Year 9 Maths

Topic 1: Working with decimals

Multiplying and dividing by decimals

Example 1: Multiplying

0.34 x 0.4

1)Count the decimal places (3dp)
2) Multiply without d.p. 34 x 4=136
3) Insert 3 decimal places back in 0.136

Example 2: Division

0.7 ÷ 0.14 =

1) Write as a fraction
2) Make denominator an integer
3) Divide

$\frac{0.7}{0.14} \times 100 \times 100 = \frac{70}{140} = \frac{1}{2}$

Bank statements

Credit - Money in **Debit – money out** **Balance – what's in the account**

Date	Description	Credit	Debit	Balance
	Starting Balance			_____
13/06/19	Inventory		334.56	450.42
13/06/19	June Income	_____		2,309.19
18/06/19	Salaries		_____	1,678.12
27/06/19	Tax		78.04	_____

Rounding and approximation

Rounding to 1 significant figure

3540 ≈ 4000 (1000)

0.0436 ≈ 0.04 (2dp)

Rounding to 2nd significant figure

3540 ≈ 3500 (100)

0.0436 ≈ 0.044 (3dp)

0.406 ≈ 0.41 (2dp)

Example: Estimating/Approximating

Example - Estimate the answer by rounding to 1 signigicant figure

$\frac{39 \times 6.98}{0.52} \approx \frac{40 \times 7}{0.5} = \frac{280}{0.5} \times 10 = \frac{2800}{5} = 560$

Questions to try

Round these to 1sf and 2sf

4213 435

23.65 43.89

0.0465 0.009051

Approximate these 6.05 x 14.82

$\frac{164.7 \times 4.2}{8.24 \times 2.09}$ $\frac{12.3 \times 45}{18.8}$ $\frac{186.3 \times 88.6}{27.2 \times 22.8}$

Recurring Decimals to Fractions

Example 1

0.555555555555555... (1 number repeating)

Ex1 Convert $0.\dot{5}$ to a fraction.

Solution

Let $x = 0.\dot{5}$,

$10x = 5.\dot{5}$

$9x = 5$

$\div 9$: $x = \frac{5}{9}$

$5.\dot{5} - 0.\dot{5} = 5.0$

Example 2

0.121212121212121... (2 numbers repeating)

Ex2 Convert $0.\dot{1}\dot{2}$ to a fraction.

Solution

Let $x = 0.\dot{1}\dot{2}$,

$100x = 12.\dot{1}\dot{2}$

$99x = 12$

$\div 99$: $x = \frac{12}{99} = \frac{4}{33}$

$12.\dot{1}\dot{2} - 0.\dot{1}\dot{2} = 12.00$

Example 3

0.427427427427427... (3 numbers repeating)

Ex3 Convert $0.\dot{4}2\dot{7}$ to a fraction.

Solution

Let $x = 0.\dot{4}2\dot{7}$,

$1000x = 427.\dot{4}2\dot{7}$

$999x = 427$

$\div 999$: $x = \frac{427}{999}$

$427.\dot{4}2\dot{7} - 0.\dot{4}2\dot{7} = 427.000$

Questions to try: Convert these decimals to fractions

$0.3\dot{4}$

$0.5\dot{1}\dot{2}$

$1.2\dot{4}$

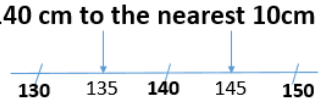
$0.\dot{3} - 0.0\dot{5}$

$0.1\dot{4} + 0.\dot{2}\dot{3}$

Example Error interval

140m has been rounded to the nearest 10m. Complete the error interval

140 cm to the nearest 10cm



LB = 135

UB =145

$135 \leq x < 145$

Complete the error interval for

8.4 cm rounded to the nearest to the tenth

LB = 8.35

UB =8.45

$8.35 \leq x < 8.45$

Lily rounds a number, y, to the nearest whole number.
Her result is 5
Write down the error interval for y

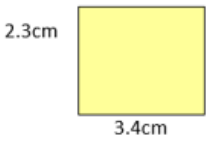
Freya rounds a number, y, to one decimal place.
Her result is 6.4
Write down the error interval for y

Oscar rounds a number, y, to the nearest integer.
His result is 100
Write down the error interval for y

A number, n, is rounded to 1 decimal place.
The result is 1.3
Using inequalities, write down the error interval for n.

A number, n, is rounded to 2 decimal places.
The result is 6.27
Using inequalities, write down the error interval for n.

A rectangle is measured as 3.4cm by 2.3cm, both measurements correct to 1d.p.



	Lower bound	Upper bound
Perimeter		
Area		

Write the error interval for the area and perimeter of the rectangle.

Topic 2: Percentages

Multipliers

Examples

21% = 21/100 = 0.21

21% increase = 100% + 21%
= 121% = **1.21**

21% decrease = 100% – 21%
79% = **0.79**

Practice – Write the multiplier

12%

12% increase

12% decrease

34% increase

13% decrease

3%

3% increase

3% decrease

0.5% increase

1.2% decrease

Percentage of amount

Work out 5% of £120
Amount £120
Rate 5%
Multiplyier 0.05
Sum 0.05 x 120 = £6

Percentage increase

Increase £120 by 5%
A £120
R 105%
M 1.05
S 1.05 x 120 = £126

Percentage decrease

Reduce £120 by 5%
A £120
R 100% - 5% = 95%
M 0.95
S 0.95 x 120 = £114

15% of 80ml

4% of £360

2.5% of 60cm

Increase 60ml by 70%

Increase 80ml by 4%

Increase £230 by 1.5%

Decrease 9kg by 6%

Reduce 225kg by 35%

£8,000 depreciated by 4.5%

Percentage change

A percentage change always relates back to the original amount. The formula is:

$$\frac{difference}{original} \times 100$$

Example

Savings increase from £400 to £450. Work out the percentage increase.

original	new	difference
400	450	450-400 = 50

$$\frac{difference}{original} \times 100 = \frac{50}{400} \times 100 = 12.5\%$$

Questions to try

a A t-shirt is reduced from £25 to £20. Find the percentage discount.

b A coat is reduced from £80 to £64. Find the percentage discount.

a Jenna's savings increase from £400 to £450. Calculate the interest rate.

b Amanda's antique vase increases in value from £600 to £720. Calculate the percentage increase in value.

A car costs £9750 when new. 5 years later it is sold for £4500. What is the average percentage loss each year?

Reverse percentage

Example
After 15% off, a jacket costs £34. What was the original price of the jacket?

O: ?
R: 100% -15% = 85%
M: 0.85
S: ? x 0.85 = 34

Inverse operations

Original = $\frac{£34}{0.85}$ =£40

1) A camera costs £180 in a 10% sale. What was the pre-sale price?

2) After fuel prices rose by 15%, a family's annual heating bill was £1654. What would the bill have been without the price increase?

3) The cost of a holiday, including VAT at 20% is £540. What is the pre-VAT price?

4) The world's tiger population has decreased by 95% since 1910 and is now believed to be as low as 3200. If these figures are correct, what was the tiger population in 1910?

5) The sale price of a television is £420 after a 15% reduction. What was the price before the sale?

Example

Expressing as a percentage

Write 11 as a percentage of 25

Step 1 Write as a fraction $\frac{11}{25}$

Step 2 Convert into percentage $\frac{11}{25} = \frac{44}{100} = 44\%$

Write

a £3 out of £6 as a percentage

c 4p out of 10p as a percentage

e £80 out of £400 as a percentage

g 7 kg out of 35 kg as a percentage

i 90 km out of 100 km as a percentage

Example

Simple interest

I put £1000 into an account that pays 3% simple interest per annum. What is the balance after 4 years?

3% of £1000 = 0.03 x 1000 = £30

4years x £30 = £120

£120 + £1000 = **£1120**

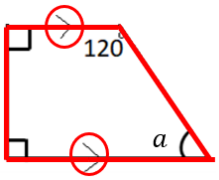
1. Hannah put £340 into her saving account, which pays 4% simple interest per year. What will be the balance after 3 years?

2. Work out my bank balance at 3 years if I initially save £5600 and get 4% interest p.a.

3. Carolyn invested £700 for 3years at 2% per annum simple interest. Work out the total amount of interest Carolyn earned.

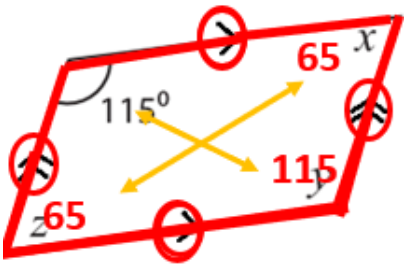
Topic 3: Angles in Parallel Lines and Properties of Shapes

Angles in a trapezium



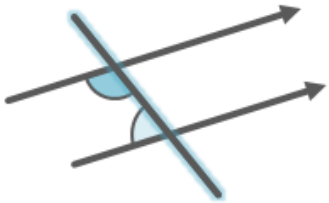
1. Connect the arrows to make a "capital C"
2. The two angles are called Co-interior angles. they add up to 180° (90 and 90 make 180)
3. This works if the capital C is reversed
4. 180° – 120° = 60°

Angles in a Parallelogram

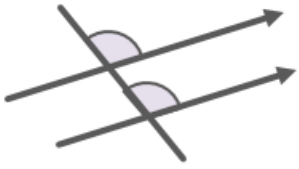


Opposite angles in a parallelogram are equal.

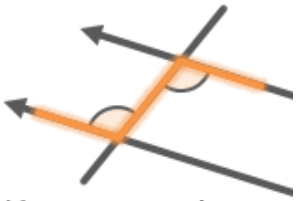
Angles in parallel lines



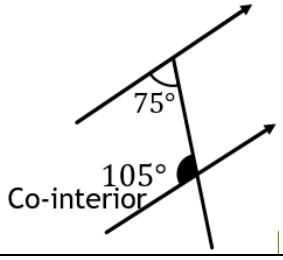
Co-interior angles sum to 180°.

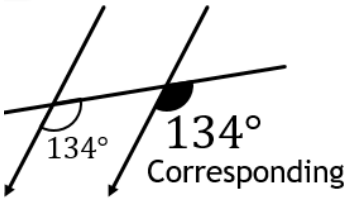


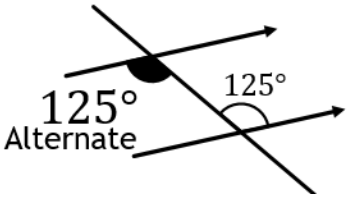
Corresponding angles are equal



Alternate angles are equal

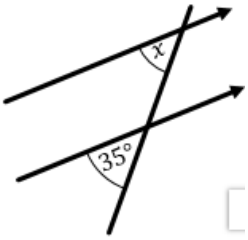


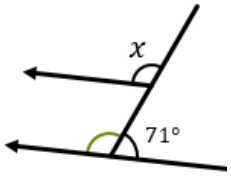


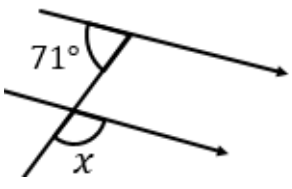


Questions to try

Find the value of each angle and state the reason

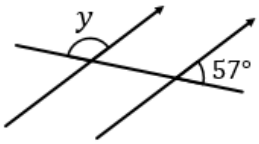




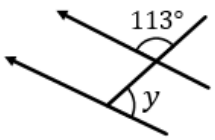


Find the value of y.

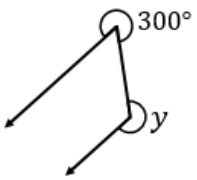
a

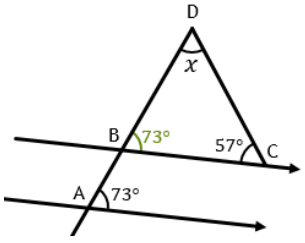


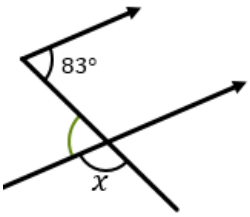
b

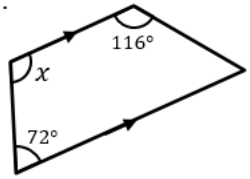


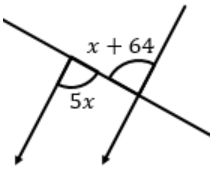
c

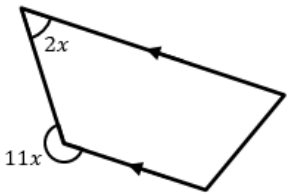












Topic: 4: Indices, Powers and Roots (Index Laws)

$a^0 = 1$ $\quad a^{-1} = \frac{1}{a}$ $\quad a^{\frac{1}{2}} = \sqrt{a}$ $\quad a^{\frac{1}{3}} = \sqrt[3]{a}$

Multiplication rule	Division rule	Brackets rule
$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$	$(a^m)^n = a^{m \times n}$

Examples (multiplication rule)	Examples (division rule)	Examples (brackets rule)
Simplify	**Simplify**	
$y^{11} \times y^5 = y^{16}$	$y^5 \div y^2 = y^3$	$(y^3)^7 = y^{21}$
$6y^3 \times 2y^5 = 12y^5$	$8y^3 \div 2y = 4y^2$	$(3y^4)^2 = 9y^8$
$4^6 \times 4^3 = 4^9$	**Evaluate**	
	$4^6 \div 4^3 = 4^3$	
	$4^3 = 64$	

Questions to try:

Simplify:	Simplify:	Simplify:
6^2 x 6^8	$a^5 \div a^2$	$(2^3)^5 =$
a^3 x a^8	$\frac{4^{-2}}{4^5}$	$(x^2)^3 =$
$2x^3 \times 3x^5 =$	**$8a^6 \div 2a^3$**	$(y^7)^8 =$
$2x^3y^2 \times 3x^5y^2 =$	**$15a^2b^4 \div 3a^4b^3$**	$(5y^4)^3 =$
$x^{-3} \times x^{-10}$	$\frac{12a^2b^3}{4ab^7} =$	**$(3a^{-2}b^3)^2 =$**
$\frac{a^3 \times a^5}{a^6}$		**$(5a^4b^2)^3 =$**
$\frac{5^4 \times 5^5}{5^2 \times 5^3}$	$\frac{12x^6}{2x^3 \times 3x^5}$	**$(2a^5b^{-2})^{-2} =$**

Topic 5: Standard Form

Standard form is written in the form of

$a \times 10^n \qquad 1 \le a < 10$

where a is a number bigger than or equal to 1 and less than 10.

n can be any positive or negative whole number.

