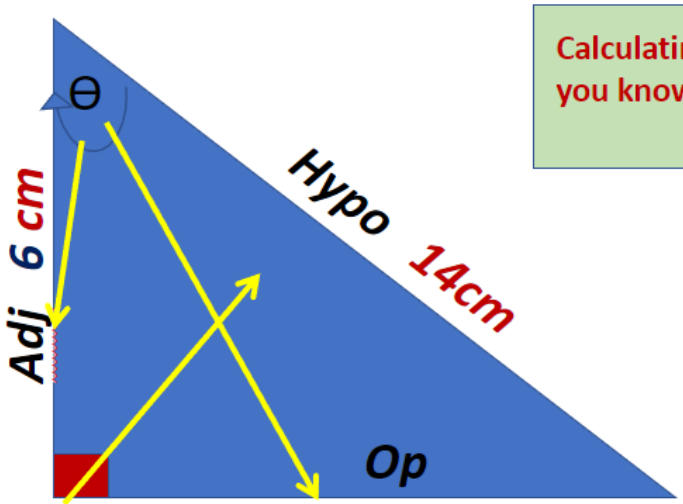
Square root both sides!





1. *Label*
2. *Information*
3. *Rule*
4. *Substitution*
5. *Work out the answer*
6. *Round if needed*

O A O
S H C H T A



Calculating an angle if you know two sides

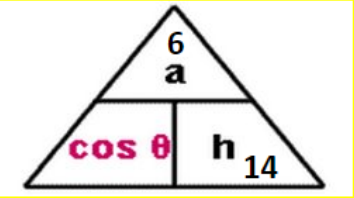
2) Θ = X
A = 6 cm
H = 14 cm

SOH C AH TOA
SOH CAH TOA

$Cos\ \Theta = \frac{A}{H}$

4) $Cos\ \Theta = \frac{6}{14}$

5) Θ = shift Cos 6/14
Θ = 64.62306647



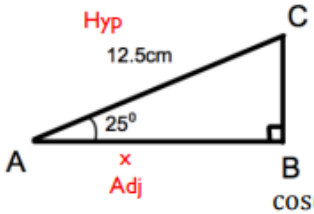
6) Θ = 64.6 cm
(3 SF)

What you need to know:

Trigonometry – Finding a side 1

SOH	CAH	TOA
SINE = $\frac{OPP}{HYP}$	COSINE = $\frac{ADJ}{HYP}$	TANGENT = $\frac{OPP}{ADJ}$

Calculate the length of AB.



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverse operations to rearrange & isolate x.

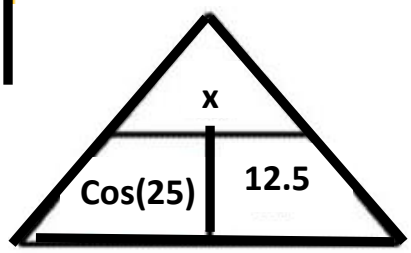
$\cos(25) = \frac{x}{12.5}$

The inverse of dividing by 12.5 is multiplying.

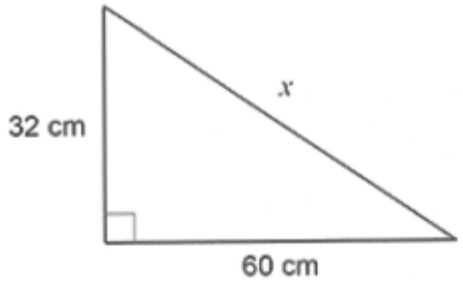
$\cos(25) \times 12.5 = x$

$x = 11.32884734\ cm$

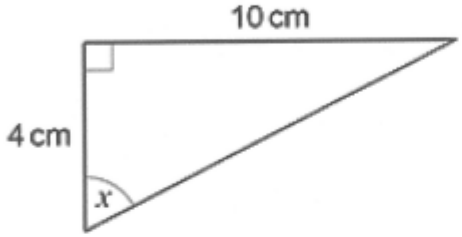
Don't round your answer, you get no marks for this!



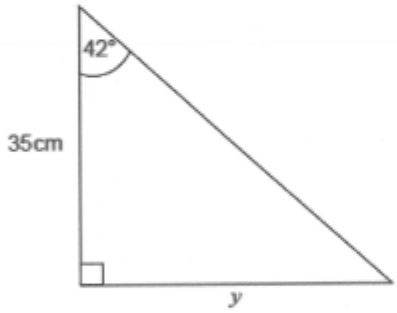
1)



2)

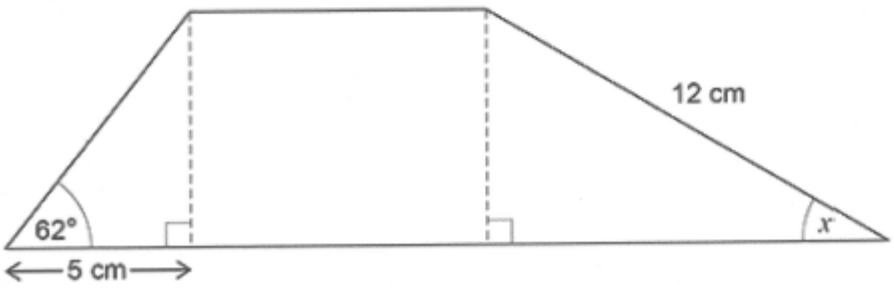


3)



Not drawn accurately

4)



Work out the size of angle x.

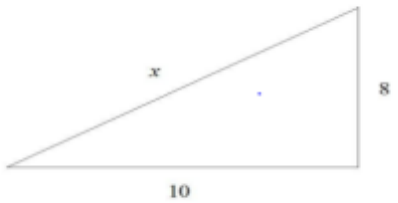
1. Use your calculator to work out

a) 4.3^2

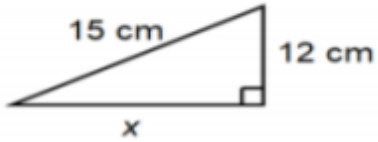
b) $7^2 + 9^2$

c) $\sqrt{31.36}$

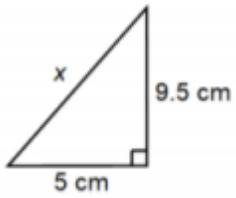
2. Find the value of x in questtions 2-5



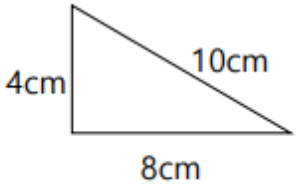
3. Find the value of x in the following



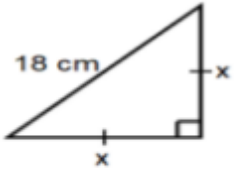
4.



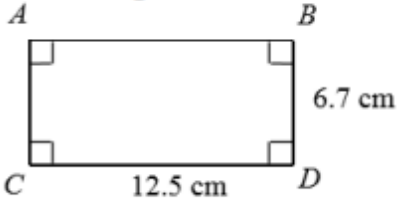
5. Is this triangle right-angled?



6. Find the value of x in the following



7. Find the length BC



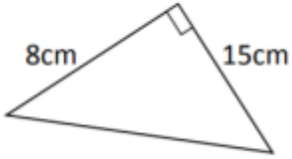
8. ABC is a right -angled triangle.

AB = 15.5 m and AC = 12 m.

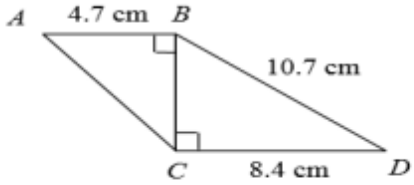
Find the length of BC.

Give your answer to 1 decimal place.

9. Find the perimeter of the following



10. Find the length AC

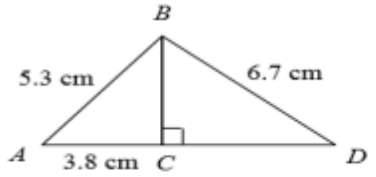


11. Triangle ABC has perimeter 12 m.

AB = 5 m and BC = 4 m.

Is triangle ABC a right-angled triangle?
Show clearly how you decide.

12. Find the length CD



Yr 11 (H) Revision - Sine Rule

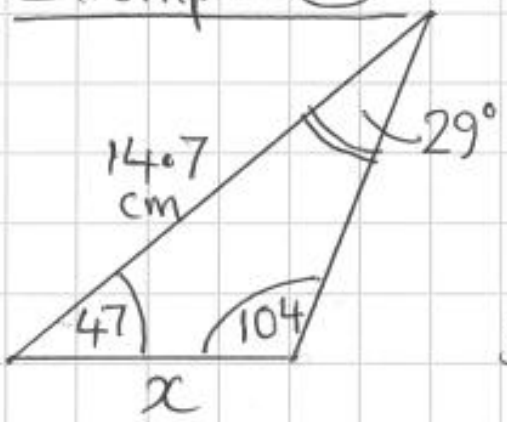
To calculate a length

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

To calculate an angle

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Example (1)

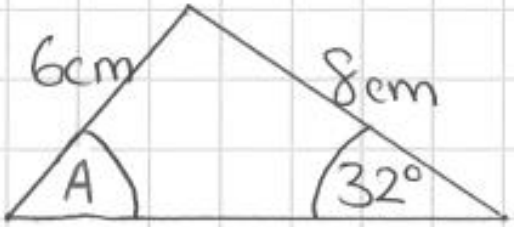


firstly work out the missing angle.... $180 - 47 - 104 = \underline{29}$

$$\frac{x}{\sin(29)} = \frac{14.7}{\sin(104)}$$

$$x = \frac{14.7}{\sin(104)} \times \sin(29) = \underline{7.34\text{cm}}$$

Example (2)

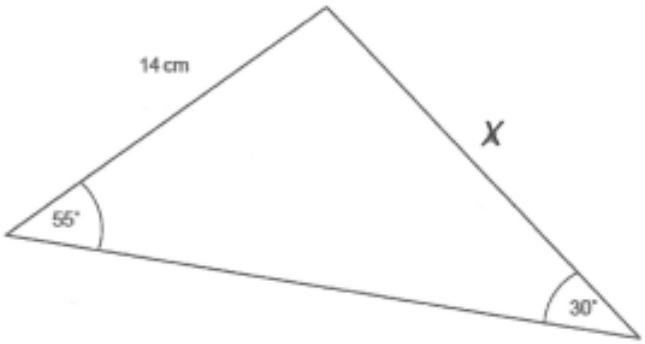


$$\frac{\sin(A)}{8} = \frac{\sin(32)}{6}$$

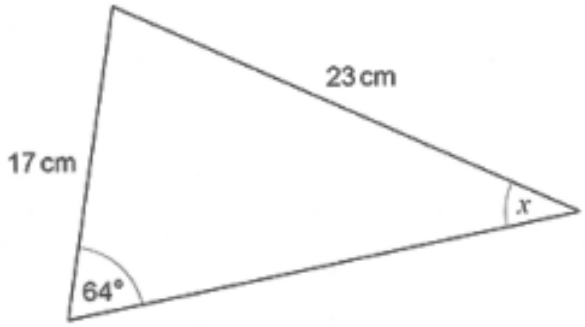
$$\sin(A) = \frac{\sin(32)}{6} \times 8.$$

then $\sin^{-1}(\text{ANS}) = \underline{45^\circ}$

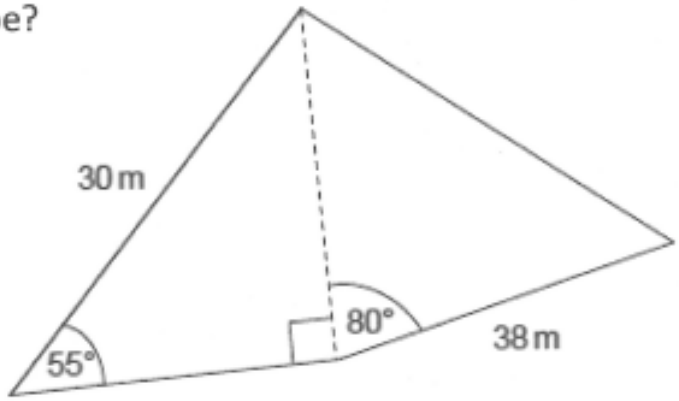
1)



2)

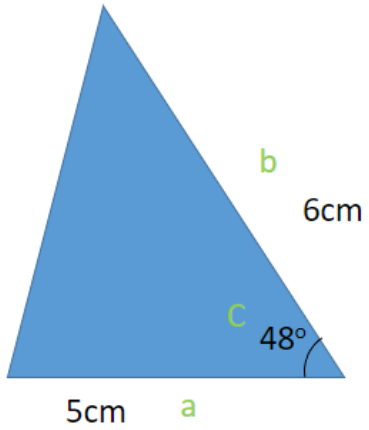


3) Perimeter of shape?



Example – find the area

Learn the formula:- Area$= \frac{1}{2}abSinC$



Find the area of the triangle to 1dp.

1) Label sides
2) Substitute in values:

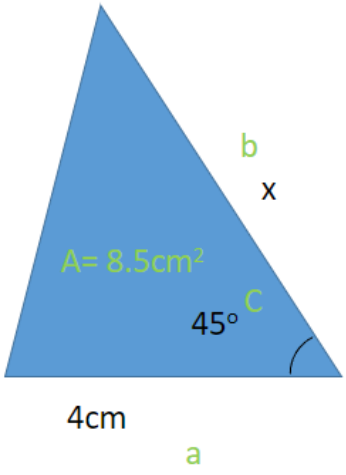
$$\text{Area} = \frac{1}{2} \times 5 \times 6 \times Sin(48)$$

3) Type in to your calculator:

11.1cm to 1dp

Example – missing side

Learn the formula:- Area$= \frac{1}{2}abSinC$



Find the length of 'x' 1dp.

1) Label sides
2) Substitute in values:

$$8.5 = \frac{1}{2} \times 4 \times ? \times Sin45$$

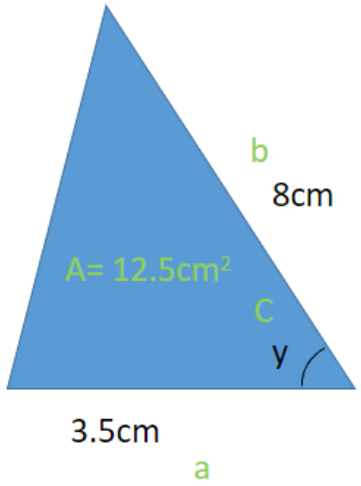
3)Divide through by everything except the ?.

$$\frac{8.5}{\frac{1}{2} \times 4 \times sin45} = ?$$

4) Type in to calculator:6.0cm to 1dp

Example – missing angle

Learn the formula:- Area$= \frac{1}{2}abSinC$



Find the length of 'y' 1dp.