Write in standard form

First , Dot, Bounce
(Bounce Right-positive; Left-negative)

$42500 = 4.25 \times 10^4$

$4243.5 = 4.2435 \times 10^3$

$0.0045 = 4.5 \times 10^{-3}$

Write as ordinary numbers
If the power is:
Bigger than 1 -> positive power
Smaller than 1 -> negative power

2.9×10^3

2.900 =2900

9.1×10^{-5}

0.000091 =0.000091

Fix the standard form
First, Dot, Bounce
Bounce right once – add 1
Bounce left once – subtract 1

$82.1 \times 10^3 = 8.21 \times 10^4$ (+1)

$0.82 \times 10^3 = 82.1 \times 10^2$ (−1)

$820 \times 10^3 = 8.2 \times 10^5$ (+2)

Write in standard form:

70,000

54000

1,506,000

0.00054

0.00103

0.5

Write as ordinary numbers

9×10^4

1.2×10^3

4.06×10^4

8×10^{-2}

8.64×10^{-4}

6.07×10^{-3}

Write these in correct standard form

84×10^6

0.46×10^5

24×10^{-5}

65213×10^{-7}

0.0055×10^{-2}

Multiplying with standard form

$6 \times 10^4 \times 7 \times 10^5$

$= 6 \times 7 \times 10^4 \times 10^5$

$= 42 \times 10^{4+5}$

$= 42 \times 10^9$

$= 4.2 \times 10^{10}$

Dividing with standard form

(a) $(14 \times 10^8) \div (7 \times 10^5)$

$= \dfrac{14 \times 10^8}{7 \times 10^5}$

$= 14 \div 7 \times 10^8 \div 10^5$

$= 2 \times 10^3$

Write the anser in standard form

a) $(2.5 \times 10^3) \times (4 \times 10^2)$

b) $(3 \times 10^{-4}) \times (7 \times 10^2)$

c) $(9 \times 10^{-4}) \times (7 \times 10^{-4})$

Write the answer in standard form

$(16 \times 10^7) \div (4 \times 10^9)$

$(4.8 \times 10^{15}) \div (3 \times 10^4)$

$(1.44 \times 10^8) \div (12 \times 10^{-5})$

Put these in asecnding order

$3.2 \times 10^2 \qquad 4.5 \times 10^{-4} \qquad 2.9 \times 10^2$

Work these out and write the answer in correct standard form

$(2 \times 10^6)^3$

$3.1 \times 10^4 + 6.6 \times 10^3$

$\dfrac{(6 \times 10^7) + (4 \times 10^6)}{(2 \times 10^4)}$

Topic 6: Ratio Calculations

Sharing in a ratio

There are 3 circles for every 2 diamonds. If there are 25 shapes in total, how many are circles?

	C	D	T
R	3	2	5
Mx	x5	x5	x5
A	15	10	25

Multiplier: $\frac{25}{5} = 5$

Circles = 15

Given one part

In a bag there are 2 red counters for every 5 yellow counters. There are 20 red counters in the bag.
How many counters were in the bag in total?

	R	Y	T
R	2	5	7
Mx	x10	x10	x10
A	20	50	70

Multiplier: $\frac{20}{2} = 10$

Total is 70 counters

Given difference

Ratio of A : B is 2: 5
B is 12 more than A. Work out the total of A and B.

	A	B	T	diff
R	2	5	7	3
M	x4	x4	x4	x4
A	8	20	28	12

Multiplier: $\frac{12}{3} = 4$

Total is 28

Overlapping ratio

A:B = 3:6 and B:C = 9:10

Q) Find ratio of A:C

Write out the ratios.
Common letter in middle

A : **B** and **B** : C
3 : **6** and **9** : 10

Find LCM of **B**
LCM of 6 and 9 is 18 so

A : **B** and **B** : C
3 : **6** **9** : 10
: **18** **18** :

Ratio of A : B : C is 9 : 18 : 20

So ratio of A : C 9 : 20

Ratios, Fractions and Equations

From the ratio A : B = 2 : 3
we can work out the following facts

a) A is 2/5 of the total

b) B is 3/5 of the total

c) 3A = 2B

d) $A = \frac{2}{3}B$

e) $B = \frac{3}{2}A$

Questions to try:

a) Charlie and Dave share some money in the ratio 2:7. Charlie gets £14.
How much money did they share?

b) Itisham has four times as much money as Hussain. Itisham has £1600. How much do they have in total?

c) A metal alloy block of mass 56kg consists of copper, zinc and tin in the ratio 4:3:1. Find the mass of each metal.

d) Two numbers are in the ratio 4:3. Their sum is 70. Find the numbers.

e) Amy and Arman share £140 in the ratio 2:5 How much more does Arman get?

f) Two numbers have a ratio 7 : 5. Their difference is 12. Find the numbers.

g) Simplify ratio of 2.5 : 3 2m : 40cm

h) A : B = 3 : 4 What fraction of the total is A? Write A as a fraction of B.

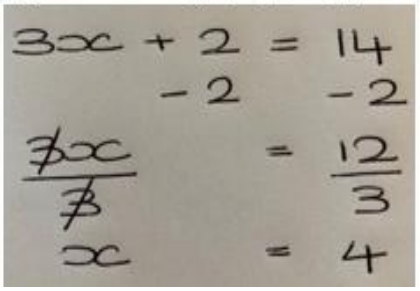
i) The ratio of a : b is 2 : 3 The ratio of b : c is 1 : 4 What is the ratio of a : b : c?

j) Adam and Barry are sharing some money in the ratio 3:5. Barry gets £24 more than Adam. How much money do they share?

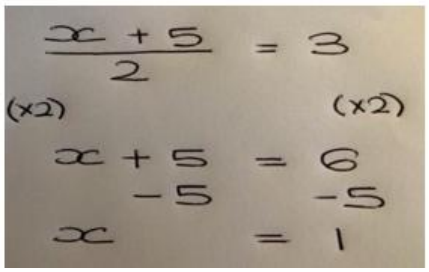
Topic 7: Solving and Rearranging Linear Equations

Linear equations

Solve $3x + 2 = 14$

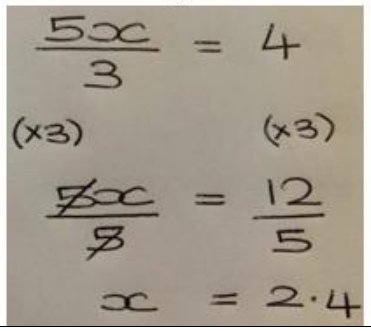


$\frac{x+5}{2} = 3$

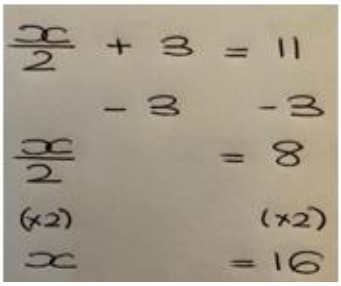


Equations with fractions

Solve $\frac{5x}{3} = 4$



Solve $\frac{x}{2} + 3 = 11$



Unknowns on both sides

Solve: 6x + 2 = 3x – 5

Pick on the smaller one

-3x -3x

3x + 2 = -5

-2 -2

$\frac{3x}{3} = \frac{-7}{3}$

$X = -\frac{7}{3}$

(x4) $3x - 3 = \frac{2x+1}{4}$ (x4)

4(3x -3) = 2x + 1

Expand the bracket

12x -12 = 2x + 1

-2x -2x

10x – 12 = 1

+12 + 12

10x = 13

$X = \frac{13}{10} = 1\frac{3}{10}$

Questions to try

Solve $2b + 9 = 23$

Solve $\frac{9t}{3} = 6$

Solve $\frac{x-9}{4} = 6$

Solve $\frac{x}{4} - 9 = 6$

$9x - 1 = 6x + 4$

$\frac{2x+1}{3} = x - 2$

$\frac{3x}{7} - 6 = 10$

$8 - 4x = 2 - 7x$

$4 = \frac{4(2k-3)}{2} - 2$

Rearranging to make "x" the subject

Rearrange y = ax + b to make "x" the subject

-b -b

y – b = ax

÷a ÷a

$\frac{y-b}{a} = x$

Rearrange $y = \frac{x}{a} - b$ to make "x" the subject

+b +b

$y + b = \frac{x}{a}$

a(y + b) = x

Rearrange these to make "x" the subject

a) x + c = w

b) x – b = w

c) ax = b

d) xy = b

e) $\frac{x}{a} = b$

f) xy + b = c

g) xy – b = c

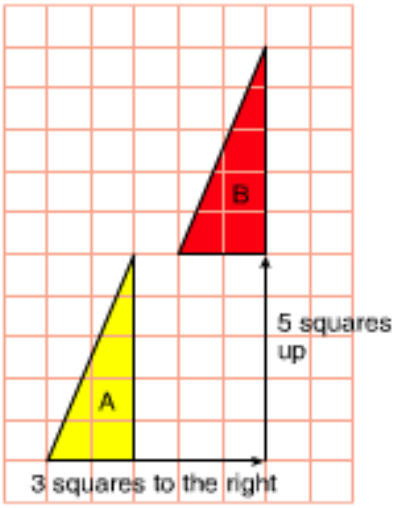
h) $\frac{x}{a} + b = c$

i) $\frac{x-b}{a} = y$

Topic 8: Transformations (TRRE)

Translation

In the diagram below, shape **A** has been mapped onto shape **B** by a **translation**. In a translation, all points of the shape move the same distance in the same direction. All points of shape **A** move three squares to the right and five squares up.



In a translation

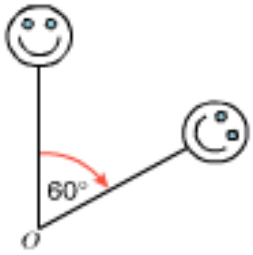
- the lengths of the sides of the shape do not change
- the angles of the shape do not change
- the shape does not turn.

The diagram shows a translation of the triangle by vector $\begin{pmatrix}3\\5\end{pmatrix}$

Rotation

To **rotate** means to turn. A bicycle wheel, the hands of a clock and the drum of a washing machine all turn or rotate.

This face on a stick has rotated 60° **clockwise** (↻) about the point O. The size of the face has not changed.



To describe a rotation give

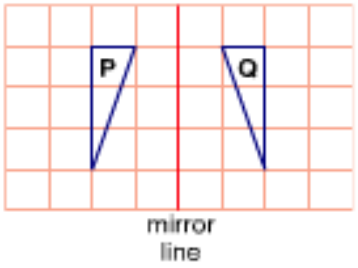
- the angle of turn
- the direction of turn (**clockwise** or **anticlockwise**)
- the point the shape turns about (the **centre of rotation**).

In a rotation

- the lengths of the sides of the shape do not change
- the angles of the shape do not change
- the shape turns
- the centre of rotation does not move.

Reflection

Reflecting each corner of triangle **P** in the mirror line gives the corners of triangle **Q**.



Triangle **Q** is the **reflection** of triangle **P** in the mirror line. Also, triangle **P** is the reflection of triangle **Q** in the mirror line. (In mathematics, mirror lines are like two-way mirrors.)

In a reflection

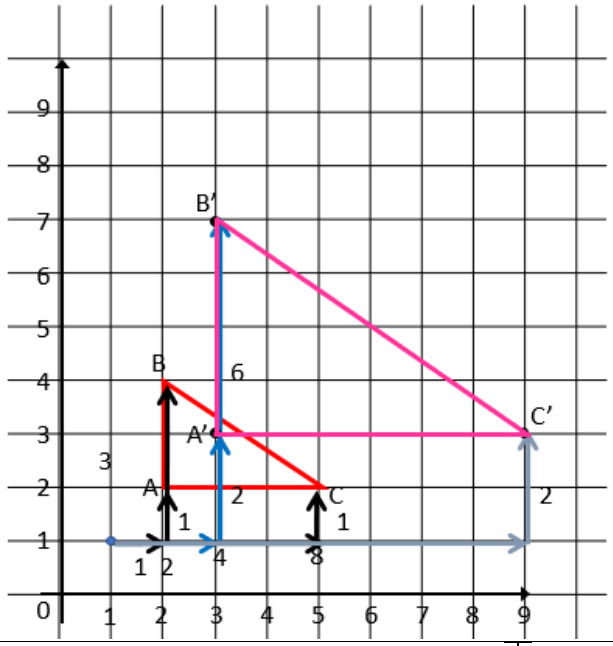
- the lengths of the sides of the shape do not change
- the angles of the shape do not change
- the image is as far behind the mirror line as the shape is in front.

To describe a reflection give

- the mirror line.

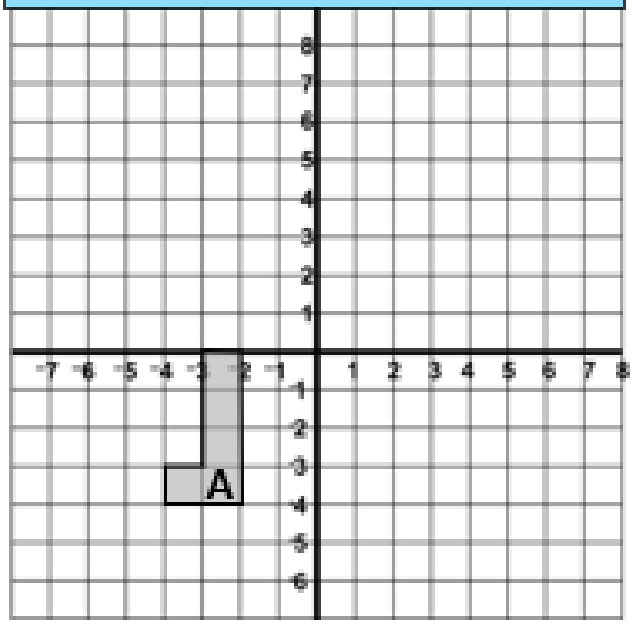
Enlargement

Enlarge triangle ABC using a scale factor of 2, centre of enlargement (1, 1).

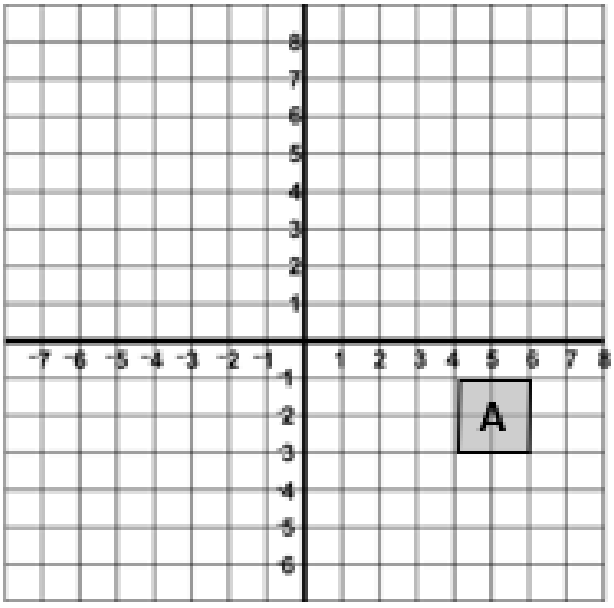


A scale factor of 2 means the length of the shape is multiplied by 2 **and** the distance from the centre of enlargement to each **vertex** is multiplied by 2.

Enlargement, by scale factor two from (-6,-5)



Enlargement, by scale factor four from (7,-4)



Topic 9: Linear graphs

Plotting linear graphs: examples

Plot y = 3x + 2 for the values of x $-3 \leq x \leq 3$

x	-1	0	1	2
y	-1	2	5	8

Gradient

Y - Intercept

1) Substitute

$y = 3(-1)+2 = -3+2 = -1$

$y = 3(0)+2 = 0+2 = 2$

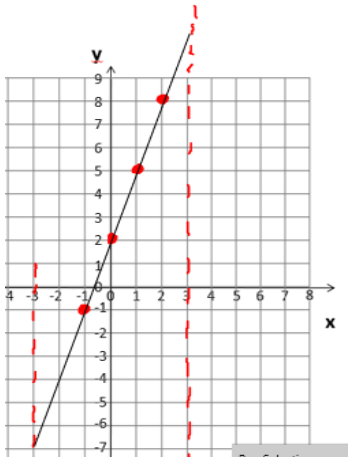
$y = 3(1)+2 = 3+2 = 5$

$y = 3(2)+2 = 6+2 = 8$

2) Plot

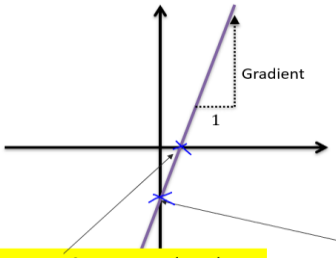
(-1,-1), (0,2), (1,5), (2,8)

3) Draw the line.



Gradient and y-intercept

The steepness of a line is known as the **gradient**.
It tells us what y changes by as x increases by 1.



x – intercept $(x, 0)$
the point at which the line crosses the x-axis

The equation of a straight line is of the form:

$$y = mx + c$$

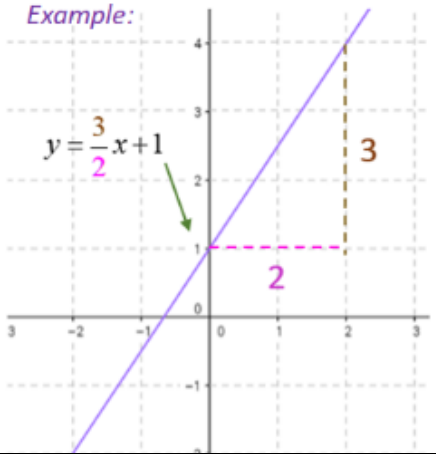
m is a gradient

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$$

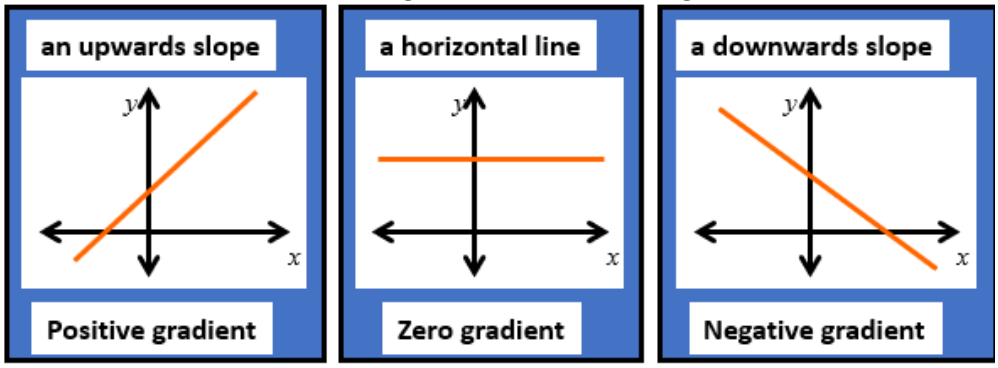
c is the **'y-intercept'** (0, y)

the point at which the line crosses the y-axis

Example:



We "read" gradients from left to right.



y = mx + c Examples

Y = 2x + 4 m = 2 c = 4

Y = -3x – 4 m = -3 c = -4

Y = 4 - x m = -1 c = 4

Example: Lines must always be in correct format before we find the gradient and intercept

2y = 6x – 8 Rearrange to y = 3x – 4
m = 3 c = -4

x + y = 5 Rearrange to y = 5 – x
y = - x + 5
m = -1 c = 5

Questions to try:

Draw the following graphs:

Values of x from -3 to 5

a) y= 3x-1

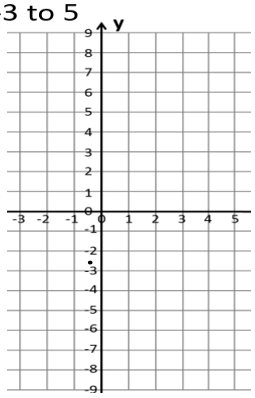
b) y= 2x-3

c) y= -2x+5

d) y= -x-2

e) y= $\frac{1}{2}x$-1

f) x + y = 1



Copy and complete the following table.

Gradient	y-intercept	Equation
2	1	$y = 2x + 1$
4	-3	
1	-1	
$\frac{1}{2}$	1	
		$y = x$
		$y = 1$

Write down the equation of the line with ….

(a) gradient of 3 and y–intercept of 6

(b) gradient of 2 and y–intercept of –1

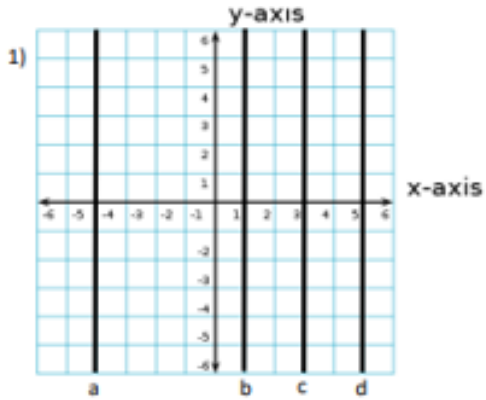
(c) gradient of –4 and y–intercept of 3

(d) gradient of 8 and y–intercept of 4

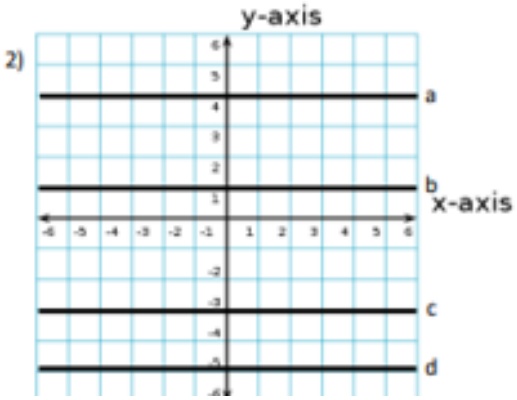
Horizontal and Vertical graphs

Find the equation of each line

1)



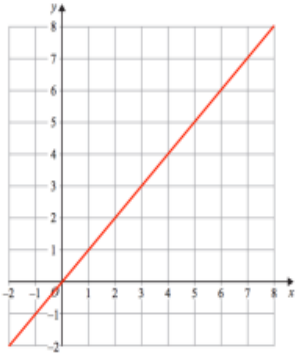
2)



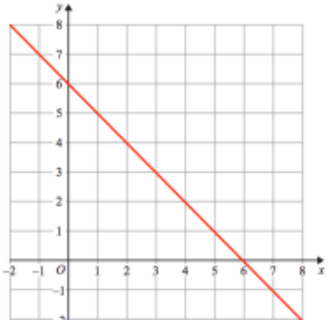
Find the equation of each line

(b)

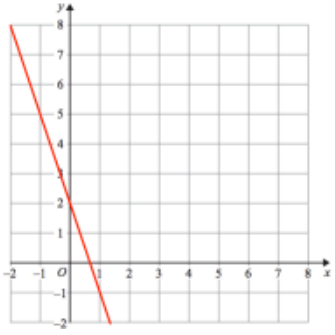
(c)



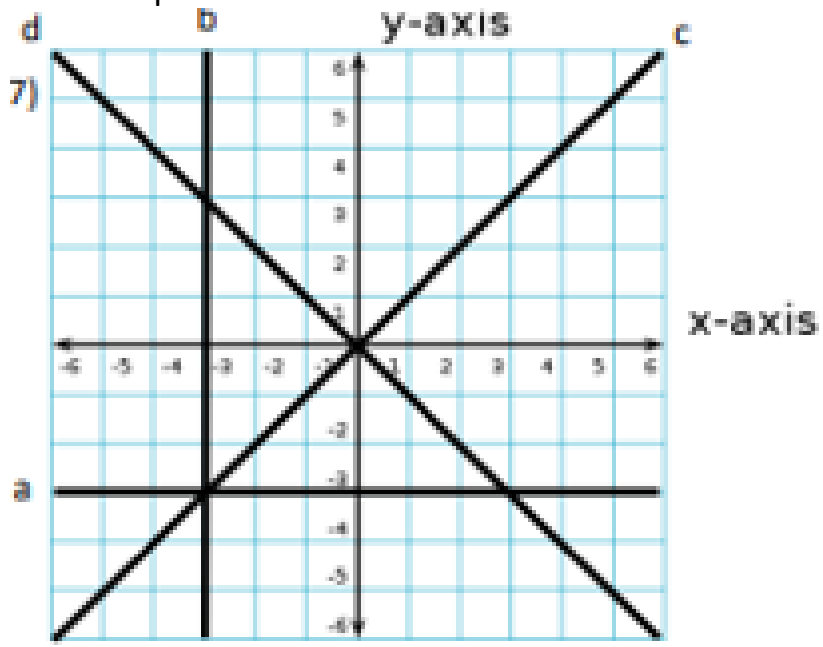
(e)



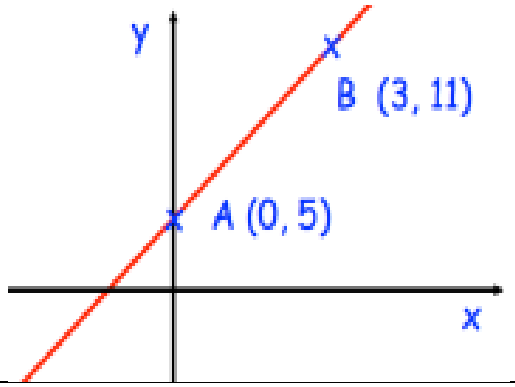
(f)

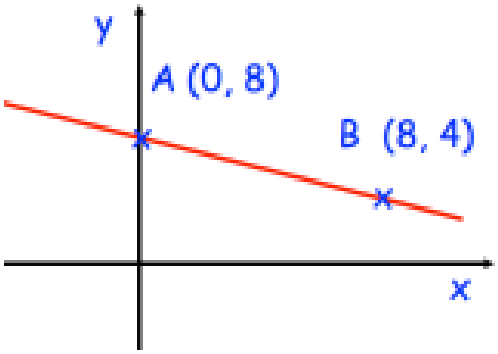


Find the equations of each line



Find the equation of these lines.