1) Label sides
2) Substitute in values:

$$12.5 = \frac{1}{2} \times 3.5 \times 8 \times Sin(?)$$

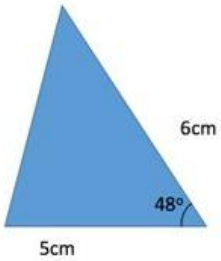
3)Divide through by everything except the sin(?).

$$\frac{12.5}{\frac{1}{2} \times 3.5 \times 8} = \sin(?)$$

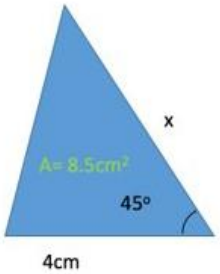
4) Type in to calculator and don't forget to sin $^{-}$1:62.3° to 1dp

Area = $\frac{1}{2}$ abSinC

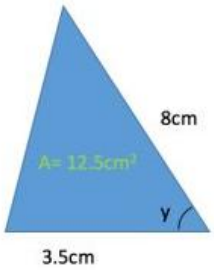
Find the **area**



Find the **missing side**



Find the **missing angle**



Angles in polygons

Facts

$$\frac{360}{number\ of\ sides} = exterior\ angle$$

$$\frac{360}{exteriro\ angles} = number\ of\ sides$$

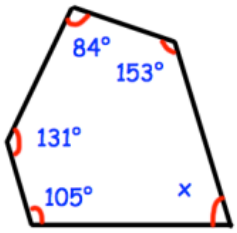
$$Interior + Exterior = 180°$$

$$(n-2)\ X\ 180 = angle\ sum$$

Angles in a straight line = 180°

- Angles in a triangle = 180°
- Angles in a quadrilateral = 180°

Examples



Angle Sum = $(5-2) \times 180$

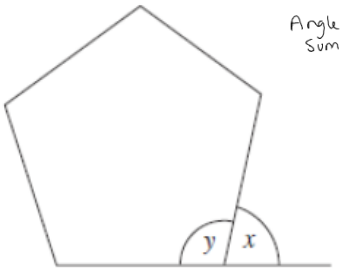
3×180

$= 540°$

$540 - (84 + 153 + 131 + 105)$

$= 67°$

This is a regular pentagon.



Angle Sum = $(5-2) \times 180$

$= 3 \times 180$

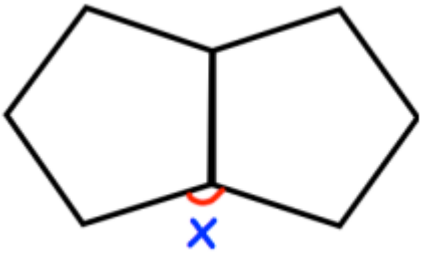
$= 540$

$540 \div 5 = 108$ (interior) $y = 108$

Interior + exterior = 180°

$180 - 108 = 72°$

$x = 72°$



Angle sum = $(5-2) \times 180$

$= 3 \times 180$

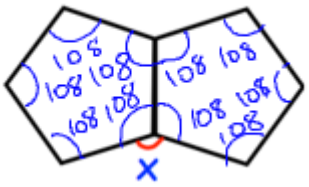
$= 540$

$540 \div 5 = 108°$

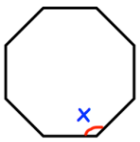
$108 + 108 = 216$

$360 - 216$

$= 144° = x$

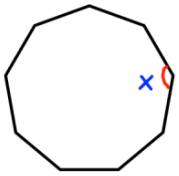


Questions

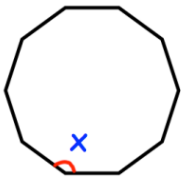


regular octagon

regular hexagon



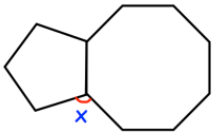
regular nonagon

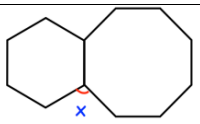


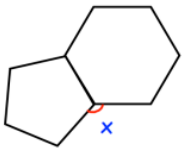
regular decagon

regular pentagon

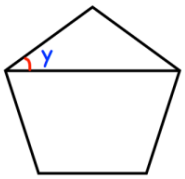
regular hexagon







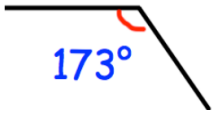
Shown is a regular pentagon.
Find y.



A regular polygon has 18 sides.
Calculate the size of each interior angle.

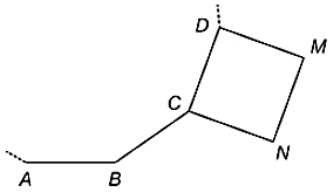
A regular polygon has 30 sides.
Calculate the size of each interior angle.

Explain why this cannot be an interior angle from regular polygons.



A polygon has an interior angle that is five times larger than the exterior angle. How many sides does it have?

AB, *BC* and *CD* are sides of a regular 12-sided polygon.
CDMN is a square.

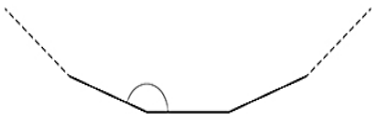


Not drawn accurately

Prove that points *A*, *B* and *N* lie on a straight line.

Part of a regular polygon with 15 sides is shown.

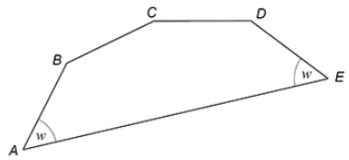
Not drawn accurately



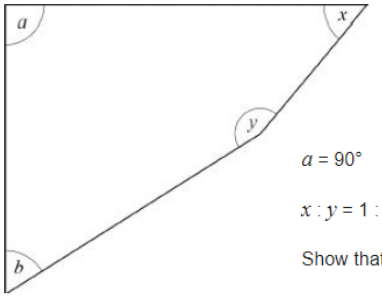
Work out the size of an **interior** angle.

AB, *BC*, *CD* and *DE* are four of the sides of a regular decagon.

Not drawn accurately



Work out the size of angle w.



$a = 90°$ and $a : b = 5 : 3$

$x : y = 1 : 3$

Show that $b = x$

SURDS

Examples

$\sqrt{5} \times \sqrt{5} \times \sqrt{5} \times \sqrt{5}$

$\sqrt{5} \times \sqrt{5} \times \sqrt{5} \times \sqrt{5}$

5×5

$= 25$

$\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$

$\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$

$= 4\sqrt{3}$

$3\sqrt{7} + 5\sqrt{7}$

$3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$

$12\sqrt{2} - 8\sqrt{2}$

$12\sqrt{2} - 8\sqrt{2} = 4\sqrt{2}$

Simplify $(\sqrt{3})^5$

$\sqrt{3} \times \sqrt{3} \times \sqrt{3}$

$3 \times \sqrt{3}$

$3\sqrt{3}$

Simplify $\sqrt{5} \times \sqrt{4}$

$\sqrt{5 \times 4} = \sqrt{20}$

$\sqrt{20}$

$2 \quad 10$

$2 \quad 5$

$\sqrt{2} \times \sqrt{2} \times \sqrt{5}$

$2 \times \sqrt{5} = 2\sqrt{5}$

Simplify $\sqrt{450}$

$\sqrt{450}$

$10 \quad 45$

$2 \quad 5 \quad 5 \quad 9$

$3 \quad 3$

$\sqrt{2} \times \sqrt{3} \times \sqrt{3} \times \sqrt{5} \times \sqrt{5}$

$\sqrt{2} \times 3 \times 5$

$15\sqrt{2}$

Simplify $18\sqrt{14} \div 6\sqrt{2}$

$18 \div 6 = 3$

$\sqrt{14} \div \sqrt{2} = \sqrt{7}$

$$\frac{18\sqrt{14}}{6\sqrt{2}} = 3\sqrt{7}$$

Simplify $6\sqrt{50}$

$\sqrt{50} = \sqrt{2} \times \sqrt{5} \times \sqrt{5}$

$5 \quad 10$

$2 \quad 5$

$\sqrt{2} \times 5$

$5\sqrt{2}$

$6 \times 5\sqrt{2}$

$= 30\sqrt{2}$

Simplify $4\sqrt{80} + 3\sqrt{45}$

$\sqrt{80} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{5}$

$8 \quad 10$

$2 \quad 4 \quad 2 \quad 5$

$2 \quad 2$

$2 \times 2 \times \sqrt{5}$

$4\sqrt{5}$

$\sqrt{45} = \sqrt{3} \times \sqrt{3} \times \sqrt{5}$

$5 \quad 9$

$3 \quad 3$

$3 \times \sqrt{5}$

$3\sqrt{5}$

$4 \times 4\sqrt{5} + 3 \times 3\sqrt{5}$

$16\sqrt{5} + 9\sqrt{5}$

$= 25\sqrt{5}$

Expand

$5\sqrt{3}(\sqrt{3} + 2\sqrt{4})$

- $5\sqrt{3} \times \sqrt{3}$
  $= 5 \times 3 = 15$
- $5\sqrt{3} \times 2\sqrt{4}$
  $= 10\sqrt{12}$
  $= 10 \times 2\sqrt{3}$
  $= 20\sqrt{3}$

$\sqrt{12} = \sqrt{2} \times \sqrt{2} \times \sqrt{3}$

$2 \quad 6$

$2 \quad 3$

$2\sqrt{3}$

$5\sqrt{3}(\sqrt{3} + 2\sqrt{4})$

$15 + 20\sqrt{3}$

Expand

$(3\sqrt{5} + 7)(2\sqrt{5} + 1)$

- $3\sqrt{5} \times 2\sqrt{5}$
  $= 6 \times 5 = 30$
- $7 \times 1 = 7$
- $7 \times 2\sqrt{5} = 14\sqrt{5}$
- $3\sqrt{5} \times 1 = 3\sqrt{5}$

$(3\sqrt{5} + 7)(2\sqrt{5} + 1)$

$30 + 7 + 14\sqrt{5} + 3\sqrt{5}$

$37 + 17\sqrt{5}$

Rationalise $\frac{9}{\sqrt{6}}$

$\frac{9}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$\frac{9\sqrt{6}}{6} \div 3 = \frac{3\sqrt{6}}{2}$

Rationalise $\frac{4}{1+\sqrt{5}}$

$\frac{4}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$\frac{4(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{4-4\sqrt{5}}{-4}$

$(1+\sqrt{5})(1-\sqrt{5})$

$1-1\sqrt{5}+1\sqrt{5}-5$

$= -4$

Write $\frac{18}{\sqrt{2}} - \frac{12}{\sqrt{32}}$ in the form $\frac{a\sqrt{2}}{b}$ where a and b are integers.

$\frac{18}{\sqrt{2}} - \frac{12}{\sqrt{32}}$

Rationalise

$\frac{18}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{18\sqrt{2}}{2}$

$\sqrt{32} = \sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$

$2 \times 2 \times \sqrt{2}$

$4\sqrt{2}$

2 | 16

2 | 8

2 | 4

2 | 2

$\frac{12}{4\sqrt{2}} \div 4 = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

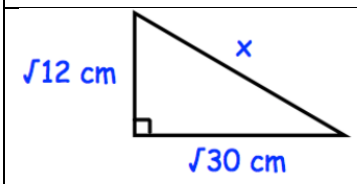
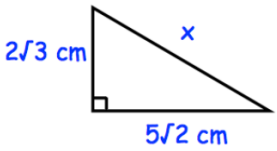
$\frac{18\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}$

$= \frac{15\sqrt{2}}{2}$

Questions

$(\sqrt{7})^2$	$(\sqrt{10})^2$
$(\sqrt{10})^4$	$(\sqrt{2})^3$
$(\sqrt{2})^5$	$(\sqrt{10})^3$
$\sqrt{8} \times \sqrt{2}$	$\sqrt{5} \times \sqrt{20}$
$\sqrt{10} \times \sqrt{7} \times \sqrt{3}$	$2\sqrt{3} \times 3\sqrt{5}$
$2\sqrt{2} \times 3\sqrt{8}$	$4\sqrt{3} \times 2\sqrt{3}$

$\sqrt{48} \div \sqrt{8}$	$12\sqrt{10} \div 3\sqrt{5}$
$36\sqrt{54} \div 18\sqrt{6}$	$\frac{15\sqrt{14}}{3\sqrt{2}}$
Simplify $\sqrt{48}$	**Simplify** $\sqrt{180}$
Simplify $4\sqrt{50}$	**Simplify** $15\sqrt{32}$
$\sqrt{6} \times \sqrt{10}$	$5\sqrt{10} \times 2\sqrt{30}$
$\sqrt{50} + \sqrt{8}$	$\sqrt{75} + \sqrt{27}$
$6\sqrt{75} - 2\sqrt{12}$	$4\sqrt{80} + 3\sqrt{45}$
Write $\frac{10}{\sqrt{2}} + \sqrt{18}$ in the form $a\sqrt{b}$ where b is a prime number.	$y = \frac{5\sqrt{x}}{2}$ Work out the expression for y^2
$\sqrt{5}\,(3\sqrt{2} - \sqrt{5})$	$\sqrt{12}\,(7 - \sqrt{3})$
$(2 + \sqrt{3})(1 + \sqrt{3})$	$(3\sqrt{5} + 7)(2\sqrt{5} + 1)$

$(10-\sqrt{2})^2$	$(4+\sqrt{3})(4-\sqrt{3})$
$\frac{2}{\sqrt{3}}$	$\frac{5}{\sqrt{2}}$
$\frac{3}{2\sqrt{5}}$	$\frac{5\sqrt{5}}{\sqrt{20}}$
$\frac{6}{3-\sqrt{2}}$	$\frac{\sqrt{2}}{\sqrt{10}+\sqrt{8}}$
$\frac{3\sqrt{6}}{\sqrt{6}-3}$	$\frac{\sqrt{11}-\sqrt{3}}{\sqrt{11}+\sqrt{3}}$
	

Direct and inverse proportion

Examples

Directly – k is next to	Inversely – k is over
When one goes up, the other one goes up	**When one goes up, the other one goes down**

A is directly proportional to B.
When A = 12, B = 3
(a) Find a formula for A in terms of B.
(b) Find the value of A when B = 5
(c) Find the value of B when A = 36

(a) Directly → k next to

$A = BK$ ($A = 4B$)

$\frac{12}{3} = \frac{3K}{3}$

$4 = K$

(b) $A = 4B$

$A = 4 \times 5 = 20$

$A = 20$

(c) $A = 4B$

$\frac{36}{4} = \frac{4B}{4}$

$9 = B$

T is inversely proportional to N.
When T = 30, N = 5.
(a) Find an equation connecting T and N.
(b) Work out the value of T when N = 10
(c) Work out the value of N when T = 25