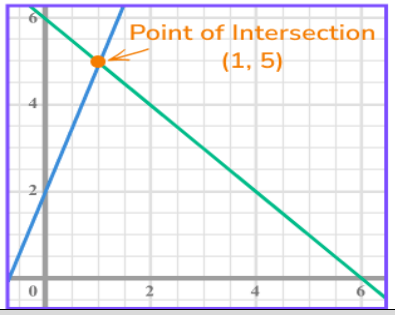




Topic 10: Simultaneous equations

Example – solving graphically

Solving simultaneous equations graphically allows us to solve two or more algebraic equations that share variables by sketching their graphs. The point or points of intersection gives the solution to the simultaneous equations.



Example

$$x + y = 6$$
$$-3x + y = 2$$

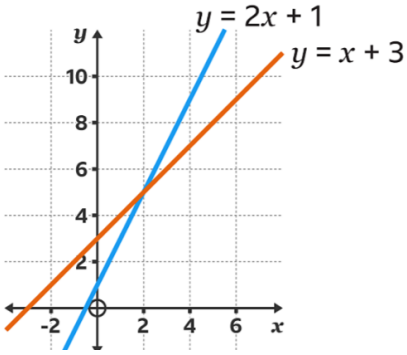
The graphs of these two equations intersect at (1, 5).

So the solution to the simultaneous equations is:
$x = 1$ and $y = 5$

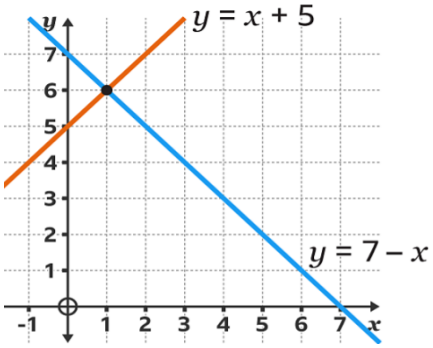
THIRD SPACE LEARNING

Questions to try

What is the solution to the pair of equations y= 2x+1 and y=x+3?



What is the solution to the pair of equations y= x+5 and y=7-x?

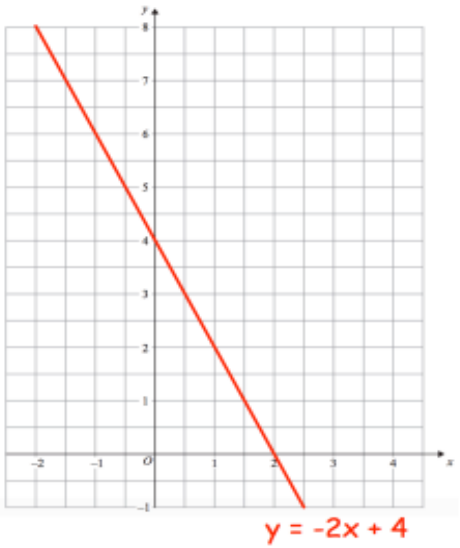


Question 5: The straight line $y + 2x = 4$ has been drawn on the grid.

(a) On the same grid, draw the graph of $y = x + 1$

(b) Use the graphs to solve the simultaneous equations

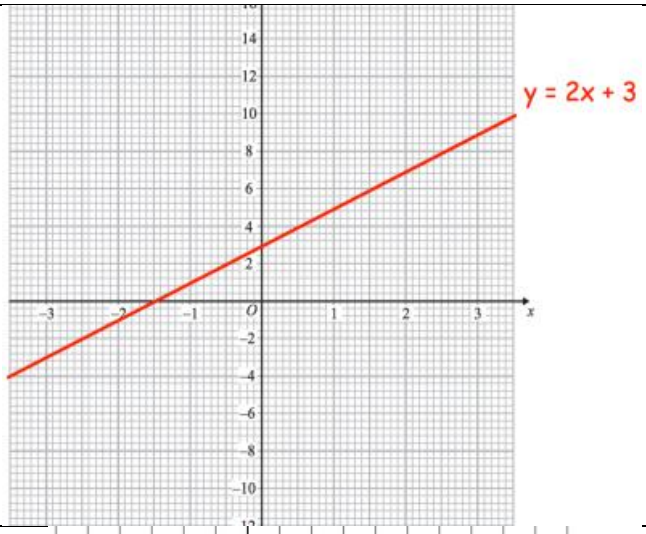
$$y + 2x = 4$$
$$y = x + 1 .$$



(a) On the same grid, draw the graph of $x + y = 9$

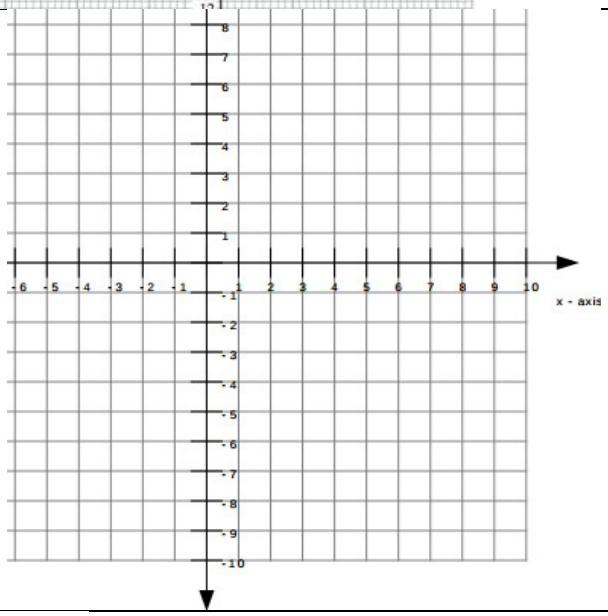
(b) Use the graphs to solve the simultaneous equations

$3x + y = 8$
$x + y = 9$



By drawing the graphs of $y = 3x + 1$ and $x + y = 7$

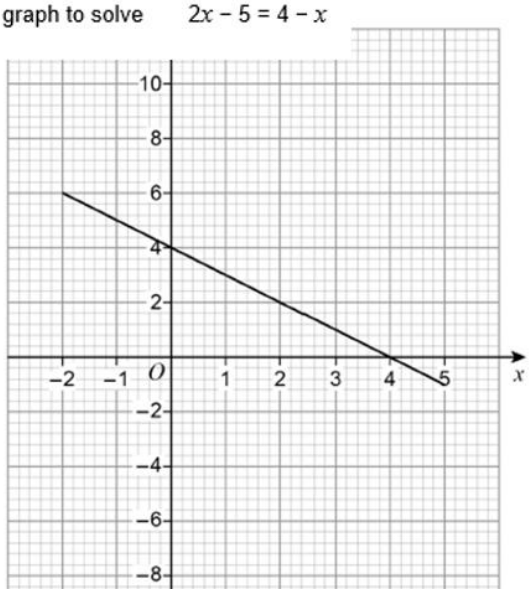
Solve the simultaneous equations
$y = 3x + 1$
$x + y = 7$



The graph of $y = 4 - x$ for values of x from −2 to 5 is shown on the gri

(a) On the grid, draw the graph of $y = 2x - 5$ for values of x from −2 to 5

(b) Use your graph to solve $2x - 5 = 4 - x$



Solving simultaneous equations algebraically

Example 1: both equations are y =

1. $y = 5x + 1$
   $y = 2x + 10$

$5x+1=2x+10$
$-2x \quad -2x$
$3x+1=10$
$-1 \quad -1$
$\frac{3x}{3} = \frac{9}{3}$
$x = 3$
$y = 5x+1$
$y = 5(3)+1 = 16$
$x=3, y=16$

Check : 16 = 5(3) + 1 ✓

Example 2: one equation is y =

$y = 2x - 3$
$5x - 2y = 8$

$5x-2(2x-3)=8$
$5x-4x+6=8$
$x+6=8$
$-6 \quad -6$
$x=2$

$y=2x-3$
$y=2(2)-3=$
$4-3=1$
$x=2, y=1$

Check : 5(2) – 2(1) = 8 ✓

Example 3: Solving where one needs scaling

2x + y = 12 (x3)
3x – 3y = 9

6x + 3y = 36
3x – 3y = 9 +
9x = 45
x = 5

2(5) + y = 12
10 + y = 12 y = 2

Example 4: Solving where both need scaling

5x + 3y = 22 (x4)
2x + 4y = 20 (x3)

20x + 12y = 88
6x + 12y = 60 -
14x = 28
x = 2

5(2) + 3y = 22
10 + 3y = 22
3y = 12 y = 4

Try these questions

Solve	y = 4x – 6 y = x + 3	Solve	x + y = 5 y = 3x + 1
Solve	y = 3x + 4 y = x - 2	Solve	2y + 6 = 6x y = - x + 3
Solve	6x + y = 18 4x + y = 14	Solve	9x – 4y = 19 4x+ 4y = 20
Solve	3x + 2y = 23 2x – y = 6	Solve	5x –3y = 33 3x – 9y = 63
Solve	3x + 2y = 7 2x + 9y = 43	Solve	5x – 3y = 18 2x + 4y = 54

Topic 11: Proportion

Direct Proportion Examples

If 8 chocolate bars cost £5, calculate the cost of 10 chocolate bars.

$\times\frac{1}{8}$ (8 bars = £5) $\times\frac{1}{8}$

1bar = 0.625

$\times 10$ (10 bars = £6.25) $\times 10$

A pipe 2.5m long has a mass of 35kg. What would be the mass of 5.5m of the same type?

$\times\frac{5.5}{2.5}$ (2.5m = 35kg) $\times\frac{5.5}{2.5}$

5.5m = 77kg

Oliver has 30Euros left from his holiday in France. Assuming the exchange rate is £1 = €1.14, how many pounds can he exchange his euros for?

$\times 1.14$

£1 = €1.14

£26.32 = €30

$\times 1.14$

Questions to try

Q1) Here are the prices of doughnuts at two different shops.

Shop A	Shop B
3 doughnuts for £2	5 doughnuts for £3.50

I want to buy **15** doughnuts.

In which shop are the doughnuts **cheaper**?

You **must** show your working.

Q2)

You can buy jars of the same jam in two sizes.

A

454g for £1.59

B

340g for £1.25

Which jar is better value for money?

You **must** show working to explain your answer.

Q3) Kate buys **24 cans** of lemonade.
She buys the cans in **packs of 4**
Each pack costs **£1.20**

Pack of 4 Cost £1.20

Steve buys **24 cans** of lemonade.
He buys the cans in **packs of 6**
Each pack costs **£1.60**

Pack of 6 Cost £1.60

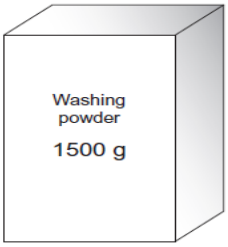
Kate pays more for her 24 cans than Steve pays for his 24 cans.

How much more?

Q5)

Washing powder is sold in two sizes, 600 grams and 1500 grams.

£3.30



Was £9.60
Now 15% off

Which size is better value for money?
You **must** show your working.

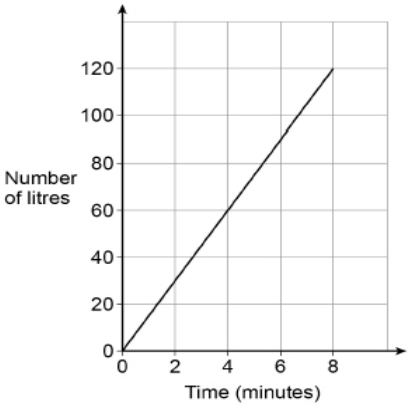
Q6)

b **Crisps**	
18 pack (30g each packet) of crisps for £3.40	200g share bag of crisps. Buy one, get one half price. Normal price £2

Direct Proportion Graph Example

Water is poured into a tank.

The graph shows the number of litres of water in the tank.



How much water is poured in the tank in 10 minutes?

8 min = 120 L

÷4 ÷4

2min = 30L

10min = 150L

How long does it take to fill up the tank with 200L of water?

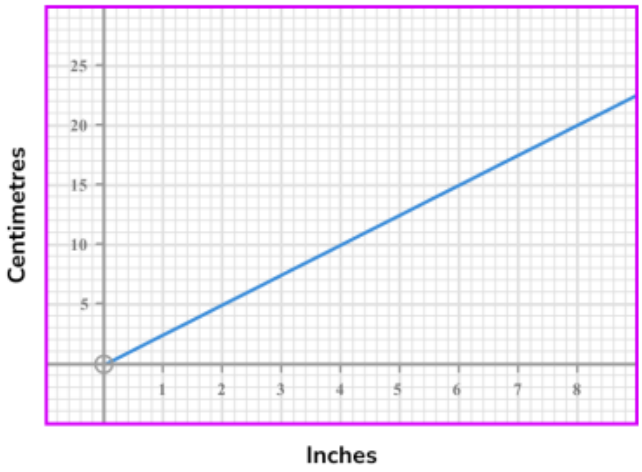
8 min = 120 L

$\times \frac{200}{120}$ $\times \frac{200}{120}$

13.3 min = 200L

Convertsion graphs: Questions to try

Q1) The graph shows the conversion between Inches and Centimetres

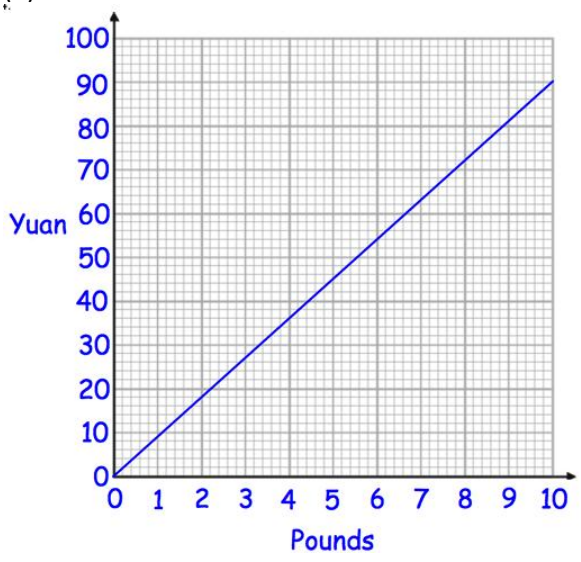


Use the graph to convert

a. 3 inches to *cm*
b. 16 *cm* to inches

How could you use the graph to covert 12inches into centimetres?

Q2) The graph shows the conversion between Pounds (£) and Yuan.

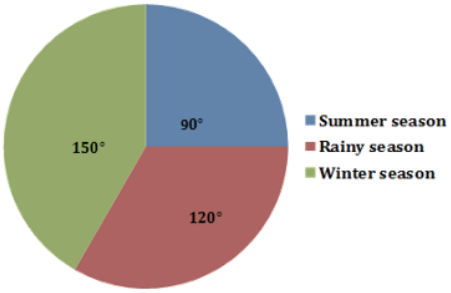


(a) Use the graph to convert £8 into Yuan.

(b) Use the graph to convert 45 Yuan into Pounds (£).

(c) Convert 900 Yuan into Pounds (£).

Pie Charts



$120° = 60$

$\div 12$ $\quad$ $\div 12$

$10° = 5$

$\times 15$ $\quad$ $\times 15$

$150° = 75$

$120° = 60$

$\div 120$ $\quad$ $\div 120$

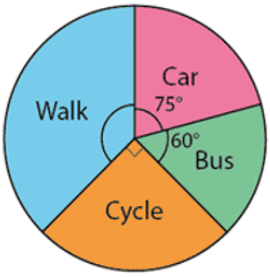
$1° = 0.5$

$\times 360$ $\quad$ $\times 360$

$360° = 180$

The pie chart shows the seasons that people took holidays.
60 people took holidays in the rainy season.
How many people took holidays in winter season?
How many people were asked in total?

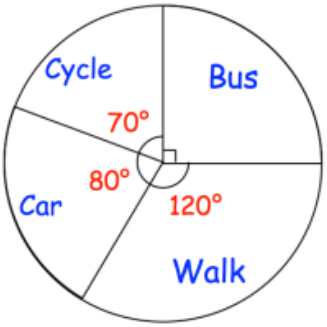
The pie chart shows how pupil in class 8C travelled to school one morning.
5 pupils in class 8C travelled by car.



a)How many pupils walked to school?

b) How many pupils were included in the survey?

The pie charts shows how a group of students travel to school.

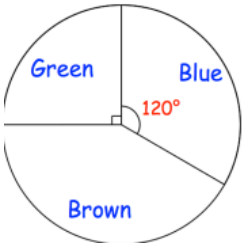


There are 36 students in the group.

(e) How many students caught the bus?

(f) How many students walked?

There are 24 students in a class.
The pie chart shows information about their eye colour.

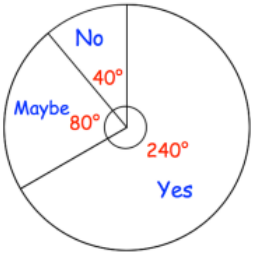


How many students have green eyes?

How many students have blue eyes?

How many students have brown eyes?

90 students were asked if they wanted to attend a history trip.
The results are shown in the pie chart.



(a) How many students said No?

(b) What fraction of the students said Yes?

(c) How many students said Yes?

(d) How many students said Maybe?

Topic 12: Compound Units (Speed, Distance and Time)

Distance

Amina is driving at 45mph.
How far would she drive in 20mins?

Distance	Time
45 miles	60 mins
÷ 3	÷ 3
15miles	20mins

Chelsea is driving at 45mph.
How far would she drive in 36mins?

Distance	Time
45 miles	60 mins
÷ 5	÷ 5
9miles	12mins
x3	x3
27miles	36mins

Questions
Work out the distance travelled when I travel at:

- 12mph for 10 minutes
- 12mph for 40 minutes
- 48mph for 36 minutes

Arman travels 135miles in 5hours.
Daisy travels 8 miles in 12minutes.
Who has the greater average speed?

Speed

Dawud travels 240km in 3 hours.
What is his speed in km per hour?

Distance	Time
240 km	3 hours
÷ 3	÷3
80 km	1 hour

80km in 1 hour
80km/h

Hannah travels 10 miles in 20 minutes hours. What is her speed in mph?

Distance	Time
10 miles	20 mins
x3	x3
30 miles	60mins

30miles in 1 hour
30mph

Questions
Work out the speed if I travel

- 20miles in 4 hours
- 8 miles in 12 minutes
- 1.6 miles in 24 minutes

Arman travels 10km in 4hours.
Daisy travels 900m in 20minutes.
Who has the greater average speed?

Scaling up

Fred flies a distance of 2km in 10 seconds. What is his speed in km per h?

Distance	Time
2km	10 secs
x 6	x 6
12km	1 min
x 60	x 60
720 km	60 min

Speed = 720 km/h

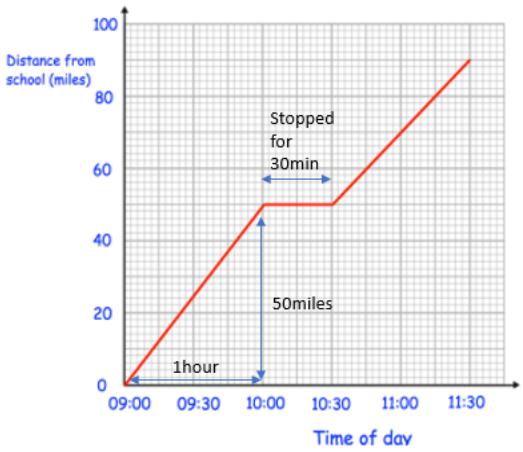
Questions
Fred flies a distance of 4 km in 15 seconds. What is his speed in kph?

Ibtisam runs at a speed of 8km/hour.
How long does she take to complete 100m? Give your answer in seconds.

Arman travels 10km in 4hours.
Daisy travels 900m in 20minutes.
Who has the greater average speed?

Speed from a Distance Time graph

The distance-time graph shows class 8A's journey to the zoo.
They stopped for a picnic on the way to the zoo.



(a) What time did the bus leave school? 9am

(b) What time did they stop for a picnic? 10am

(c) How far had they travelled when they stopped for a picnic? 50miles

(d) How long did they stop for? 30min

e) At what speed did they travel before they stopped for a picnic?

$$\text{Speed} = \frac{change\ in\ distance}{change\ in\ time}$$

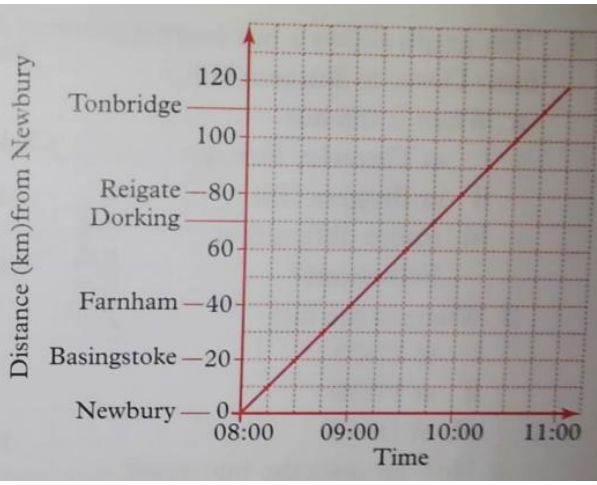
$$\text{Speed} = \frac{50miles}{1hour} = 50\text{mph}$$

This graph shows a car journey from Newbury to Tonbridge.

a Find the time when the car is
- **i** at Reigate
- **ii** at Basingstoke
- **iii** at Tonbridge.

b Where is the car
- **i** at 09:00
- **ii** at 09:45
- **iii** after travelling for two hours?

c Find the average speed of the car over these three hours.



This graph shows a car journey from Albridge to Chington and back again.

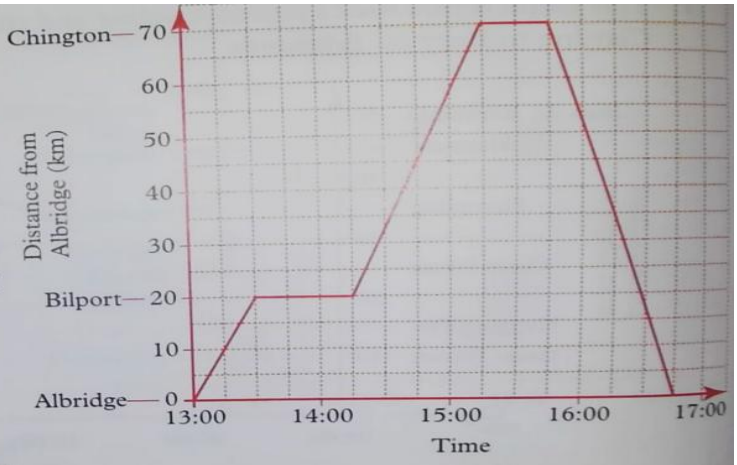
a How long does the car stop at Bilport?

b When does the car arrive at Chington?

c When does the car leave Bilport?

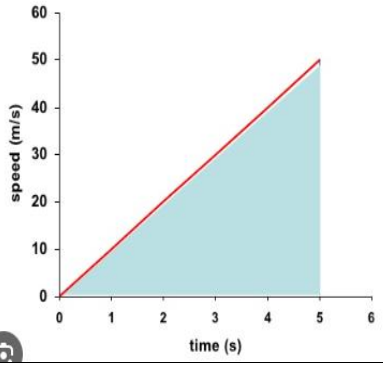
***d** At what speed does the car travel
- **i** from Albridge to Bilport
- **ii** from Bilport to Chington
- **iii** from Chington back to Albridge?

e At what time on the return journey is the car exactly halfway between Chington and Albridge?



Example

The area under a velocity-time graph = distance travelled



$$\text{area} = \frac{b \times h}{2}$$

$$= \frac{5 \times 50}{2}$$

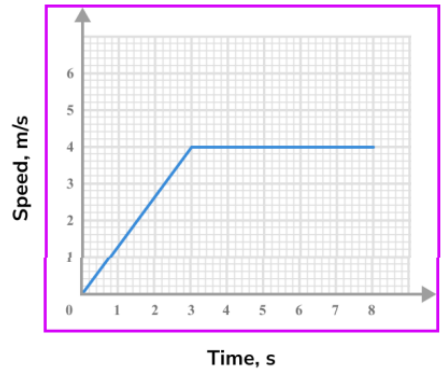
$$= 125$$

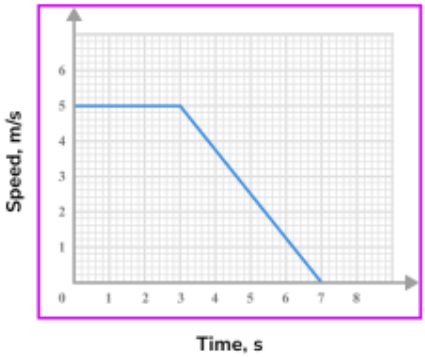
Distance travelled = 125 m

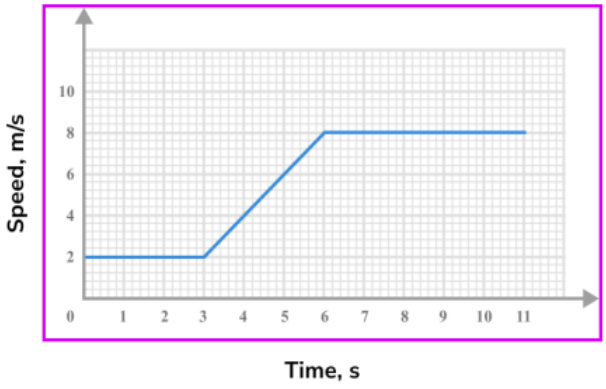
$$\text{Acceleration} = \frac{change\ in\ speed}{change\ in\ time}$$

$$\text{Acceleration} = \frac{50m/s}{5s} = 10\ \text{m}/s^2$$

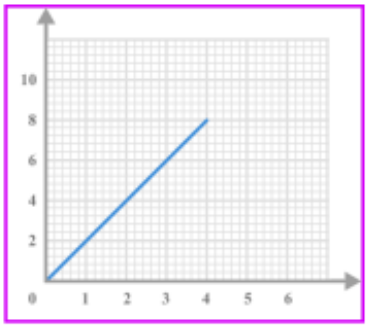
Find the total distance from the speed -time graphs.