(a) Inversely → K over

$T = \frac{K}{N}$ → $T = \frac{150}{N}$

$30 = \frac{K}{5}$ (×5, ×5)

$150 = K$

(b) $T = \frac{150}{N}$

$T = \frac{150}{10}$

$T = 15$

(c) $T = \frac{150}{N}$

$\frac{25}{1} = \frac{150}{N}$

$\frac{150}{25} = \frac{25N}{25}$

$N = 6$

y is directly proportional to the square of x
When y = 6.4, x = 4
(a) Find a formula for y in terms of x
(b) Find the value of y when x = 8
(c) Find the value of x when y = 78.4

(a) Directly → K next to

$y = x^2K$ → $y = 0.4x^2$

$6.4 = 4^2K$

$\frac{6.4}{16} = \frac{16K}{16}$

$0.4 = K$

(b) $y = 0.4x^2$

$y = 0.4(8)^2$

$y = 0.4(64)$

$y = 25.6$

(c) $y = 0.4x^2$

$\frac{78.4}{0.4} = \frac{0.4x^2}{0.4}$

$196 = x^2$

$\sqrt{196} = x$

$x = 14$

y is inversely proportional to the cube root of x
When y = 2500, x = 64
(a) Find the value of y when x = 8
(b) Find the value of x when y = 2000

(a) Inversely → K over

$y = \frac{K}{\sqrt[3]{x}}$ → $y = \frac{10000}{\sqrt[3]{x}}$

$2500 = \frac{K}{\sqrt[3]{64}}$

$2500 = \frac{K}{4}$ (×4, ×4)

$K = 10000$

(b) $y = \frac{10000}{\sqrt[3]{x}}$

$y = \frac{10000}{\sqrt[3]{8}}$

$y = \frac{10000}{2}$

$y = 5000$

(c) $y = \frac{10000}{\sqrt[3]{x}}$

$\frac{2000}{1} = \frac{1000}{\sqrt[3]{x}}$

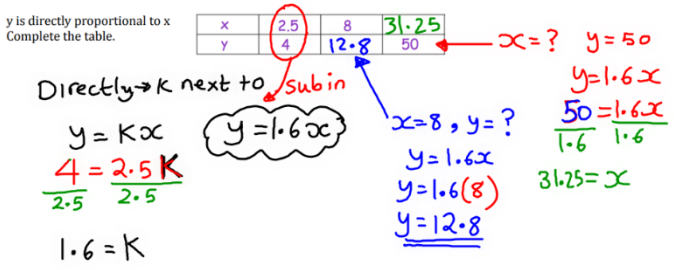
$\frac{1000}{2000} = \frac{2000\sqrt[3]{x}}{2000}$

$0.5 = \sqrt[3]{x}$ (cube it, cube it)

$0.125 = x$

y is directly proportional to x
Complete the table.

x	2.5	8	
y	4		50



Questions

C is directly proportional to D.
When C = 125, D = 5
(a) Find an equation for C in terms of D.
(b) Find the value of C when D = 10
(c) Find the value of D when C = 75

w is inversely proportional to *f*
When *f* = 12, *w* = 40
(a) Find a formula connecting *w* and *f*
(b) Find the value of *w* when *f* = 60

B is inversely proportional to y
When B = 0.8, y = 13
(a) Find an equation for B in terms of y.
(b) Work out the value of B when y = 5

A is directly proportional to B^2
When A = 50, B = 5
(a) Find a formula for A in terms of B.
(b) Find the value of A when B = 3
(c) Find the value of B when A = 200

The cost of a circular table is directly proportional to the square of its radius.
A table with a radius of 40cm cost £90.

What is the cost of a table with a radius of 60cm?

T is inversely proportional to the cube of L
When L = 0.2, T = 5
(a) Write a formula connecting T and L.
(b) Work out the value of T when L = 0.5
(c) Work out the value of L when T = 2

q is inversely proportional to the square of t.
When q = 7.5, t = 1.6
(a) Calculate the value of q when t = 8
(b) Calculate the value of t when q = 1.875

The mass of a paperweight is *m* grams.
The length of the paperweight is *L* centimetres.
m is directly proportional to the cube of *L*.
m = 4968 when *L* = 12

(a) Work out an equation connecting *m* and *L*
(b) Work out the mass of a paperweight with a length of 4 centimetres

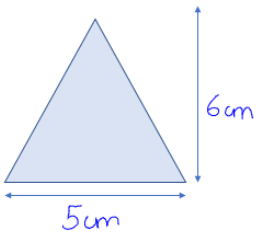
Perimeter, circumference and area

Examples

Area is the space inside the shape

Perimeter is the total length around the shape

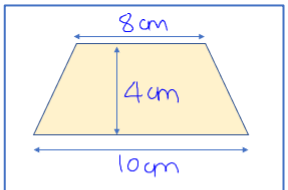
Area of triangle



Area = $\frac{\text{Base} \times \text{height}}{2}$

Area = $\frac{5 \times 6}{2} = \frac{30}{2} = 15\text{cm}^2$

Area of trapezium

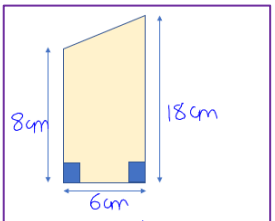


Area = $\frac{a+b}{2} \times h$

Area = $\frac{10+8}{2} \times 4$

$\frac{18}{2} \times 4$

$9 \times 4 = 36\text{cm}^2$



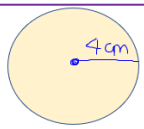
Area = $\frac{a+b}{2} \times h$

Area = $\frac{18+8}{2} \times 6$

$\frac{26}{2} \times 6$

$13 \times 6 = 78\text{cm}^2$

Area of circle



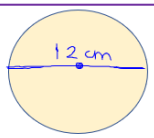
Area = $\pi \times r^2$

Radius = 4

Area = $\pi \times 4^2$

$\pi \times 16$

$= 16\pi$ or 50.3cm



Area = $\pi \times r^2$

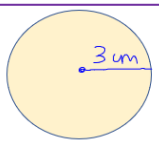
Radius = 6

Area = $\pi \times 6^2$

$\pi \times 36$

36π or 113.1cm

Circumference of circle

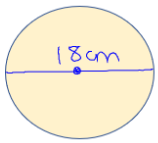


Circumference = $\pi \times d$

Circumference = 6cm

$C = \pi \times 6$

$C = 6\pi$ or 18.8cm

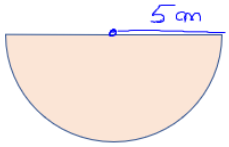


Circumference = $\pi \times d$

Circumference = 18cm

$C = \pi \times 18$

$C = 18\pi$ or 56.5cm

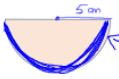


Radius = 5cm

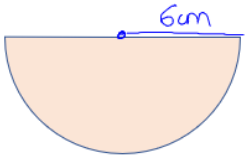
Diameter = 10cm

Arc length is part of the circumference.

Arc length = $\frac{\pi \times 10}{2} = \frac{10\pi}{2} = 5\pi$



arc length $= 5\pi$



Radius = 6cm

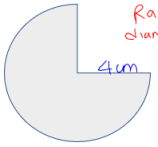
Diameter = 12cm

Perimeter: Total length around the shape.

Arc length + diameter

$\frac{\pi \times 12}{2} + 12$

Perimeter = $6\pi + 12$



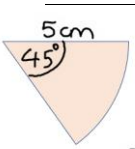
Perimeter = Arc length + radius + radius

arc length $= 6\pi$

$\frac{\pi \times 8 \times 3}{4}$

$= \frac{24\pi}{4}$ $+4$ $+4$

$6\pi + 8$ = perimeter



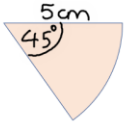
$\frac{45}{360} = \frac{1}{8}$ (÷45)

Area = $\pi \times 5^2 \times \frac{45}{360}$

$\pi \times 5^2 \times \frac{1}{8}$

$\frac{\pi \times 25}{1} \times \frac{1}{8}$

Area = $\frac{25\pi}{8}$



$\frac{45}{360} = \frac{1}{8}$ (÷45)

Perimeter = Arc length + radius + radius

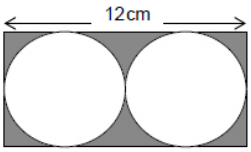
$\frac{\pi \times 10}{1} \times \frac{1}{8}$

$\frac{10\pi}{1} \times \frac{1}{8}$

$= \frac{10\pi}{8}$

Perimeter = $\frac{10\pi}{8} + 5 + 5 = \frac{10\pi}{8} + 10$

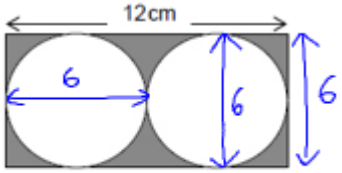
Two identical circles just fit inside a rectangle as shown.



Not drawn accurately

Work out the area of the shaded section.

Give your answer in terms of π



diameter = 6

Shaded area = Area of rectangle − Subtract Area of circles

Area of rectangle: $12 \times 6 = 72cm^2$

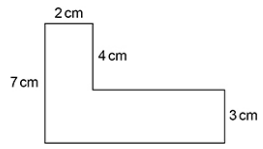
Area of circles: diameter = 6, radius = 3

Area of one circle = $\pi \times 3^2 = 9\pi$

Area of two circles = $9\pi \times 2 = 18\pi$

$72 - 18\pi$

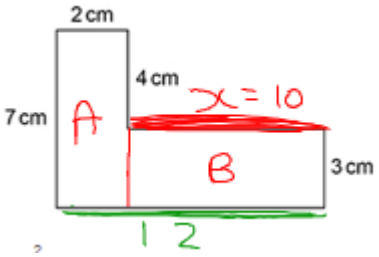
The L-shape is made from rectangles.

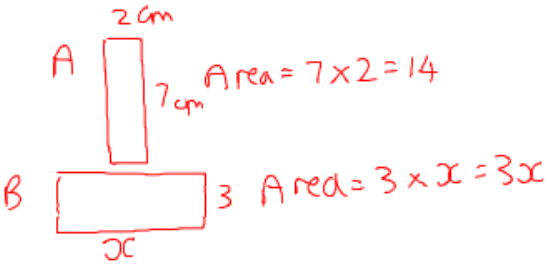


The area is 44 cm^2

Work out the perimeter.

Not drawn accurately





Area = 14 + 3x = 44

14 + 3x = 44

(−14) (−14)

$\frac{3x}{3} = \frac{30}{3}$

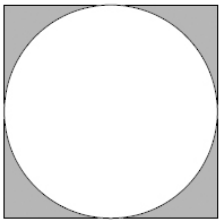
x = 10

Perimeter = 2+4+10+3+12+7

= 38cm

Questions

The diagram shows a circle of radius 5 cm inside a square.

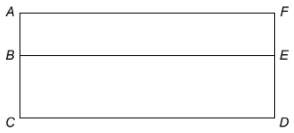


Work out the shaded area.

ABEF and *ACDF* are rectangles.

AF = 10 cm *AB* = 2 cm *BC* = 4 cm

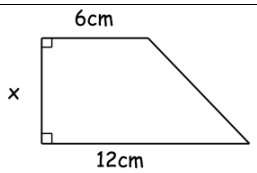
Not drawn accurately



Work out

perimeter *ABEF* : perimeter *ACDF*

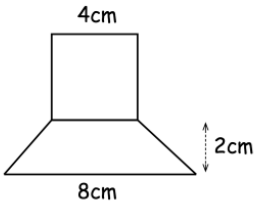
Give your answer in its simplest form.



The area of the trapezium is 63cm².

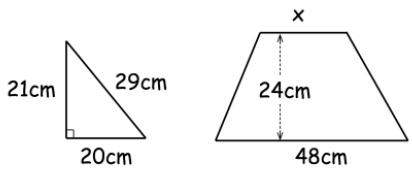
Work out the value of x.

A club logo is made from a square and a trapezium.



Calculate the area of the logo

Shown below is a triangle and a trapezium.



The area of the trapezium is four times the area of the triangle.

Find x.

A large rectangle is made by joining three identical small rectangles as shown.

Not drawn accurately

The perimeter of one small rectangle is 15 cm

Work out the perimeter of the large rectangle.

A circle has radius 4.2 cm

Work out the length of the circumference.

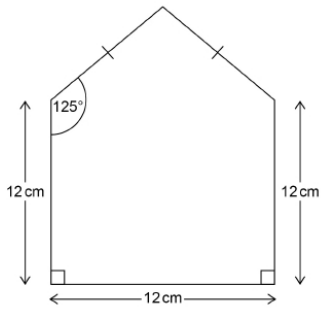
Give your answer to 1 decimal place.

The perimeter of an isosceles triangle is 25 cm
The length of each side, in cm, is a prime number.

Work out the lengths of the sides of the **two** possible isosceles triangles.

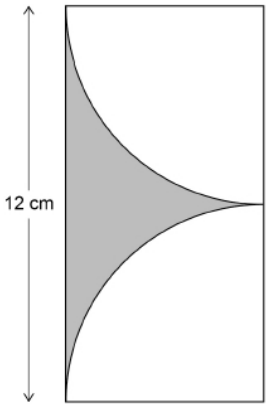
A pentagon is made from a square and an isosceles triangle.

Not drawn accurately



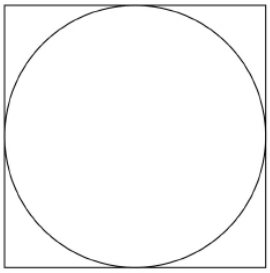
Work out the perimeter of the pentagon.

Two identical quarter circles are cut from a rectangle as shown.



Work out the shaded area.

Here is a circle touching a square.



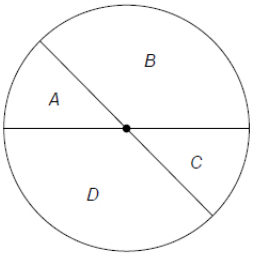
The area of the square is 64 cm^2

Work out the area of the circle.

Give your answer in terms of π.

A circle has radius 6 cm
Two diameters split the circle into four sectors, as shown.

Not drawn accurately

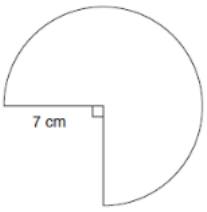


Area of sector A : Area of sector B = 1 : 3

Work out the area of sector A.
Give your answer in terms of π.

The diagram shows a sector of a circle.
The radius is 7 cm

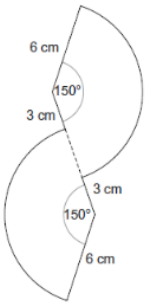
Not drawn accurately



Work out the **perimeter** of the shape.