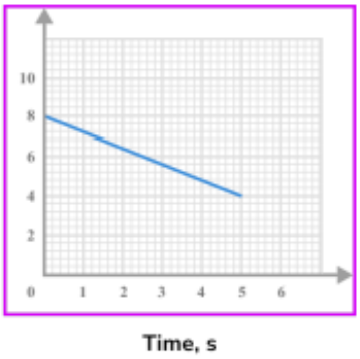




Find the acceleration



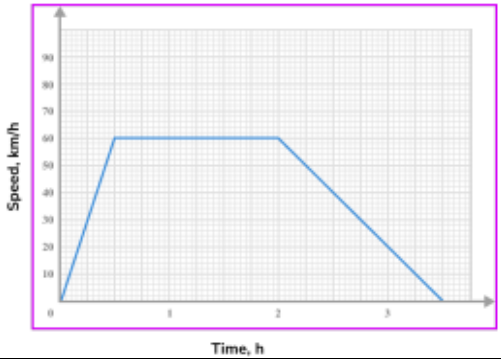
Find the acceleration



Find the acceleration in

(a) The first part of the journey

(b) The last part of the journey



Topic 13: Identities and Quadratics

Expanding brackets: Examples

Single bracket	More than one single bracket	Double brackets
$2(g+4) = 2g+8$ $2x(x+y) = 2x^2+2xy$	$5(x+3)+6(x-4)$ $5x+15+6x-24$ $11x-9$	$(x+4)(x+3)$ $x^2+3x+4x+12$ $x^2+7x+12$

Questions to try: Expand and simplify

$3x(x-2) \equiv$	$2(3x+4)+3(x-4)$	$(x-3)(x+7)$
$-(x+2) \equiv$		
$x^2(x+y) \equiv$	$8(x-1)+3(4x-3)$	$(5x+4)(3x+2) =$
$xy(x-xy) \equiv$		$(3x-2)(5x+4) =$
$2x^2(3xy+5x^3) \equiv$	$2(4x+1)-3(2x-2)$	$(7-a)^2$

Factorising: Factorising into a single bracket examplesv

$9x-18 \equiv 9(x-2)$ HCf = 9x	$6x^2-3x \equiv 3x(2x-1)$ HCf = 3x	$9xy-18x \equiv 9x(2y-2)$ HCf = 9x

Questions to try:

$-48a-24$	$-14a^2+28a$	$10xy-5y \equiv$
$-21a+27$	$56a^2+40a$	$15x^2-5x \equiv$
$-36a-54$	$-24z^2-6z$	$20x^2-10x^5 \equiv$
		$15xy^2-21xy+3x \equiv$

Factorising quadratics: Examples

Factorise
$x^2 - 7x + 12$

$ax^2 + bx + c$
$x^2 - 7x + 12$
$(x-3)(x-4)$

$a = 1$
$b = -7$
$c = 12$

ac
x | 12
−3 | −4
+ | −7
b

• 1×12
• 2×6
• 3×4

Factorise
$x^2 + 4x - 5$

$ax^2 + bx + c$
$x^2 + 4x - 5$
$(x+5)(x-1)$

$a = 1$
$b = 4$
$c = -5$

ac
x | −5
5 | −1
+ | 4
b

• 1×5

Factorise $x^2 - 81$

Difference of two squares

$x^2 - 81$ ← difference of two squares.

$(x+9)(x-9)$

Simplify
$$\frac{2x + 8}{x^2 - 16}$$

Simplifying algebraic fractions

$2x + 8 = 2(x+4)$

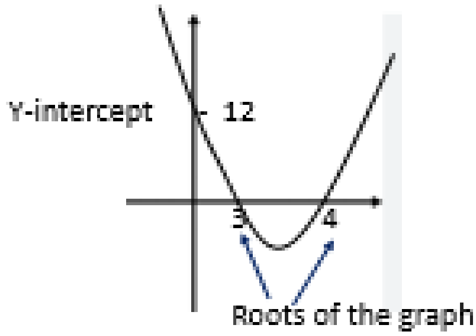
$x^2 - 16 = (x+4)(x-4)$

$$\frac{2(x+4)}{(x+4)(x-4)} = \frac{2}{x-4}$$

Questions to try:

$w^2 + 4w + 3$	$x^2 + 5x + 6$	$x^2 + 9x + 20$	$x^2 + 11x + 28$
$w^2 - 7w - 8$	$x^2 + 5x - 6$	$x^2 - x - 20$	$x^2 - 9x + 14$
$w^2 - 16$	$x^2 - 144$	$9 - y^2$	$4w^2 - 25$

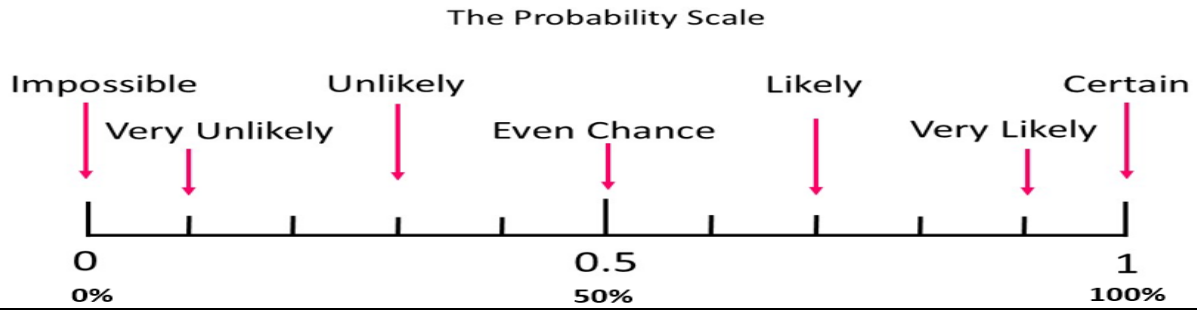
Solving quadratics

By factorising	Sketch the graph
Solve $x^2 - 7x + 12 = 0$ $ax^2 + bx + c$ $x^2 - 7x + 12$ $(x-3)(x-4)$ $a = 1$, $b = -7$, $c = 12$ ac: × 12; −3, −4; + −7 b • 1×12 • 2×6 • 3×4 $(x - 3)(x - 4) = 0$ $x - 3 = 0 \;\; or \;\; x - 4 = 0$ $x = 3 \;\; or \;\; x = 4$	
Practice - Solve and sketch the following quadratic equations.	
$x^2 + 6x + 8 = 0$	
$x^2 - 11x + 18 = 0$	
$x^2 + x - 12 = 0$	
$x^2 - x - 56 = 0$	
$x^2 - 9 = 0$	
$x^2 + 2x = -1$	
$x^2 - x - 8 = 2x + 2$	

.

Topic 14: Probability

The Probability scale



Systematic listing

Example

Marco visits a restaurant with his friends.
Shown is the menu.
Marco chooses one starter, one main and one dessert.
List all possible outcomes.

Starter	Main	Dessert
Soup	Curry	Ice Cream
Fish	Pizza	Danish
	Burger	

SCI	SPI	SBI
SCD	SPD	SBD
FCI	FPI	FBI
FCD	FPD	FBD

P(SOUP with CURRY) = 2/12 = 1/6

Questions

William is going to attend a two day summer camp at his local leisure centre.
He can take part in one activity on Monday and one activity on Tuesday.

Monday	Tuesday
Golf	Ice-skating
Football	Swimming
Rugby	Dodgeball
Hockey	Basketball

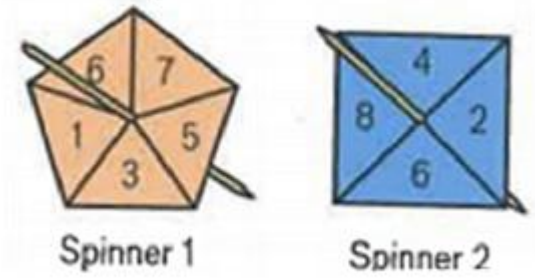
List all the possible combinations of activity he can take part in.

What is the probability that William plays golf?
What is the probability that William does Ice-skating on Tuesday?
What is the probability that William does Ice-skating on both days?

Sample space diagram

Example

We can use sample spaces (a type of table) to show all the possible outcomes of two events.



These spinners are spun and their results are added.

+	1	3	5	6	7
2	3	5	7	8	9
4	5	7	9	10	11
6	7	9	11	12	13
8	9	11	13	14	15

P(at least 7) = 17/20

Questions

Two fair, six-sided dice are rolled.
Their outcomes are multiplied together.
Complete the table.

Dice 2 \ Dice 1	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

P(Prime)
P(Square)
P(Cube number)
P(less than 9)

Expected outcome = probability x number of trials

Example

A biased coin is tossed 300 times.
The probability of it landing on heads is 0.2.

a) How many times would you **expect** to get a head?

$0.2 \times 300 = 60$

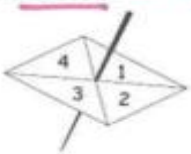
b) How many times would you **expect** to get a tail?

$\rightarrow P(T) = 1 - 0.2 = 0.8$

$0.8 \times 300 = 240$

Example

Here is a four-sided spinner. **The spinner is biased.**



This means that there is not an equal chance of landing on each number.

The table shows the probabilities that the spinner will land on 1 or on 3

Number	1	2	3	4
Probability	0.2	x	0.1	x

$x = 0.35$ $x = 0.35$ $= 1$

Questions

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

(a) Work out the probability that the counter taken is yellow

There are 40 counters in the bag.

(b) Work out the number of blue counters in the bag.

The probability that a biased dice will land on a five is 0.3

Megan is going to roll the dice 400 times.

Work out an estimate for the number of times the dice will land on a five.

Jack sows 300 wildflower seeds.
The probability of a seed flowering is 0.7

Work out an estimate for the number of these seeds that will flower.

There are only red counters, blue counters, white counters and black counters in a bag.

The table shows the probability that a counter taken at random from the bag will be red or blue.

Colour	red	blue	white	black
Probability	0.2	0.5		

The number of white counters in the bag is the same as the number of black counters in the bag.

Tania takes at random a counter from the bag.

(a) Work out the probability that Tania takes a white counter.

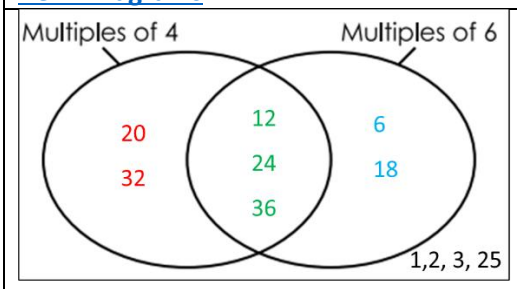
.....

There are 240 counters in the bag.

(b) Work out the number of red counters in the bag.

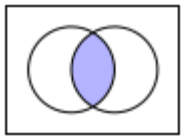
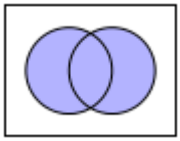
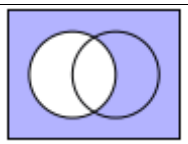
.....

Venn Diagrams

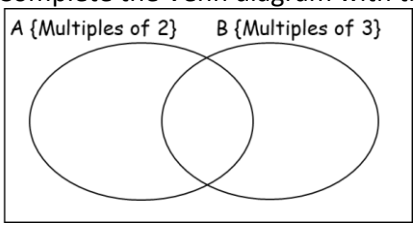


1, 2, 3, 6, 12, 18, 20, 25, 24, 32, 36

P(Multiple of 4 and 6) -> P(A∩B) = 3/11
P(Multiple of 4 or 6) -> P(A∪B) = 7/11
P(Not a multiple of 4) -> Work out P(A)' = 6/11

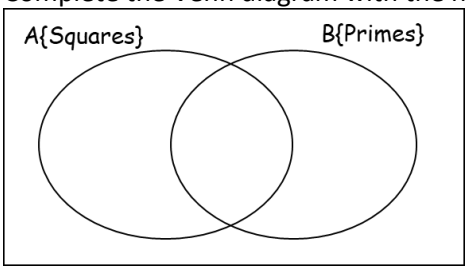
P(A∩B) Intersection of A and B "BOTH"	
P(A∪B) Union of A and B "Either/Or"	
P(A)' Complement of A "Not A"	

Complete the Venn diagram with the numbers 1 to 10

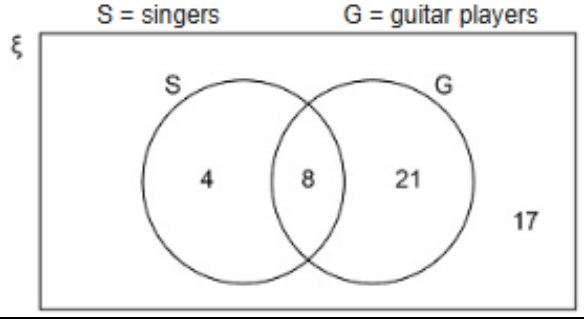


Work out P(A∩B)
Work out P(A∪B)
Work out P(A)'

Complete the Venn diagram with the numbers 1 to 10

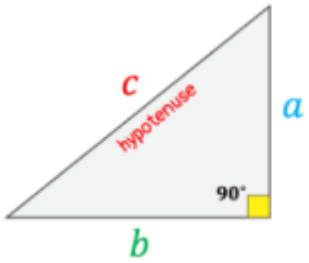


Work out P(A∩B)
Work out P(A∪B)
Work out P(A)'



The Venn Diagram shows information about 50 people who are in bands.
i) Work out ***how many*** people are only singers
ii) Work out P(S∩G)
iii) What is the probability that someone plays guitar
iv) Work out P(S)'
v) A guitarist is chosen at random. What is the probability that they also sing.