This shape is made from two identical sectors.

Not drawn accurately



Work out the perimeter of the shape.
Give your answer in terms of π.

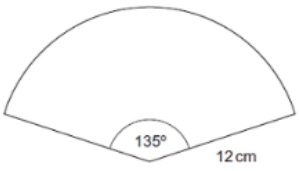
A wheel has diameter 0.7 m

(a) Work out the circumference.

(b) Work out the number of complete turns when the wheel travels 1.6 km
You **must** show your working.

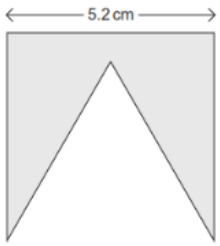
The diagram shows a sector of a circle, radius 12 cm.

Not drawn accurately



Show that the perimeter of the sector is greater than 52 cm.

This shape is made by cutting out an equilateral triangle from a square.



Not drawn accurately

Work out the perimeter of the shape.

Algebraic fractions

Examples

$\frac{3c}{4} + \frac{5c}{9}$

$\frac{3c}{4} \times \frac{9}{9} + \frac{5c}{9} \times \frac{4}{4}$

$\frac{27c}{36} + \frac{20c}{36} = \frac{47c}{36}$

$\frac{5}{6x} - \frac{1}{3x}$

$\frac{5}{6x} \times \frac{1}{1} - \frac{1}{3x} \times \frac{2}{2}$

$\frac{5}{6x} - \frac{2}{6x} = \frac{3}{6x} = \frac{1}{2x}$ (÷3)

$\frac{3x+11}{2} - \frac{2x-3}{6}$

$\frac{3x+11}{2} \times \frac{3}{3} - \frac{2x-3}{6} \times \frac{1}{1}$

$\frac{9x+33}{6} - \frac{2x-3}{6}$

$\frac{9x+33-2x+3}{6}$

$\frac{7x+36}{6}$

$\frac{2}{x+5} + \frac{3}{x+1}$

$\frac{x(x+1)}{x(x+1)} \times \frac{2}{x+5} + \frac{3}{x+1} \times \frac{x(x+5)}{x(x+5)}$

$\frac{2(x+1)}{(x+5)(x+1)} + \frac{3(x+5)}{(x+5)(x+1)}$

$\frac{2x+2}{(x+5)(x+1)} + \frac{3x+15}{(x+5)(x+1)}$

$\frac{5x+17}{(x+5)(x+1)}$

$$\frac{x+3}{2x+1} - \frac{x-2}{x-1}$$

$\times(x-1)$ $\frac{x+3}{2x+1}$ $-$ $\frac{x-2}{x-1}$ $\times(2x+1)$

$\downarrow$

$\frac{(x-1)(x+3)}{(2x+1)(x-1)} - \frac{(x-2)(2x+1)}{(2x+1)(x-1)}$

$(x-1)(x+3)$

$x^2-x+3x-3$

x^2+2x-3

$(x-2)(2x+1)$

$2x^2-4x+x-2$

$2x^2-3x-2$

$\frac{x^2+2x-3-(2x^2-3x-2)}{(2x+1)(x-1)}$

$\frac{x^2+2x-3-2x^2+3x+2}{(2x+1)(x-1)}$

$\frac{-x^2+5x-1}{(2x+1)(x-1)}$

$$\frac{x-3}{4} - \frac{x-8}{3} = 4$$

$\frac{x-3}{4}^{\times 3} - \frac{x-8}{3}^{\times 4} = 4$

$\frac{3(x-3)}{12} - \frac{4(x-8)}{12} = 4$

$\frac{3x-9}{12} - \frac{4x-32}{12} = 4$

$\frac{3x-9-4x+32}{12} = 4$

$\frac{-x+23}{12} \times \frac{4}{1}$

$1(-x+23) = 4(12)$

$-x+23 = 48$

$-23 \quad -23$

$-x = 25$

$x = -25$

$$\frac{3}{x-2} + \frac{3}{x+2} = 2$$

$x(x+2)\frac{3}{(x-2)} + \frac{3}{(x+2)}^{\times(x-2)} = 2 \times(x-2)$

$\downarrow \qquad \downarrow$

$\frac{3(x+2)}{(x-2)(x+2)} + \frac{3(x-2)}{(x-2)(x+2)} = 2$

$(x+2)(x-2)$

$x^2+2x-2x-4$

x^2-4

$\frac{3x+6+3x-6}{(x-2)(x+2)} = 2$

$\frac{6x}{x^2-4} \times \frac{2}{1}$

$1(6x) = 2(x^2-4)$

$6x = 2x^2-8$

$(-6x)$

$0 = 2x^2-6x-8$

Factorise

$2x^2-6x-8=0$

$2x^2-8x+2x-8$

$2x(x-4) \quad 2(x-4)$

$(x-4)(2x+2)=0$

$x-4=0 \qquad 2x+2=0$

$+4 \quad +4 \qquad -2 \quad -2$

$x=4 \qquad 2x=-2$

$x=\frac{-2}{2}=1$

$$\frac{x^2+x-6}{x^2-25} \times \frac{x^2+10x+25}{x^2-4}$$

x^2+x-6

$(x+3)(x-2)$

	-6
3	-2
1	

$x^2-25 = (x+5)(x-5)$

$x^2-4 = (x+2)(x-2)$

Difference of 2 squares

$x^2+10x+25$

$(x+5)(x+5)$

	25
5	5
+10	

$\frac{(x+3)(x-2)(x+5)(x+5)}{(x+5)(x-5)(x+2)(x-2)}$

$\frac{(x+3)(x+5)}{(x-5)(x+2)}$

Questions	
$\frac{w}{3} + \frac{w}{9}$	$\frac{m}{3} + \frac{2m}{7}$
$\frac{3n}{4} - \frac{5n}{9}$	$\frac{7h}{8} - \frac{5h}{12}$
$\frac{2}{fg} - \frac{4}{g}$	$\frac{m^2}{6} - \frac{9}{4m}$
$\frac{3}{4b^2} - \frac{1}{2b}$	$\frac{2}{ab^2} - \frac{3}{b^3}$
$\frac{5x-4}{2} + \frac{x+1}{3}$	$\frac{9x+1}{5} - \frac{x+2}{2}$
$\frac{4x-7}{10} - \frac{2x-9}{3}$	$\frac{3x+11}{2} + \frac{x-20}{3}$
$\frac{4}{x+5} - \frac{2}{x-1}$	$\frac{2}{x+1} + \frac{1}{x+3}$
$\frac{x}{x+7} + \frac{2x+5}{3x+1}$	$\frac{x+1}{x-2} + \frac{x+3}{x+5}$

$$\frac{x+5}{3} + \frac{x+1}{2} = 8$$

$$\frac{x+5}{10} - \frac{x+4}{2} = 1$$

$$\frac{2}{x-3} + \frac{1}{x-4} = 2$$

$$\frac{2}{x-5} - \frac{2}{x-4} = 1$$

$$\frac{x+8}{15} \times \frac{10}{x^3+8x^2}$$

$$\frac{x^2+5x+6}{4} \times \frac{2}{x+2}$$

$$\frac{x^2+2x-8}{x^2+5x+6} \times \frac{x+2}{x+4}$$

$$\frac{3x^2+8x-3}{25} \times \frac{30}{6x^2+13x-5}$$

Probability

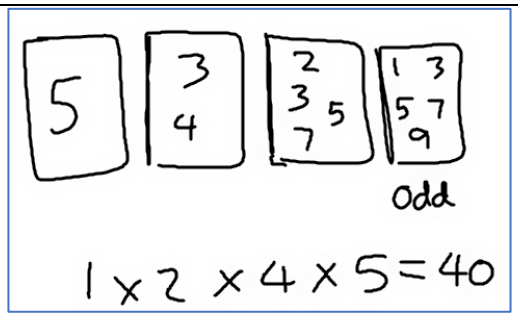
- Probability is out of 1
- Relative frequency is written as FDP
- AND means multiply
- OR means add

Example- product rule for counting

Rosie picks a 4-digit **odd** number.

The first digit is 5.
The second digit is a 3 or a 4.
The third digit is prime.

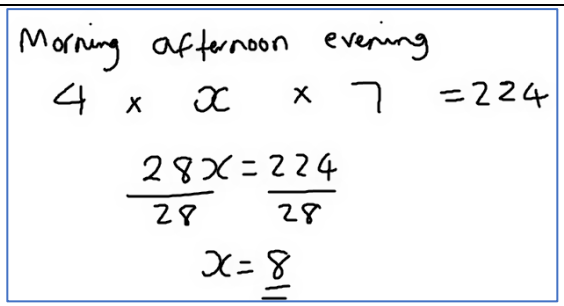
How many different 4-digit numbers could Rosie pick?



At a summer camp, children pick a morning, an afternoon and an evening activity.
There are 4 morning and 7 evening activities to pick from.

Altogether there are 224 different ways to choose their activities.

How many afternoon activities are there?



Questions

A restaurant has 4 starters and 6 main course on its menu.
Hailey orders a starter and a main course.

How many different combinations of starters and main courses are there?

Harry picks a 4 digit pin for his credit card.
Each digit is a number 0 to 9.
Harry can repeat digits.

(a) How many possible codes are there?

Harry chooses not to repeat any digits.

(b) How many possible codes are there now?

When a spinner is spun, it shows

Blue (B) or Green (G) or Red (R) or White (W).

When a coin is tossed, it shows

Heads (H) or Tails (T).

The spinner is spun and the coin is tossed.

Complete this list of possible outcomes.

BH

A vending machine has a different item in each section.

It sells

7 drinks, 3 of which are juice

5 snacks, 2 of which are fruit bars

11 meals, 4 of which are salad.

One drink, one snack and one meal are chosen at random.

Show that the probability of getting a juice, a fruit bar and a salad is **more** than 5%

In a video game, players make their own character.

They choose one of each from

8 faces

4 bodies

5 hairstyles.

(a) How many different characters can be made?

Liam is trying to remember a 3-digit code.

He knows the rule that

the first digit is a cube number

the second digit is a factor of 16

the third digit is an odd number.

Liam tries at random a code that matches the rule.

Work out the probability that this is the correct code.

Example- Relative frequency

Here are the results after 250 spins of a coin.

Heads	128
Tails	122

The coin is spun an extra 50 times.

After all 300 spins, the relative frequency of Heads is 0.49

For the **extra 50 spins**, work out number of Heads : number of Tails

Heads

New Relative Frequency $= 0.49$

$300 \times 0.49 = 147$ ← after 300 spins

128 ← after 250 spins

$147 - 128 = 19$ ← extra 50 spins

Head $= 19$

Tails $= 50 - 19 = 31$

Today, she made 120 calls.

The table shows the results.

Result of call	Frequency
Not answered	33
Answered but sale not made	81
Answered and sale made	6

(a) Write down the relative frequency that a call was **not answered**.

Answer _____

(1)

a) Relative frequency $=$ F D P

$\frac{33}{33+81+6} = \frac{33}{120}$

b) Relative Frequency of sales made $= \frac{6}{120}$

" Probability X number of trials "

$\frac{6}{120} \times 500 = 25$

(b) During the **rest of the week**, Rosie will make 500 calls.

Using the results in the table, how many sales does she expect to make during the **rest of the week**?

Adam and Bianca each throw the same biased coin.

Here is some information about their throws.

	Number of throws	Number of Heads
Adam	40	14
Bianca	60	20

Bianca says,

"My results give a better estimate of the probability of Heads than Adam's results."

Is she correct?

Tick a box.

Yes ☐ No ☐

Give a reason for your answer.

More trials = More reliable result

Adam $= \frac{14}{40}$

Bianca $= \frac{20}{60}$ ← more trials = better estimate.

Yes, she is correct.

Questions

A coin is thrown 50 times.
The coin lands on heads 30 times.

Write down the relative frequency of landing on heads.

Total number of trains	50	100	150	200	250
Total number of late trains	16	21	36	38	55
Relative frequency of late trains	0.32	0.21			

(a) Complete the relative frequency graph.

A spinner can land on red, blue or green.

After 350 spins

relative frequency of red = 0.18

relative frequency of blue = 0.62

Work out the number of times the spinner landed on green.

A biased coin is thrown 250 times.

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

0.3 0.32 0.342 0.4

A factory makes kettles.

Four samples of kettles are tested for faults.

Each sample has size 200

Here are the relative frequencies of faulty kettles in the samples.

Sample	P	Q	R	S
Relative frequency	0.03	0.035	0.015	0.01

Work out the range of the number of faulty kettles in the four samples.

Circle the relative frequency that represents 13 successes out of 50 trials.

0.13 26 13 : 50 0.26

On three days, Ali throws darts at a target.

Here are his results.

	Number of throws	**Number of hits**	**Number of misses**
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Total	90	54	36

(a) Work out **two** different estimates for the probability of Ali hitting the target.

Bag A contains 20 green balls and 12 yellow balls.

Bag B contains 15 green balls and 9 yellow balls.

John says,

"It's more likely that a green ball is chosen from Bag A than Bag B because there are more green balls in Bag A than Bag B."

Is he correct?

Yes ☐ No ☐

Give a reason for your answer.

In a bag there are only red discs, blue discs and green discs.

There are 10 red discs.

When one disc is picked at random

$$P(\text{red}) = \frac{1}{8}$$

$$P(\text{blue}) = \frac{2}{5}$$

How many **green** discs are in the bag?

The probability that a biased coin lands on heads is $\frac{2}{5}$

The coin is spun twice.

Work out the probability of two heads.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

(a) Work out the probability that the counter taken is yellow

There are 40 counters in the bag.

(b) Work out the number of blue counters in the bag.

The table shows information about some CDs.

Type	Rock	Pop	Jazz
Number of CDs	2	x	$2x + 5$

A CD is chosen at random.

The probability it is **rock** is $\frac{1}{20}$

Work out the probability it is jazz.

A biased spinner can land on A, B or C.

The table shows the probabilities, in terms of k, of A, B and C.

	A	B	C
Probability	$0.5k$	$7k - 0.15$	$2.5k$

Work out the probability of B.