Topic 15: Pythagoras' Theorem



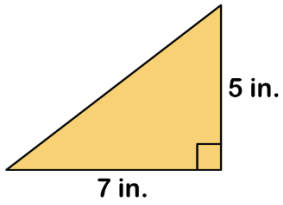
$$c^2 = a^2 + b^2$$

★ $c = \sqrt{a^2 + b^2}$

★ $a = \sqrt{c^2 - b^2}$

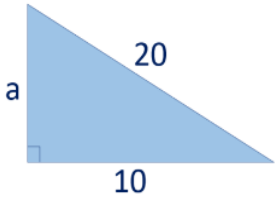
★ $b = \sqrt{c^2 - a^2}$

Examples



$$a^2 + b^2 = c^2$$
$$(5)^2 + (7)^2 = c^2$$
$$25 + 49 = c^2$$
$$74 = c^2$$
$$\sqrt{74} = c$$

c = 8.6

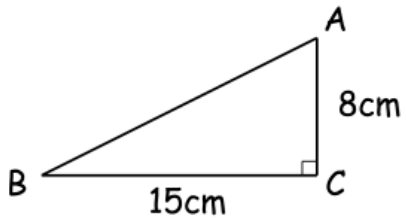


$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{20^2 - 10^2}$$
$$a = \sqrt{400 - 100}$$
$$a = 17.3$$

Questions to try

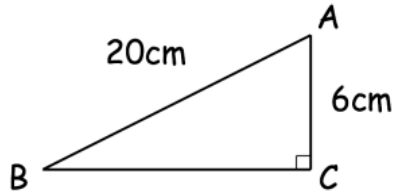
Calculate length AB

Not drawn accurately

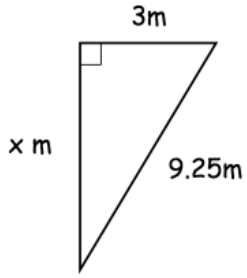


Calculate length BC

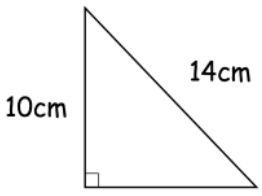
Not to scale



Work out x

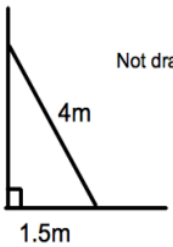


Not drawn accurately



Shown is a right-angled triangle.

Calculate the area of the triangle

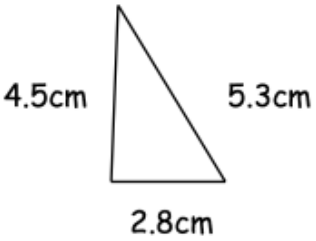


Not drawn to scale

A 4 metre ladder is placed against a vertical wall.
The base of the ladder is 1.5 metres from the base of the wall.

Work out how far the ladder reaches up the wall.

Show that the triangle is right-angled

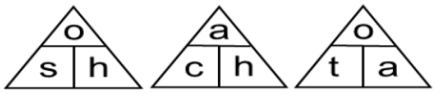


Topic 16: Right-angled Trigonometry (SOHCAHTOA)

Trigonometric ratios

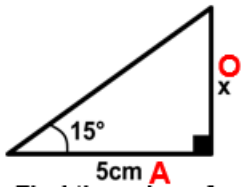
$$\sin\theta = \frac{Opposite}{Hypotenuse} \qquad \cos\theta = \frac{Adjacent}{Hypotenuse} \qquad \tan\theta = \frac{Opposite}{Adjacent}$$

O A O
S H C H T A



Examples

Find the value of side x



$\tan\theta = \frac{Opposite}{Adjacent}$

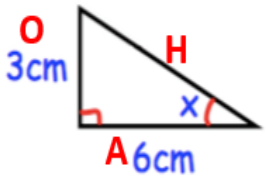
$\tan 15 = \frac{x}{5}$

$x = 5 \times \tan 15$

$x = 1.34cm(3s.f)$

Find the value of angle x

(a)

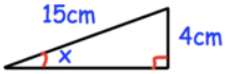


$\tan x = \frac{3}{6}$

$x = \tan^{-1}(\frac{3}{6})$

$x = 26.6°$

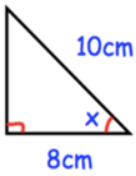
Find the missing angle



1. Label sides
2. Choose equations

3. Substitute the numbers in

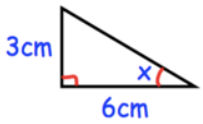
Find the missing angle



1. Label sides
2. Choose equations

3. Substitute the numbers in

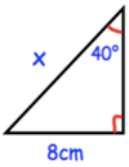
Find the missing angle



1. Label sides
2. Choose equations

3. Substitute the numbers in

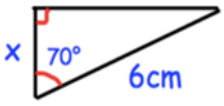
Find the length of the missing side



1. Label sides
2. Choose equations

3. Substitute the numbers in

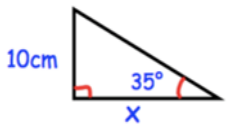
Find the length of the missing side



1. Label sides
2. Choose equations

3. Substitute the numbers in

Find the length of the missing side

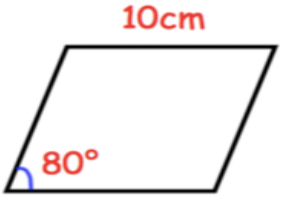


1. Label sides
2. Choose equations

3. Substitute the numbers in

Shown is a rhombus of side length 10cm.

Calculate its area.



Topic 17: Bearings

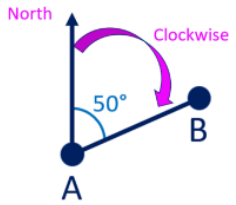
A bearing is an angle measured clockwise from north

Write bearings with 3 figures

Reverse bearings

There is always a difference of 180° between the bearing of A from B compared to B from A.

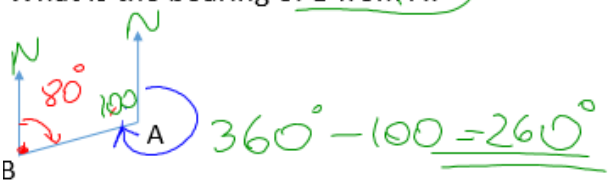
180° is always added or subtracted from the alternate angle.



The bearing of B from A is 050°

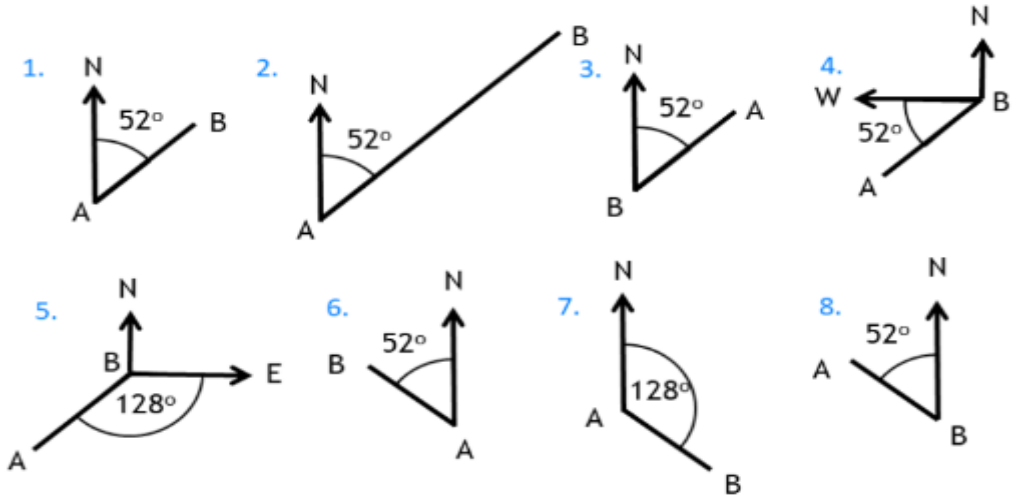
The bearing of A from B is 080°.

What is the bearing of B from A?

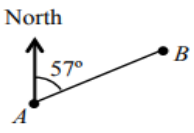


Questions to try

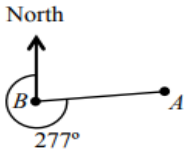
Write the bearing of B from A



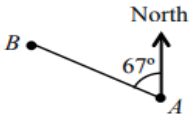
A1 Find the bearing of A from B



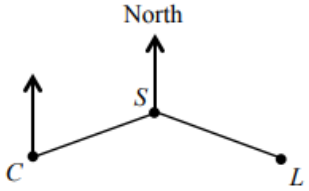
A2 Find the bearing of A from B



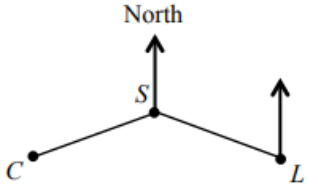
A3 Find the bearing of B from A



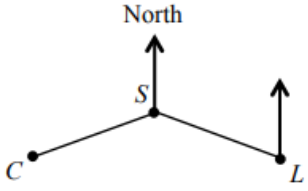
B1 The bearing of S from C is 68°. Find the bearing of C from S



B2 The bearing of L from S is 106°. Find the bearing of S from L



B3 The bearing of S from L is 289°. Find the bearing of L from S

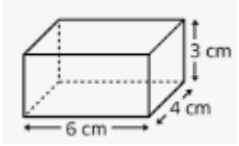


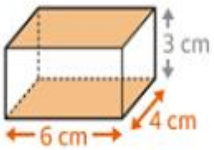
Topic 18: Surface Area

What is surface area?
The surface area of a three-dimensional shape is the total area of all of the faces. To find the surface area of a shape, we find the area of each face and add them together.

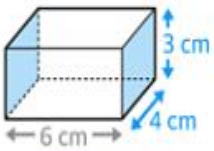
Examples

Work out the surface area of the cuboid

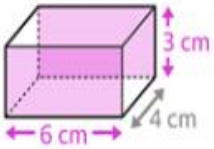




$6 \times 4 \times 2 = 48$
top and base = 48 cm²



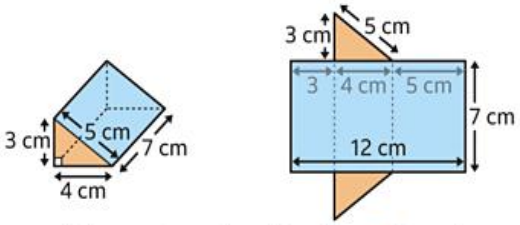
$4 \times 3 \times 2 = 24$
right and left = 24 cm²



$6 \times 3 \times 2 = 36$
front and back = 36 cm²

Surface area = 48 + 24 + 36 = $108cm^2$

Work out the surface area of the prism



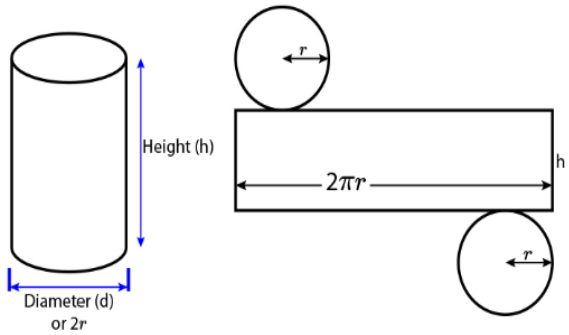
Area of the rectangle = $7 \times 12 = 84$ cm²

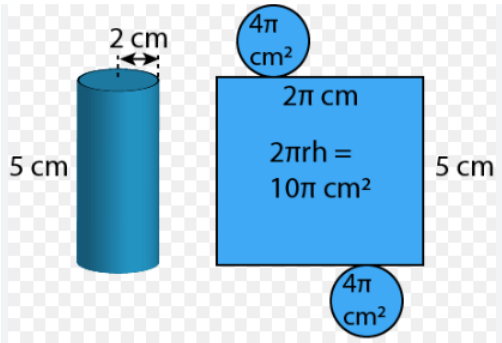
Area of two triangles = $2 \times \frac{1}{2} \times 4 \times 3 = 12$ cm²

$84 + 12 = 96$

Total surface area = **96 cm²**

Work out the surface area of a cylinder

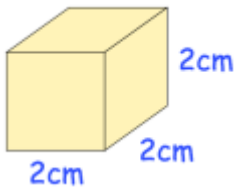


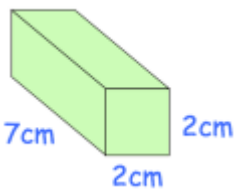


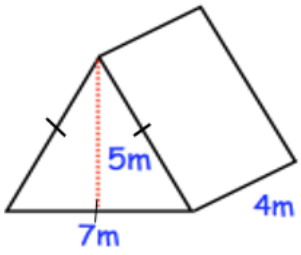
Surface Area = 10π cm² + 8π cm²

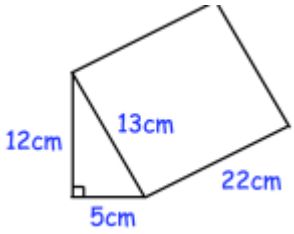
Surface Area = 18π cm²

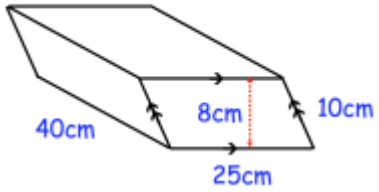
Questions – Work out the surface area of each shape

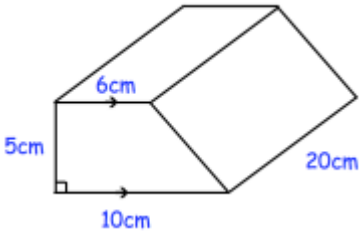


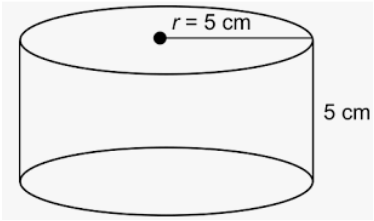


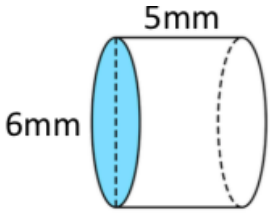

5m
4m
7m


12cm
13cm
22cm
5cm


40cm
8cm
10cm
25cm


6cm
5cm
20cm
10cm


r = 5 cm
5 cm

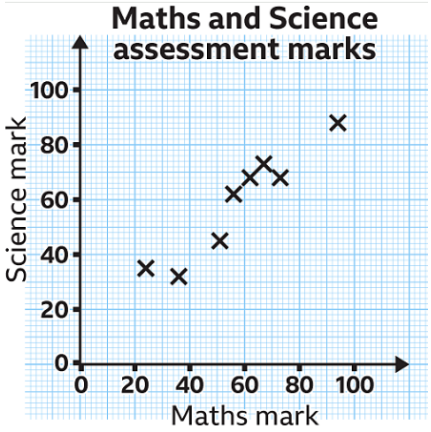

5mm
6mm

Topic 19: Data Handling: MMMR and Scatter Graphs

Scatter graphs

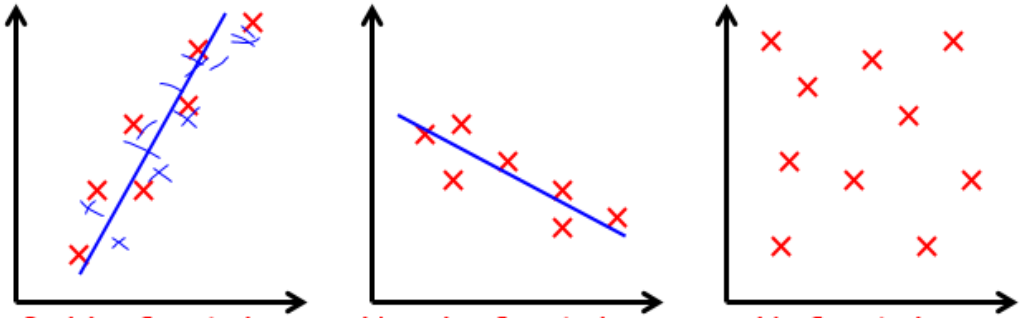
The table shows the results of eight students' maths and science marks in an assessment. Construct a scatter graph that represents these results.

Student	A	B	C	D	E	F	G	H
Maths mark	36	94	67	24	62	51	73	56
Science mark	32	88	73	35	68	45	68	62



Line of Best Fit

→ A line of best fit roughly follows the pattern of the points
→ It does NOT have to go through any points or start on the axes...

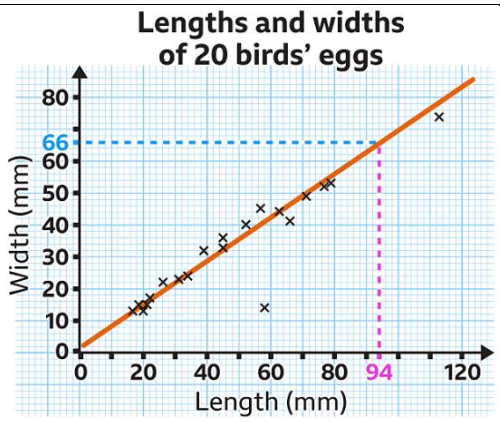


Positive Correlation
→ As one value increases, the other increases

Negative Correlation
→ As one value increases, the other decreases

No Correlation
→ There is no pattern in the data (and therefore no line of best fit...)

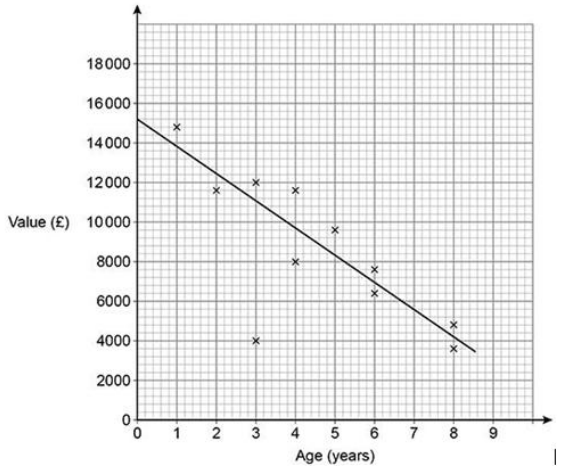
The line of best fit can be used to make predictions. For example, an egg of length 94 mm would have an expected width of 66 mm.



Questions

The scatter diagram shows the age and value of some cars in 2019

All the cars were of the same make and model.



(a) What type of correlation does the scatter graph show?

Answer _____

(b) Write down the value of the car that was an outlier.

_____

Answer £ _____

(c) Use the graph to estimate the value of a new car of this make and model in 2019

Answer £ _____

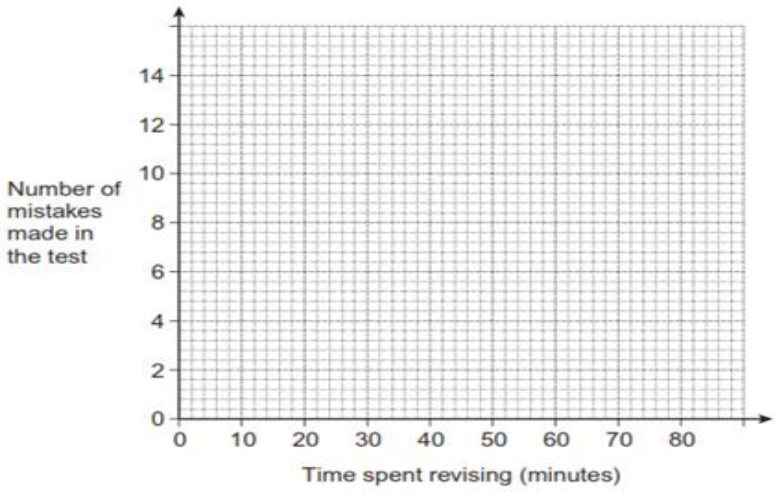
(d) A car of this make and model had a value of £5600 in 2019

Use the graph to estimate the year in which it was made.

Six pupils took a spelling test.

Time spent revising (minutes)	10	15	35	40	45	50
Number of mistakes made in the test	14	11	5	5	2	3

(a) Plot the data on the scatter diagram.



(b) A pupil revised for 25 minutes.

Use a line of best fit to estimate the number of mistakes he made.

Answer _____

(c) Another pupil in the class revised for 75 minutes.

Did she make any mistakes?

Tick a box.

Yes ☐ No ☐ Cannot tell ☐

Data Handling – Mean Median Mode Range

To find the **mean** we:

1. Find the total frequency (add)
2. Divide by how many there are (divide)

Examples

Work out the mean 5, 7, 1, 9, 8

1. 5 + 7 + 1 + 9 + 8 = 30
2. 30 ÷ 5 = 6

Mean = 6

Work out the mean -3, 7, -9, 11, 4, 8

1. -3 + 7 + -9 + 11 + 4 + 8 = 18
2. 18 ÷ 6 = 3

Mean = 3

To find the **median** we:

1. Write the numbers in ascending order
2. Find the middle numer
   If there is an even number of numbers, we add the two middle numbers and divide by 2

Examples

Work out the median 5, 8, 2, 6, 1

1. 1, 2, 5, 6, 8

Median = 5

Work out the median -1, 5, 12, -4, 8, 20

1. -4, -1, 5, 8, 12, 20

Median = $\frac{5+8}{2} = \frac{13}{2} = 6.5$

The **mode** is the most common

Examples

Work out the mode 1.2, 8, 3, 3.4, 4, 8, 5.1, 8

Mode = 8

Range = largest value – smallest value

Examples

Work out the range 15, 7, 2, 11, 9

Mode = 15 – 2 = 13

Work out the range -2, -6, 9, 3, 19, 11

Mode = 19 - -6 = 25

Find the mean of these data sets

(a) 4, 9, 7, 10, 5 (b) 2, 8, 6, 3, 12, 7, 4 (c) 3, 2, 1, 3, 2, 2, 1, 3, 1, 2, 3, 2, 1

Find the median of these data sets

(a) 5, 1, 4, 6, 8 (b) 9, 1, 3, 6, 7, 8, 9 (c) 6, 4, 7, 1, 3, 8, 1, 10

Work out the mode of these data sets

(a) 5, 6, 6, 7, 8, 10 (b) 1, 1, 1, 4, 6, 8, 12 (c) 5, 5, 7, 7, 7, 8, 8, 9

Work out the range of these data sets

(a) 5, 9, 1, 5, 7, 4, 3 (b) 6, 7, 10, 8, 9, 9 (c) 21, 15, 19, 24, 30, 26

Question 2: **A basketball team plays 8 matches.**
The number of points they score in each match are:

62, 68, 67, 79, 82, 50, 74, 62

(a) Work out the mean number of points scored

(b) Write down the modal number of points scored

(c) Write down the median number of points scored

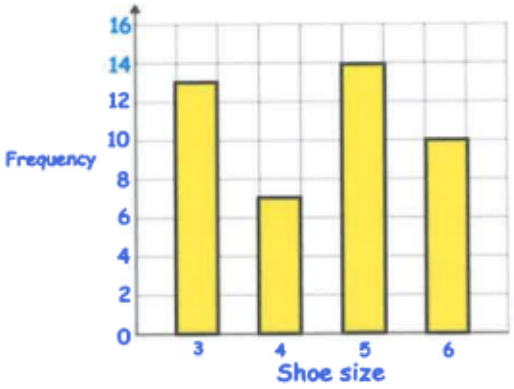
The bar chart shows the shoe sizes of a group of students.

(a) How many students in total are there?

(b) What is the modal shoe size?

(c) What is the mean shoe size

(d) What is the median show size?



Problem Solving (Reverse Mean)

Example

The mean of 5 numbers is 12.
Four of the numbers are 9, 7, 15, 21.
What is the fifth number?

call it x

$$\frac{9+7+15+21+x}{5} = 12$$

$\times 5$ $\times 5$.

$$9+7+15+21+x = 60$$

$$52 + x = 60$$

$$x = 8$$

Questions

The mean of four numbers is 10. Three of the numbers are 9, 11 and 7.
Work out the fourth number.

The mean of six numbers is 5. Five of the numbers are 6, 6, 5, 3 and 1.
Work out the sixth number.

The mean of five numbers is 8.2. Four of the numbers are 8, 10, 12 and 10.
Work out the fifth number.

Mean, median, mode and range from a frequency table

Example

The table shows the ages of different students at a summer club.
Using the table work out the mean, median, mode and range.

Age	Frequency
5	2
6	2
7	5
8	1

For this type of table, because there isn't a lot of data we can take of the data and list it

5, 5, 6, 6, 7, 7, 7, 7, 7, 8

$$\text{Mean} = \frac{5+5+6+6+7+7+7+7+7+8}{10} = \frac{65}{10} = 6.5 \text{ years old}$$

Median = 5, 5, 6, 6, 7, 7, 7, 7, 7, 8 = 7 years old

Mode = 7 years old

Range = 8 – 5 = 3 years old

Example

The table shows the number of bedrooms in houses for sale.
Work out the mean.

Too many numbers to list

Add another column called F(x)

Number of bedrooms		Frequency	F(x)
1	×	34	34
2	×	275	550
3	×	512	1536
4	×	179	716
Total		1000	2836
		2 2	1 1 1

$$\text{Mean} = \frac{\text{Total F(x)}}{\text{Total frequency}}$$

$$= \frac{2836}{1000} = 2.836$$

Questions

For each table find the mean, median, mode and range:

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

Level	Frequency
3	1
4	9
5	7
6	2
7	1

For each table find the mean:

Money Withdrawn	Frequency
£10	16
£20	19
£30	4
£40	3
£50	6
£60	2

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

Times visited	Frequency
0	131
1	873
2	599
3	205

Siblings	Frequency
0	71
1	25
2	14