Examples- Sample Space

John chooses a number at random from the digits 1 to 4

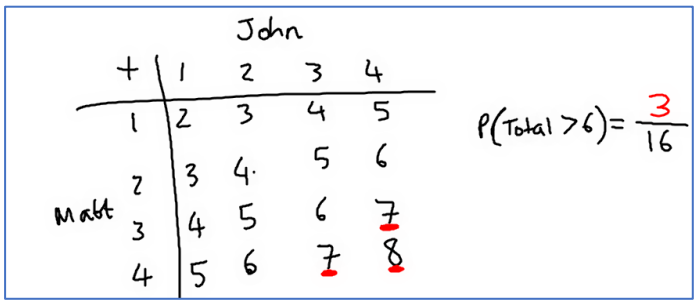
Matt also chooses a number at random from the digits 1 to 4

(a) Write down the probability that the **total** of the two numbers chosen is 10

Answer _____

(1)

(b) Work out the probability that the **total** of the two numbers chosen is more than 6



Questions

A number is picked at random from the first four **prime** numbers.

A number is picked at random from the first four **square** numbers.

The two numbers are added to get a score.

(a) Complete the table.

		Square numbers			
	+	**1**	**4**	**9**	
Prime numbers	**2**				
	3			12	
	7				

(b) What is the probability that the score is a **prime** number?

Examples- Independent events

A fair coin is flipped twice.

(a) Find the probability that the coin lands on heads twice.
(b) Find the probability that the coin lands on tails twice.
(c) Find the probability that the coin lands on heads exactly once.

a) P(Heads) and P(Heads)
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b) P(Tails) and P(Tails)
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

c) $P(H) \times P(T) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
$P(T) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
This or that $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

Penelope is playing football.
When attempting to score a penalty, the probability she scores is $\frac{2}{3}$
During the game, Penelope takes two penalties.
Find the probability that Penelope scores both.

P(Score) × P(Score)
$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Questions

The probability that a biased coin lands on heads is $\frac{2}{3}$

The coin is spun twice.

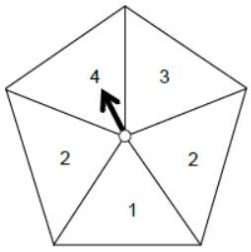
Circle the probability of two heads.

$\frac{2}{9}$ $\frac{4}{6}$ $\frac{4}{9}$ $\frac{4}{3}$

Trevor is taking part in a quiz.
The probability that he answer a question correctly is $\frac{3}{5}$
Trevor is asked two questions.

(a) Calculate the probability that Trevor answers both questions correctly.
(b) Calculate the probability that Trevor answers both questions incorrectly.

Here is a fair five sided spinner.



(a) Write down the probability of scoring an even number with one spin.

Answer _____

(1)

(b) Work out the probability of scoring a **total** of 8 with two spins.

Jackson, Frederick and Kelvin each sit a test.

The probability Jackson passes is $\frac{9}{10}$

The probability Frederick passes is $\frac{2}{3}$

The probability Kelvin passes is $\frac{1}{2}$

(a) Find the probability that Jackson and Kelvin pass, but Fredrick fails.

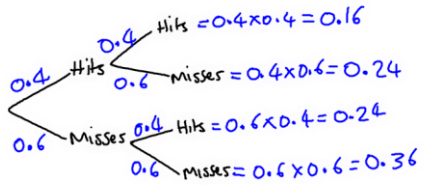
(b) Find the probability that Frederick passes, but Jackson and Kelvin fail.

(c) Find the probability that at least two boys pass.

Examples- Independent events

Mark is playing darts.
The probability he hits the bullseye is 0.4
Mark throws two darts

(a) Find the probability of Mark hitting the bullseye once.
(b) Find the probability of Mark hitting the bullseye at least once.



a) P(Hits only once) = P(H)×P(M) + P(M)×P(H)
0.24 + 0.24
= 0.48

b) P(Hit at least once) = P(H)×P(H) + P(H)×P(M) + P(M)×P(H)
0.16 + 0.24 + 0.24
= 0.64

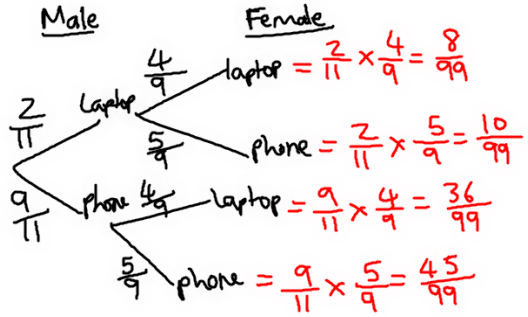
20 people were asked which device they used more often, laptop or phone.

The table shows the results.

	Laptop	Phone
Male	2	9
Female	4	5

(a) One male and one female are chosen at random.

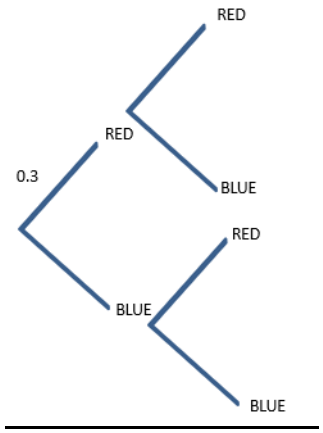
Work out the probability that **exactly** one of them said laptop.

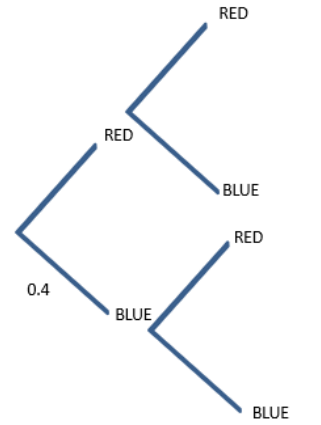


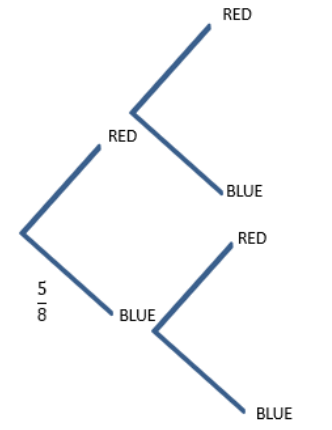
P(exactly one said laptop)
= P(L) × P(P) = 10/99
+ P(P) × P(L) = 36/99
} add

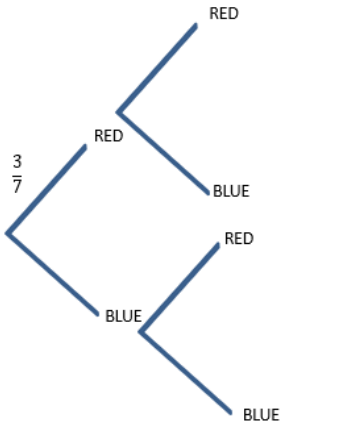
= 10/99 + 36/99 = 46/99

Questions: Complete these tree diagrams







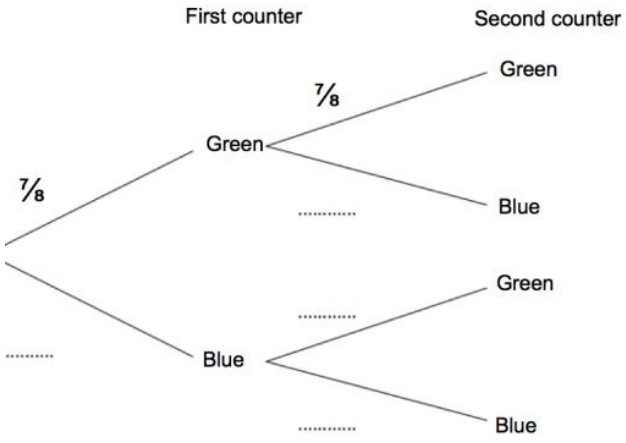


There are green and blue counters in a container.

Kevin takes at random a counter from the container.
He replaces the counter in the container.

Kevin takes at random a second counter from the container.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Kevin picks counters that are different colour.

Jennifer is playing darts.
She throws two darts aiming for a Bullseye.

The probability Jennifer hits the Bullseye on her first throw is ¼.
The probability she hits the Bullseye on her second throw ⅓.

(a) Complete the tree diagram.

Sally and Laura sit their driving tests.

The probability of Sally passing her driving test is 0.7
The probability of both Sally and Laura passing is 0.56

(a) Work out the probability of Laura passing her driving test.

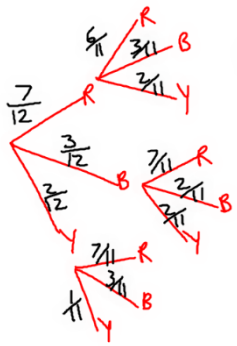
Examples – Conditional probability

A bag contains 12 discs.

7 are red
3 are blue
2 are yellow.

Two discs are taken from the bag at random, without replacement.

Work out the probability that the two discs are the same colour.



P(Two discs are same colour)

$P(R) \times P(R) = \frac{7}{12} \times \frac{6}{11} = \frac{42}{132}$

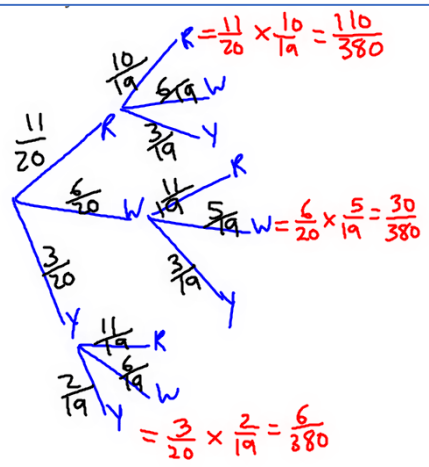
$P(B) \times P(B) = \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$

$P(Y) \times P(Y) = \frac{2}{12} \times \frac{1}{11} = \frac{2}{132}$

$\frac{42}{132} + \frac{6}{132} + \frac{6}{132} = \frac{54}{132}$

Toby has 20 counters in a bag.
11 counters are yellow.
6 counters are red.
3 counters are white.
Toby takes two counters from the bag at random.

Work out the probability that the two counters are **not** the same colour.



$R = \frac{11}{20} \times \frac{10}{19} = \frac{110}{380}$

$W = \frac{6}{20} \times \frac{5}{19} = \frac{30}{380}$

$= \frac{3}{20} \times \frac{2}{19} = \frac{6}{380}$

P(2 not the same colour)

$1 - P(\text{same colour})$

$1 - P(R) \times P(R) + P(W) \times P(W) + P(Y) \times P(Y)$

$1 - \frac{110}{380} + \frac{30}{380} + \frac{6}{380}$

$1 - \frac{146}{380}$

$\frac{380}{380} - \frac{146}{380} = \frac{234}{380}$

Questions

There are 12 sweets in a bag
7 are lemon and 5 are orange.
Two sweets are taken out at random **without** replacement.

(a) Work out the probability that the two sweets are both lemon
(b) Work out the probability that the two sweets are both orange
(c) Work out the probability that the two sweets are the same flavour
(d) Work out the probability that the two sweets are different flavours

There are three flavours of crisps in a cafe.
There are 3 packets of salt and vinegar
5 packets of cheese and onion
1 packet of roast chicken
Bella takes two packets of crisps at random.

(a) Work out the probability that she takes 2 packets of crisps that are the **same** flavour.
(b) Work out the probability that she takes 2 packets of crisps that are **different** flavours.

Here are six number tiles.

Charlie takes a tile at random without replacement.
Charlie then takes a second tile at random.

(a) Work out the probability the both tiles have the number 5 on them

(b) Work out the probability that the number on the second is less than the number on the first tile.

In dry weather, the probability of a bus being late is $\frac{1}{10}$

In rainy weather, the probability of a bus being late is $\frac{1}{4}$

In snowy weather, the probability of a bus being late is $\frac{2}{3}$

The probability of dry weather is $\frac{3}{4}$

The probability of wet weather is $\frac{1}{5}$

The probability of snow is $\frac{1}{20}$

(a) Show this information on a tree diagram
(b) Calculate the probability that the weather is dry **and** the bus is on time.
(c) Calculate the probability that the bus is late

There are 5 red pens, 3 blue pens and 2 green pens in a box.

Gary takes at random a pen from the box and gives the pen to his friend.
Gary then takes at random another pen from the box.

Work out the probability that both pens are the same colour.

Carolyn has 20 biscuits in a tin.

She has

12 plain biscuits
5 chocolate biscuits
3 ginger biscuits

Carolyn takes at random two biscuits from the tin.

Work out the probability that the two biscuits were **not** the same type.

There are three different types of sandwiches on a shelf.

There are

4 egg sandwiches,
5 cheese sandwiches
and 2 ham sandwiches.

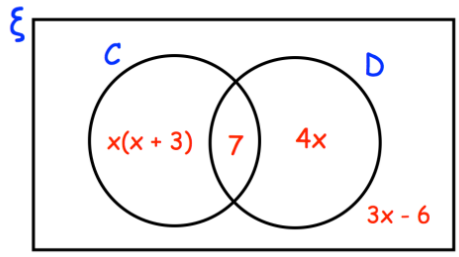
Erin takes at random 2 of these sandwiches.

Work out the probability that she takes 2 different types of sandwiches.

Examples – Venn diagram

The Venn diagram shows information about the pets owned by 40 students

ξ = 40 students
C = students who own a cat
D = students who own a dog



$x(x+3)+7+4x+3x-6=40$
$x^2+3x+7+4x+3x-6=40$
$x^2+10x+1=40$
$-40\ \ -40$
$x^2+10x-39=0$
$(x+13)(x-3)=0$
$x=-13$ or $x=3$

Can't have negative number of people.

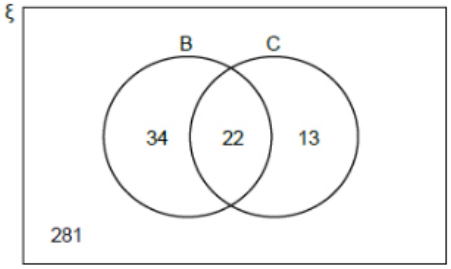
$P(\text{own dog})=\frac{19}{40}$

$7+4x$
$7+4x^3$
19
dog circle

A student is chosen at random.
They own a cat.
Work out the probability that they own a dog.

In the Venn diagram

ξ = 350 students in a college
B = students who take Biology
C = students who take Chemistry



(a) One student is chosen at random.

Work out the probability that the student takes Biology.

_____

Answer _____

(b) One student who takes Chemistry is chosen at random.

Work out the probability that the student **also** takes Biology.

a) $34+22=56$ biology circle

$P(\text{Biology})=\frac{56}{350}$

b) Total = chemistry

$22+13=35$

Biology

22 13

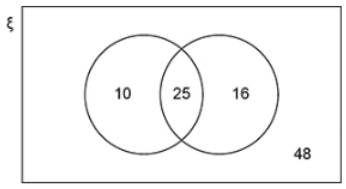
$P(\text{also takes biology})=\frac{22}{35}$

In a group of 98 students

25 study both Art and French
10 study Art but do not study French
41 study French.

Joel draws this Venn diagram to represent the information.

ξ = the group of 98 students
A = the students who study Art
F = the students who study French



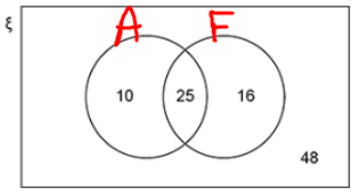
Make **two** criticisms of his diagram.

Criticism 1 _____

_____

Criticism 2 _____

_____



Make **two** criticisms of his diagram.

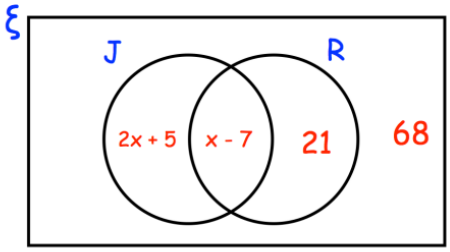
Criticism 1 48 should be 47.
because $10+25+16=51$, $98-51=47$.

Criticism 2 Venn diagram is not labelled with Art and French.

Questions– Venn diagram

The Venn diagram shows information about the cars in a car park.

ξ = 150 cars in the car park
R = red cars
J = cars manufactured in Japan



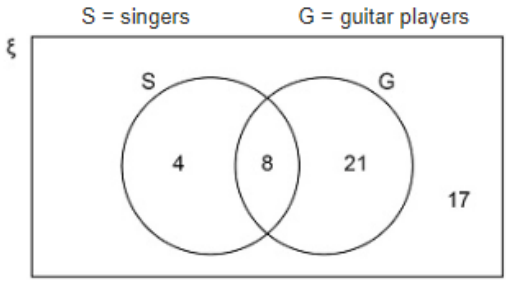
A car is chosen at random.
Work out the probability that it is red.

A group of friends have been surveyed.

38% have been to Canada.
80% have been to France.
11% have been to neither Canada or France.

Find the percentage of the group that have been to Canada and France.

The Venn diagram shows information about 50 people who are in bands.



(a) How many of the people are guitar players?

Answer _____

(b) How many of the people are singers but **not** guitar players?

Answer _____

(c) One of the people is chosen at random.

Write down the probability that the person is

not a singer

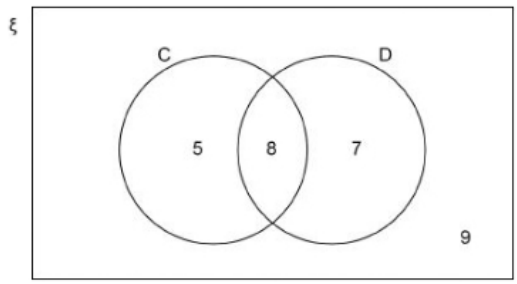
and

not a guitar player.

ξ = 29 students in a class

C = students who own a cat

D = students who own a dog



(a) A student is chosen at random.

Circle the probability that the student owns a cat or a dog but not both.

$\frac{12}{29}$ $\frac{13}{29}$ $\frac{15}{29}$ $\frac{20}{29}$

(b) A student who owns a dog is chosen at random.

Circle the probability that the student also owns a cat.

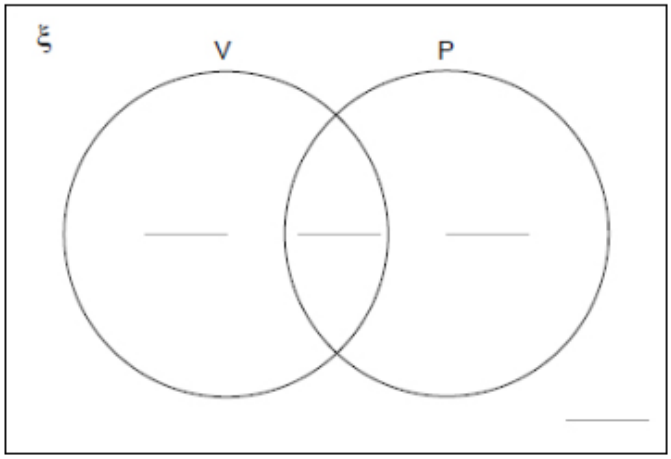
$\frac{7}{15}$ $\frac{8}{15}$ $\frac{7}{29}$ $\frac{8}{29}$

In the Venn diagram

ξ = 75 students in a Year Group

V = students that play the violin

P = students that play the piano



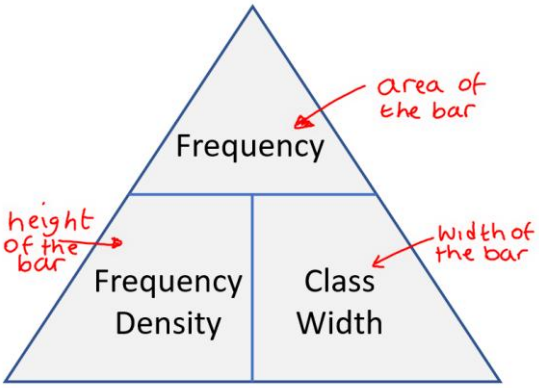
54 students play **only** the violin or **only** the piano.

$\frac{5}{6}$ of these 54 students play **only** the violin.

The number of students that play the violin is four times the number that play the piano.

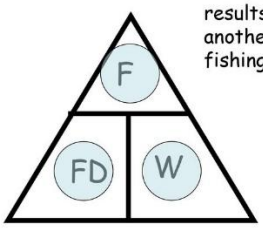
Complete the Venn diagram.

Histograms



Frequency = Freq. Density x Class Width

- Freq. Density = Frequency ÷ Width
- Width = Frequency ÷ Freq. Density



Here are the results for another angler's fishing catch

Time	Fish
0 ≤ t < 2	8
2 ≤ t < 6	4
6 ≤ t < 7	10

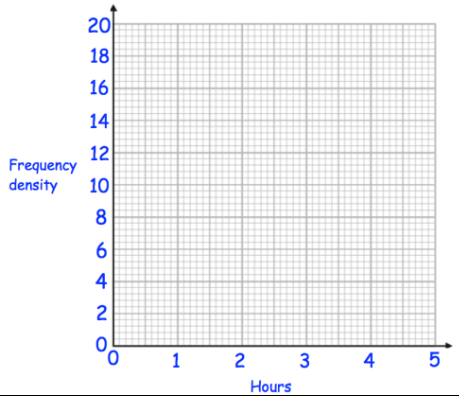
Calculate the frequency densities for the data in this table and draw a histogram for the data

Question

2. The waiting times, *h* hours, for 40 patients at an accident and emergency department in one evening is shown below.

Waiting time, h	Frequency
0 < h ≤ 0.5	8
0.5 < h ≤ 1	10
1 < h ≤ 1.5	7
1.5 < h ≤ 3	9
3 < h ≤ 5	6

Draw a histogram for this data.



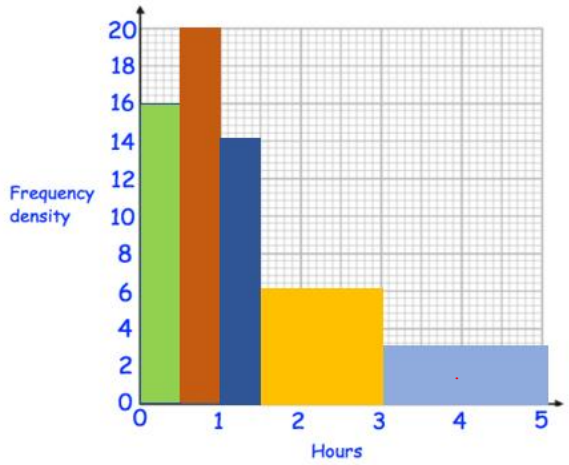
Answer

2. The waiting times, *h* hours, for 40 patients at an accident and emergency department in one evening is shown below.

Waiting time, h	Frequency	Class width	Frequency density
0 < h ≤ 0.5	8	0.5	8 ÷ 0.5 = 16
0.5 < h ≤ 1	10	0.5	10 ÷ 0.5 = 20
1 < h ≤ 1.5	7	0.5	7 ÷ 0.5 = 14
1.5 < h ≤ 3	9	1.5	9 ÷ 1.5 = 6
3 < h ≤ 5	6	2	6 ÷ 2 = 3

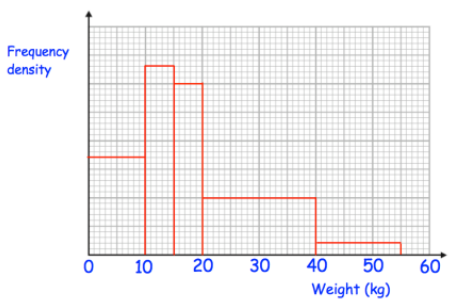
width

Draw a histogram for this data.



Question

12. The incomplete table and histogram give some information about the weights of dogs.

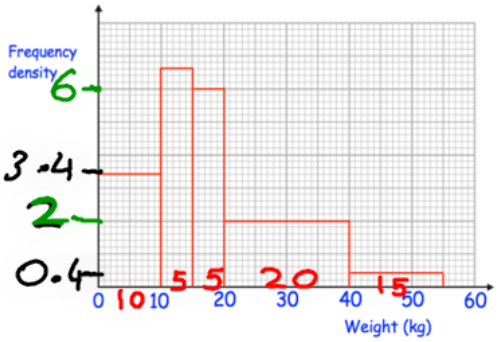


Use the information in the histogram to complete the frequency table.

weight (w kg)	Frequency
0 < w ≤ 10	34
10 < w ≤ 15	33
15 < w ≤ 20	
20 < w ≤ 40	
40 < w ≤ 55	6

Answer

12. The incomplete table and histogram give some information about the weights of dogs.



Use the information in the histogram to complete the frequency table.

weight (w kg)	Frequency	CW	FD
0 < w ≤ 10	34	10	34 ÷ 10 = 3.4
10 < w ≤ 15	33	5	33 ÷ 5 = 6
15 < w ≤ 20	5 x 6 = 30	5	2
20 < w ≤ 40	20 x 2 = 40	20	
40 < w ≤ 55	6	15	6 ÷ 15 = 0.4

class width

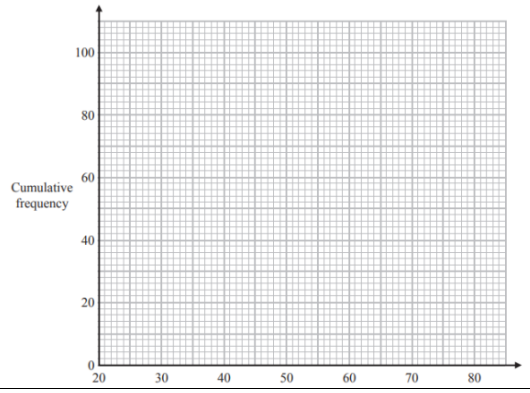
Cumulative Frequency & Box-Plots

Question

The frequency table shows the time taken for 100 people to travel to an event.

Time (minutes)	Frequency
$20 < t \leqslant 30$	9
$30 < t \leqslant 40$	16
$40 < t \leqslant 50$	20
$50 < t \leqslant 60$	29
$60 < t \leqslant 70$	15
$70 < t \leqslant 80$	11

(a) On the grid, plot a cumulative frequency graph for this information.

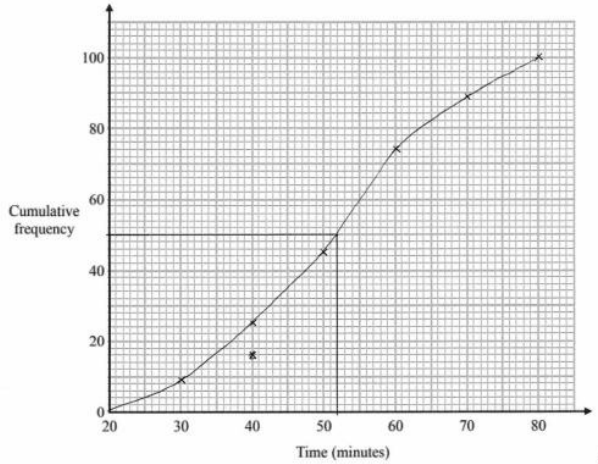


Answer

The frequency table shows the time taken for 100 people to travel to an event.

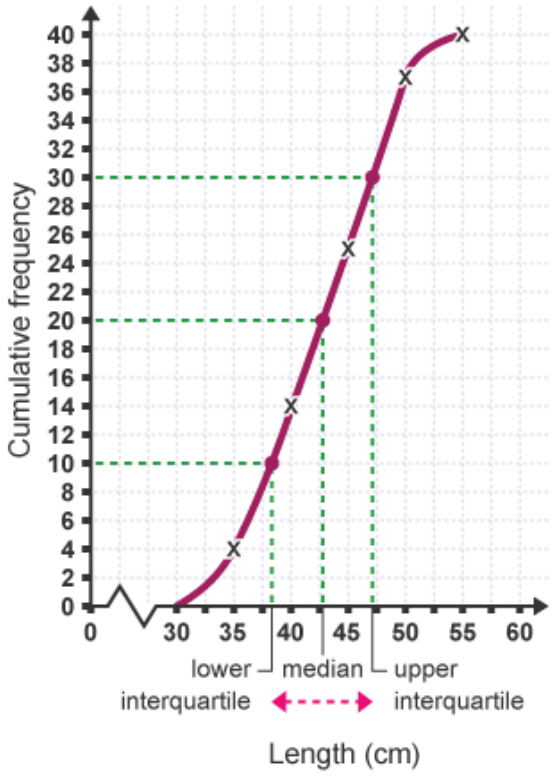
Time (minutes)	Frequency	Cumulative Frequency
$20 < t \leqslant 30$	9	9
$30 < t \leqslant 40$	16	9+16=25
$40 < t \leqslant 50$	20	25+20=45
$50 < t \leqslant 60$	29	45+29=74
$60 < t \leqslant 70$	15	74+15=89
$70 < t \leqslant 80$	11	89+11=100

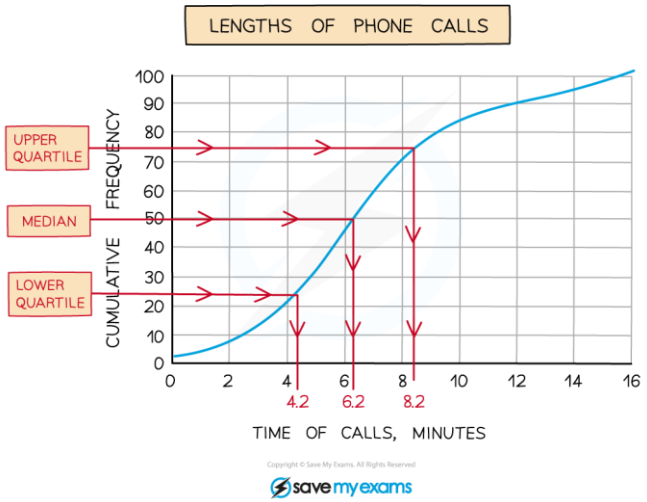
(a) On the grid, plot a cumulative frequency graph for this information.



(2)

Examples





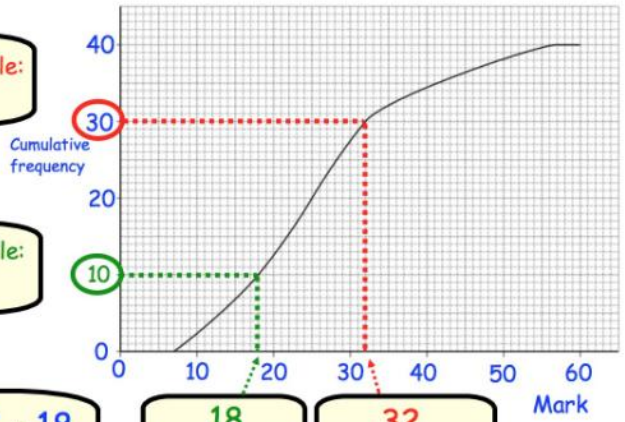
Corbett maths

Interquartile Range from a Cumulative Frequency Curve

Interquartile Range = Upper Quartile - Lower Quartile

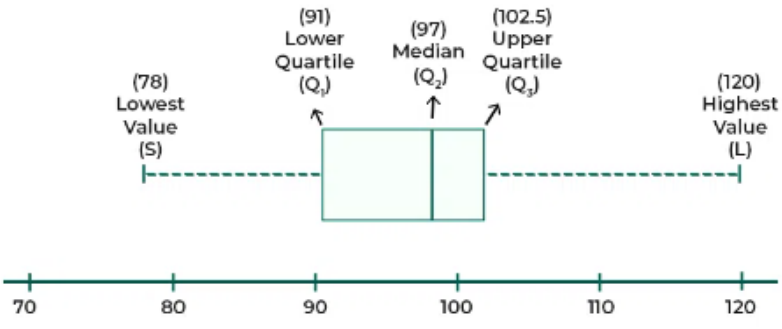
Position of the Upper Quartile:
¾ of 40 = 30th value

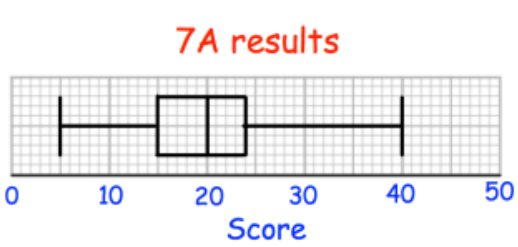
Position of the Lower Quartile:
¼ of 40 = 10th value

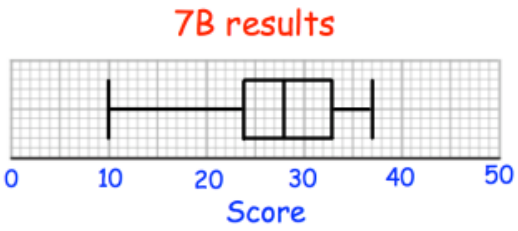


Interquartile Range = 32 - 18
= 14

Box plots







Comapre the results

- The median for class 7B (28) is higher than median for class 7A (20). This means on average class 7B had higher score.
- The IQR class 7A is 9 [24-15=9). The IQR for 7B is also 9 [33-24=9]. Lower IQR means more consistent result so this means both classes are equally consistent.

Question

Calculate the mean.

Length (x cm)	Frequency
$10 < x \leq 20$	17
$20 < x \leq 30$	26
$30 < x \leq 40$	11
$40 < x \leq 50$	6

Answer

Length (x cm)	Frequency	Midpoint	FX
$10 < x \leq 20$	17	15	17x15=255
$20 < x \leq 30$	26	25	26x25=650
$30 < x \leq 40$	11	35	11x35=385
$40 < x \leq 50$	6	45	6x45=270
	Total = 60		Total = 1560

$$\text{Mean} = \frac{1560}{60} = 26$$

Complete the table:

Score	Freq.	CW	FD
3≤s<5	10		
5≤s<8	6		
8≤s<13	15		

Complete the table:

Score	Freq.	CW	FD
3≤s<6	15		
6≤s<10			3
10≤s<16	12		

Construct a cumulative frequency table from:

Score	Frequency
3≤s<6	2
6≤s<9	5
9≤s<12	2

Construct a cumulative frequency table from:

Score	Frequency
3≤s<5	2
5≤s<7	5
7≤s<9	1

Calculate an estimate of the mean score to 1dp:

Score	Frequency
3≤s<6	2
6≤s<9	3
9≤s<12	5

Calculate an estimate of the mean score to 1dp:

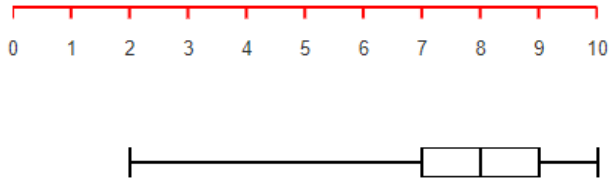
Score	Frequency
3≤s<5	2
5≤s<7	3
7≤s<9	5

Estimate the median from:

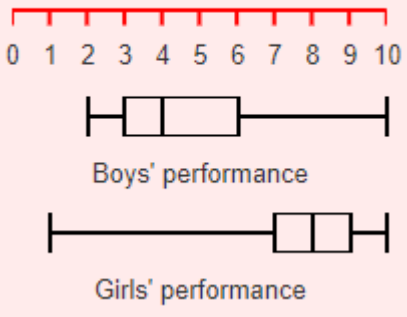
Estimate the interquartile range from:

Looking at the box plot below,

Find the median and IQR



Make 2 comparisons:



Percentages

Non-calc	Calc
35 % of £180	35 % of £180
35% = 10% x 3 + 5% 10% = £18 x3 ↓ 30% = £54 ↓ x3 ÷2 → 5% = £9 ← ÷2 35% = 30% + 5% £63 = £54 + £9	÷100 ↓ 35% as a multiplier is 0.35 0.35 x 180 = £63

Non-calc	Calc
Increase £120 by 3%	Increase £120 by 3%
x3 ↓ 1% = £1.20 3% = £3.60 ↓ x3 Increase means add. £120 + £3.60 = £123.60	Increase ↑ 100% + 3% = 103% ÷100 ↓ 103% as a multiplier is 1.03 1.03 x 120 = £123.60

Peter's weight decreases from 80kg to 64kg.

Calculate the percentage decrease in Peter's weight.

$$\frac{80-64}{80} \times 100$$

$$\%\text{ change} = \frac{\text{change}}{\text{original}} \times 100 \qquad = \frac{16}{80} \times 100 = 20\%$$

Jacob buys a watch costing £84
This cost includes VAT at a rate of 20%.

How much is the watch without VAT?

VAT means Value added tax.
+

100% + 20% = 120%

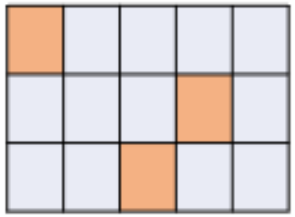
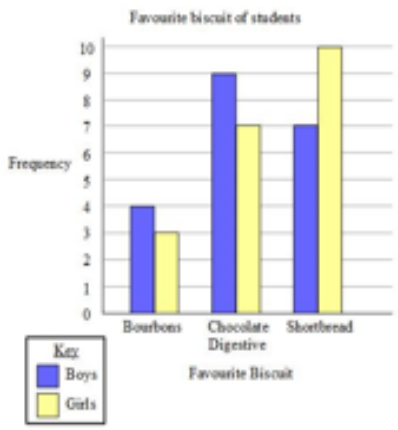
÷120 ↓ 120% = £84
÷120 ↓ 1% = 0.7
x100 ↓ 100% = £70
÷120 ↓ x100

Matt wants to invest £8000 for three years. He can choose between Bank A and Bank B.

Bank A	**Bank B**
1.2% compound interest per annum	2% compound interest in the first year 1% compound interest for each extra year

Which bank will give Matt the most interest after three years.
You must show your working.

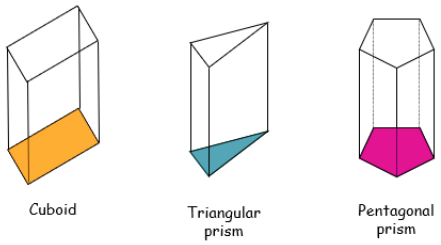
Bank A	**Bank B** most interest
Amount x Multiplier^{Time} Amount = 8000 Multiplier = 100% + 1.2% = 101.2% = 1.012 Time = 3 years $= 8000 \times 1.012^{3} =$ £8291.47	Amount x Multiplier^{Time} x Multiplier^{Time} Amount = 8000 Multiplier = 1.02 ← For 1 year. Multiplier = 1.01 ← For 2 years. Time = Total of 3 years. $8000 \times 1.02^{1} \times 1.01^{2} =$ = £8324.02

Percentages		
A (2 marks)	B (3marks)	C (3marks)
Increase £45 by 3.5%	A bank account pays 5% simple interest. Andrea invests £2400 for 6 years. How much interest does she earn.	The value of a car decreases by 16% in a year. It is now worth £8648. How much was it worth last year?
Decrease £45 by 3.5%	The value of a car decreases from £12,500 to £11,750. Work out the percentage reduction.	The price of a train ticket has increased by 9% to £49.05. How much was it originally?
Find 3.5% of £4	The average price of a holiday has increased from £1800 in 2002 to £2400 in 2018. Work out the percentage increase.	A bank account pays 5% compound interest. Andrea invests £2400. Assuming no withdrawals, what is the balance after 6 years?
What percentage is shaded? 	What percentage of pupils said chocolate digestives were their favourite? 	Craig does two science exams. He scores 40 out of 60 on paper 1. He scores 50 out of 80 on paper 2. Work out his overall percentage.

Volume of 3D shapes

Volume of prisms = area of cross-section X length

A prism is a 3D shape with uniform cross section.



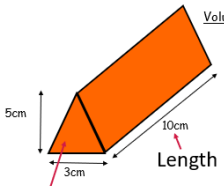
Cuboid | Triangular prism | Pentagonal prism

What is the link between Faces , Edges and Vertices in the **Polyhedra**?

Shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Tetrahedron	4	6	4
Square-based Pyramid	5	8	5
Pentagonal-based Pyramid	6	10	6
Triangular Prism	5	9	6
Hexagonal Prism	8	18	12

Volume of a Prism

Cross-sectional area (Slice) x length



$7.5cm^2$ X $10cm$

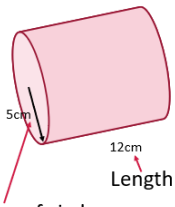
$= 75cm^3$

Area of triangle=$\frac{3X5}{2}$

$=7.5cm^2$

Volume of a Prism

Cross-sectional area (Slice) x length



25π X 12

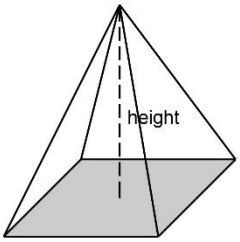
$= 300\pi$

$=394cm^3$

Area of circle =

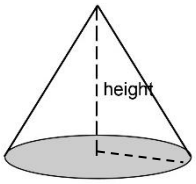
$\pi X 5^2 = 25\pi$

Volume of non - prisms



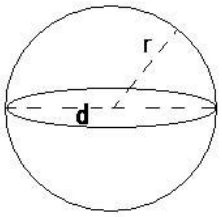
$$V = \frac{1}{3}Bh$$

Note The height must be the **perpendicular height** from the base.



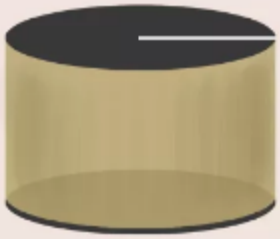
$$V = \frac{1}{3}\pi r^2 h$$

Note The height must be the **perpendicular height** from the base.



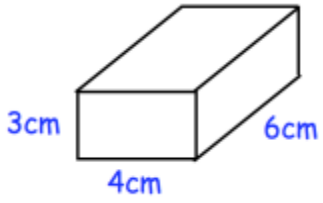
$$V = 4\pi r^2$$

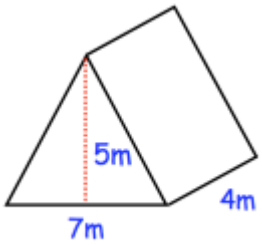
Cylinder

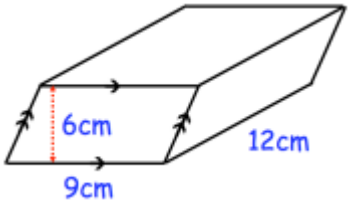


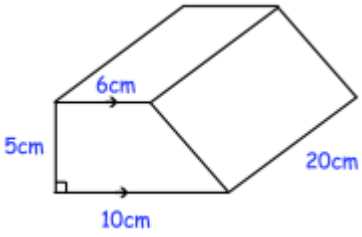
$v = \pi r^2 h$

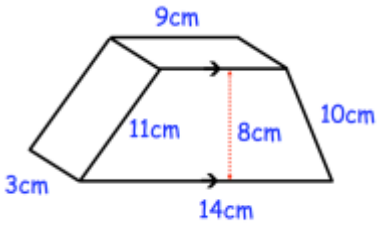
Find the volume of these shapes

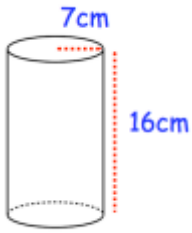


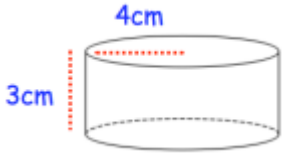


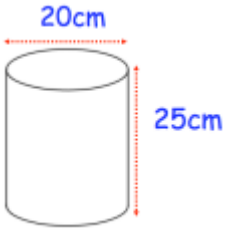


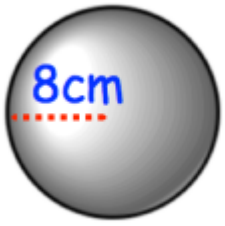


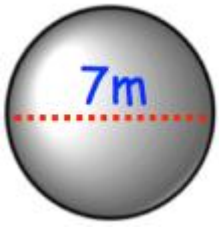


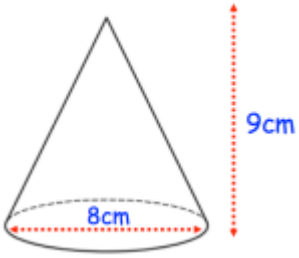


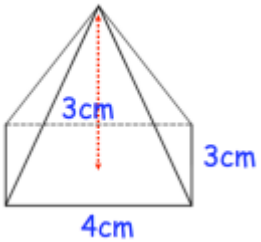










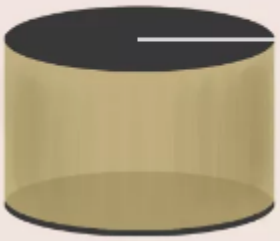


Surafce area of 3D shapes

Sphere

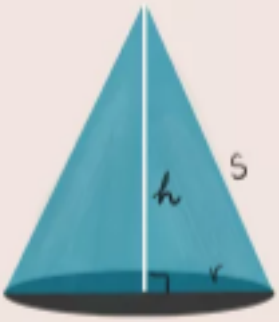
$SA = 4\pi r^2$

Cylinder



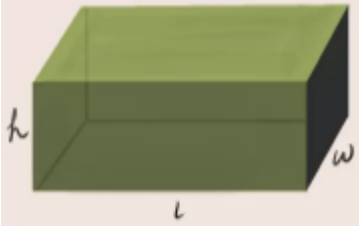
$SA = 2\pi r^2 + 2\pi rh$

Cone



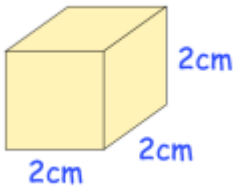
$SA = \pi rs + \pi r^2$

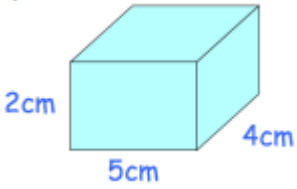
Rectangular Prism

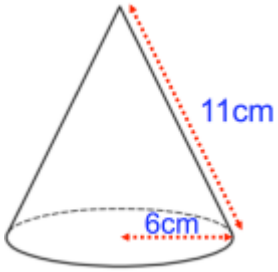


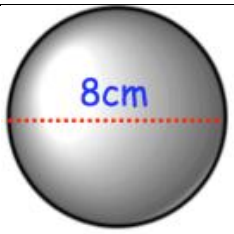
$SA = 2(lw+lh+wh)$

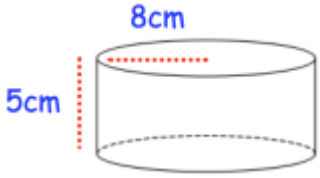
Find the surface area of these shapes











Similar Shapes

Linear scale factor = x

Area scale factor = x^2

Volume scale factor = x^3

Linear scale factor = $\frac{3}{4}$

Area scale factor = $(\frac{3}{4})^2 = \frac{9}{16}$

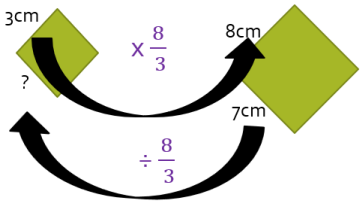
Volume scale factor = $(\frac{3}{4})^3 = \frac{27}{64}$

Linear scale factor = 3

Area scale factor = $3^2 = 9$

Volume scale factor = $3^3 = 27$

Shape A and B are mathematically similar.
Calculate the missing length

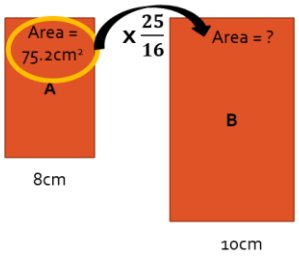


Linear scale factor = $\frac{8}{3}$

$7 \div \frac{8}{3} = \frac{21}{8}$

$= 2.625cm$

Shape A and B are mathematically similar.
Calculate the area of shape B.

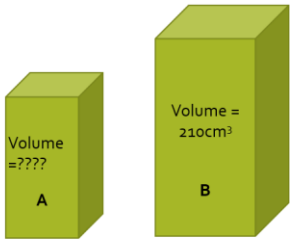


Linear scale factor = $\frac{10}{8} = \frac{5}{4}$

Area scale factor = $(\frac{5}{4})^2 = \frac{25}{16}$

$75.2 \times \frac{25}{16}$ $=117.5cm$

Shape A and B are mathematically similar.
Calculate the volume of shape B.

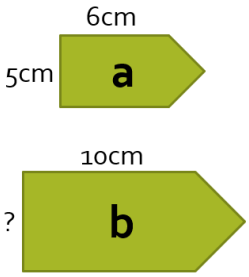


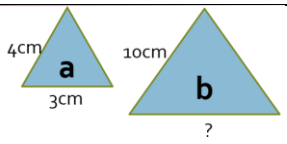
Linear scale factor = $\frac{5}{3}$

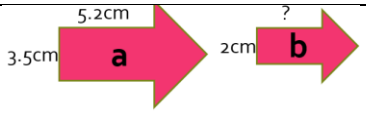
Volume scale factor = $(\frac{5}{3})^3 = \frac{125}{27}$

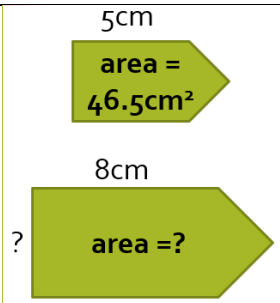
$210 \div \frac{125}{27} = 45.36cm^3$

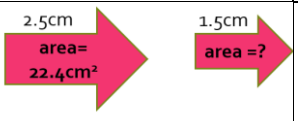
Questions – Similar Shapes!

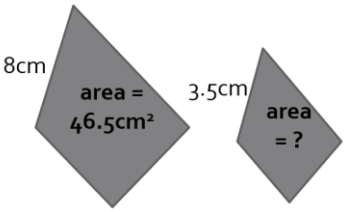


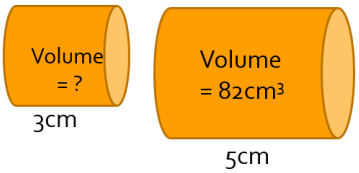


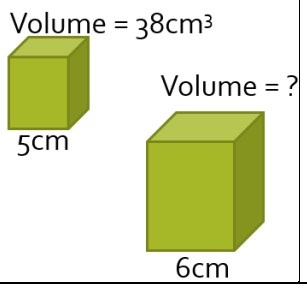


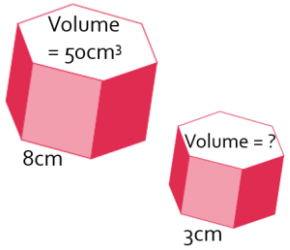












Triple Brackets, Equating Coefficients & Rearranging

Example: **(x-6)(x-8)(x-9)**

Expand (x-6)(x-8)(x-9) — Focus on (x-8)(x-9)

(x-8)(x-9) = $(x^2-17x+72)$ — Substitute

$(x+5)(x^2-17x+72)$

$(x-6)(x^2-17x+72) = x(x^2-17x+72)-6(x^2-17x+72)$

$= x^3-17x^2+72x - 6x^2+102x-432$

$= x^3-23x^2+174x-432$

Be Careful with the minus signs!

$(x-6)(x-8)(x-9) = x^3-23x^2+174x-432$

Expand Example 4: **(x-4)(x+7)(x-7)**

$(x-4)(x^2-49)$ — Substitute — (x^2-49)

$(x-4)(x^2-49) = x(x^2-49)-4(x^2-49)$

$= x^3-49x - 4x^2+196$

$= x^3-4x^2-49x+196$

Will still give you a cubic expression

$(x-4)(x+7)(x-7) = x^3-4x^2-49x+196$

Questions – Expand triple brackets

Expand and simplify

$(x+1)(x+3)(x-4)$

Expand and simplify

$(x+5)^2(x+3)$

Expand and simplify

$(3x-1)(x+3)(x-3)$

Expand and simplify

$(2x-3)^2(4-x)$

Equating Coefficients

$5(2x+a) \equiv 10x+45$

$10x + 5a \equiv 10x + 45$

$+5a \equiv +45$

$\div 5 \quad \div 5$

$a \equiv 9$

$(ax+1)(x+c)+b \equiv 5x^2+11x+8$

$ax^2 + c + x + acx + b \equiv 5x^2 + 11x + 8$

$ax^2 \equiv 5x^2$

$a \equiv 5$

$x + acx \equiv 11x$

$x(1+ac) \equiv 11x$

$1+ac \equiv 11$

$1+5c \equiv 11$

$-1 \quad -1$

$5c \equiv 10$

$\div 5 \quad \div 5$

$c \equiv 2$

$c + b \equiv 8$

$2+b \equiv 8$

$-2 \quad -2$

$b \equiv 6$

Questions – Equating Coefficients

$8(ax+2) \equiv 24x+16$

$3(9x+1) \equiv 27x+a$

$4(7x+2)+a \equiv 28x+10$

$5(3x+4)+ax \equiv 19x+20$

$2(ax+5)+3(2x+b) \equiv 18x+13$

$9(ax+b)+4(x-2) \equiv 13x+37$

$(ax+5)(x-b)+4x \equiv 2x^2+cx-10$

$2x^2+12x-9 \equiv a(x+b)^2+c$

Rearranging

$5(2 + t) = w$

$5(2 + t) = w$ — Expand the bracket

$10 + 5t = w$ — -10 from both sides

$5t = w - 10$ — Divide both sides by 5

$t = \frac{w - 10}{5}$

- **Inverse**
- **balance**
- **Cross out**

$c = \frac{3d}{d + 2}$ — Multiply by (d + 2)

$c(d + 2) = 3d$ — Expand the bracket

$cd + 2c = 3d$ — - cd

$2c = 3d - cd$ — Factorise

$2c = d(3 - c)$ — Divide by (3 - c)

$\frac{2c}{3 - c} = d$

Questions - Rearranging

Make a the subject of

$x = 3a + b$

Make b the subject of

$x = a + bc$

Make r the subject of

$p = \frac{q + r}{2}$

Make n the subject of

$d = (mn)^2$

Make n the subject of

$m = \sqrt{n - 2p}$

Make q the subject of

$L = 4pq^2$

Make t the subject of $5(t - g) = 2t + 7$

Make t the subject of the formula $m = \frac{t + 1}{t - 3}$

Number Sequences (ALL)

Linear

-3 +3 +3 +3

2, 5, 8, 11, 14, ...

Di=3
N=n
O=2

$3n + 2$

Geometric

1, 2, 4, 8, 16, ... (x2 x2 x2 x2 x2)

64, 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$ (÷4 ÷4 ÷4 ÷4 ÷4)

Multiply or divide by the same number

Quadratic

5, 14, 29, 50, 77

9, 15, 21, 27

6, 6, 6

$2a = 6$

$a = 3$

$3a + b = 9$

$3(3) + b = 9$

$3 + b = 9$

$b = 0$

$a + b + c = 5$

$3 + 0 + c = 5$

$c = 2$

Sequence is $3n^2+2$

Fibonacci

RULE: Fibonacci sequence: the next term in the sequence is the sum of the two previous terms.

2, 3, 5, 8, 13, 21, 34

2 + 3, 3 + 5, 5 + 8, 8 + 13, 13 + 21

Find the first 6 terms of the Fibonacci sequence:

$x_n = x_{n-1} + x_{n-2}$

$a, b, a + b$

The 4th term: $b + a + b = a + 2b$

The 5th term: $a + b + a + 2b = 2a + 3b$

The 6th term: $a + 2b + 2a + 3b = 3a + 5b$

$a, b, a + b, a + 2b, 2a + 3b, 3a + 5b$

Questions – Find the nth term of these sequences

9, 14, 19, 24, … …	13, 22, 31, 40, … …
10, 7, 4, 1, … …	-10, -13, -16, -19, … …
2, 5, 10, 17, 26 … …	3, 14, 29, 48, 71 … …

2) Continue the following geometric sequences:

a) 1, 2, 4, 8, ……. , ……..

b) 5, 50, 500, 5000, …… , ……

c) 3, 9, 27, 81, …… , ……..

d) 4, 20, 100, …… , ……

e) 4, 16, 64, …… , ……..

f) 64, 32, 16, 8, 4, …… , ……

Find the next three terms of the following Fibonacci-style sequences

(a) $2, 4, 6, 10, \ldots$

(b) $3, 6, 9, 15, \ldots$

(c) $4, 8, 12, 20, \ldots$

(d) $15, 23, 38, 61, \ldots$

(e) $5, 12, 17, 29, \ldots$

(f) $-3, 5, 2, 7, \ldots$

For each of the following Fibonacci-style sequences, find the next 4 terms.

(a) $a, 4a, 5a, 9a, \ldots$

(b) $3x, 3x + y, 6x + y, 9x + 2y, \ldots$

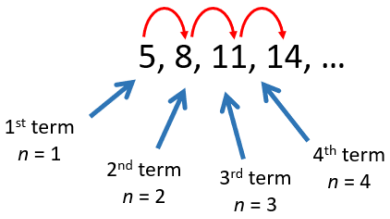
(c) $6a, -2a, 4a, 2a, \ldots$

(d) $2y, y + z, 3y + z, \ldots$

(e) $4x - 5y, 2x - y, 6x - 6y, \ldots$

(f) $-x, x + y, y, \ldots$

Examples– generating sequences



3n +5

When n = 1, 3(1)+5 =8
When n = 2, 3(2)+5 =11
When n = 3, 3(3)+5 =14
When n = 4, 3(4)+5 =17
When n = 5, 3(5)+5 =20

Sequence: 8,11,14,17,20

$2n^2 - 3$

When n = 1, $2(1^2) - 3 = -1$
When n = 1, $2(2^2) - 3 = 5$
When n = 1, $2(3^2) - 3 = 15$
When n = 1, $2(4^2) - 3 = 29$
When n = 1, $2(5^2) - 3 = 47$

Sequence: -1,5,15,29,47

Questions – generating sequences

The nth term for some sequences are given below.
Find the first 5 terms for each sequence.

(a)	5n + 3	(b)	2n + 9	(c)	3n – 2
(d)	10n – 6	(e)	9n + 10	(f)	n + 8
(g)	–7n + 20	(h)	50 – 5n	(i)	3.5n + 4

For each n^{th} term, work out the first five terms of the sequence.

(a) $n^2 + n$ (b) $n^2 + 2n$ (c) $n^2 - n$ (d) $n^2 - 3n$

(e) $n^2 + n + 2$ (f) $n^2 - 2n + 5$ (g) $n^2 + 4n - 10$ (h) $2n^2 + n$

(i) $3n^2 - n + 6$ (j) $10n^2 + 5n - 7$