# Year 8 Maths

## Topic 1 – Prime Factorisation

### Primes

A number with exactly two factors, 1 and itself.

The first ten prime numbers are

2, 3, 5, 7

11, 13, 17, 19

23, 29

**Questions**

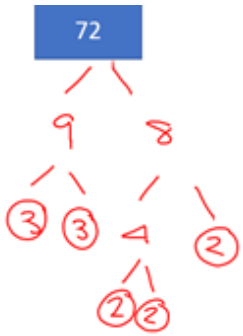
- Is 9 a prime number? Explain your answer.
- Find two prime numbers that sum to make a square number.
- Work out the prime factors of 30
- a and b are prime numbers.

  a + b = 12

  Work out 2 possible values of a – b
- Two prime numbers are multiplied, the answer ends in 0. What are the numbers?

### Product of prime factors

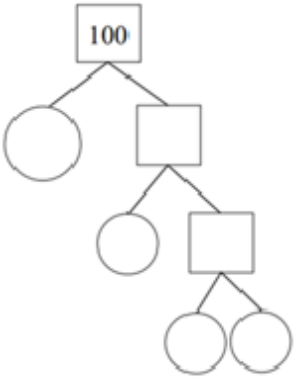
Divide the number by a factor. If it is prime, circle and stop.
Collect the primes and don't forget to "times".




Single index form means write it with powers so
72 = 2 x 2x 2 x 3 x 3
$72 = 2^3 \times 3^2$

**Questions**

Complete this prime factor tree



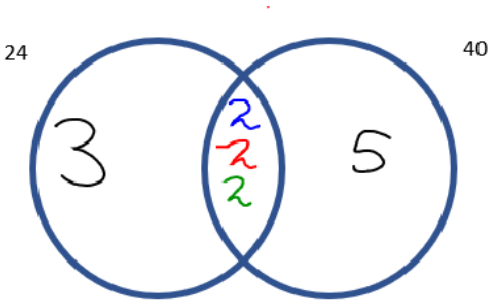

Write 270 as the product of its prime factors

### HCF and LCM

Find the HCF and LCM of ___

1. Prime Factors
2. Fill in Venn
3. Multiply middle for HCF
4. Multiply everything for LCM

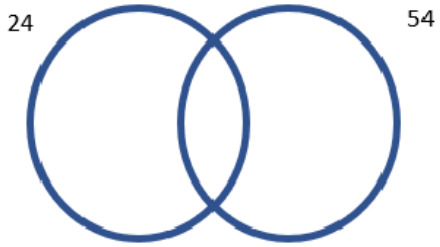
24 = 2 x 2 x 2 x 3
40 = 2 x 2 x 2 x 5




HCF of 24 and 40 = 2 x 2 x 2 = 8
LCM of 24 and 40 = 3 x 2 x 2 x 2 x 5 = 120

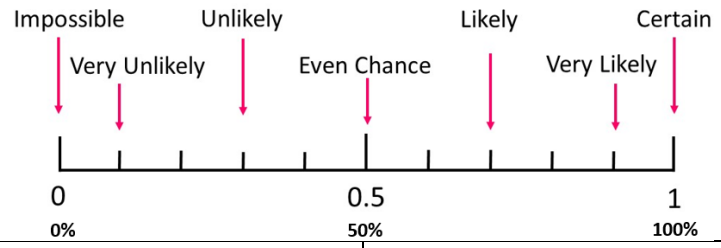
**Questions**

Use the Venn Diagram to find the HCF and LCM of 24 and 54



**Topic 2 – Probability**

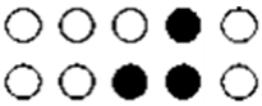
## Probability scale




### Notation and basics

P(Rains)
Means "The probability that it rains"
P(Rains)'
Means "The probability that it does **not** rain"

We measure probability using fractions, decimals or percentages



P(White) = 7/10 or 70% or 0.7

P(White)' = 3/10 or 30% or 0.3

Probability of A + Probability of NOT A = 1

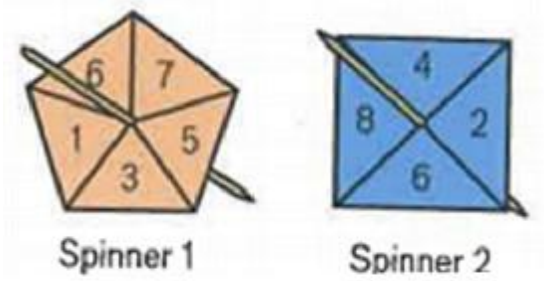
3) The table shows the probabilities of picking counters from a bag. P(Blue) is the same as P(Yellow). Complete the table.

| Colour | Red | Blue | Purple | Green | Yellow |
|---|---|---|---|---|---|
| (P) | 0.12 | | 0.4 | 0.24 | |

0.12 + 0.40 + 0.24 = 0.76 1-0.76 = 0.24
0.24 ÷ 2 = 0.12 Blue and Yellow are both 0.12

### Sample spaces

We can use sample spaces (a type of table) to show all the possible outcomes of two events.

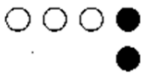



These spinners are spun and their results are added.

| + | 1 | 3 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 8 | 9 |
| 4 | 5 | 7 | 9 | 10 | 11 |
| 6 | 7 | 9 | 11 | 12 | 13 |
| 8 | 9 | 11 | 13 | 14 | 15 |

P(at least 7) = 17/20

### Questions



P(Black) =

P(Black)' =

2) The probability of a football team winning is 0.4, losing is 0.25. What is the probability of them drawing?

3) The table shows the probabilities of picking counters from a bag. P(Purple) is twice as likely as P(Yellow). Complete the table.

| Colour | Red | Blue | Purple | Yellow |
|---|---|---|---|---|
| Probability | 0.44 | 0.2 | | |

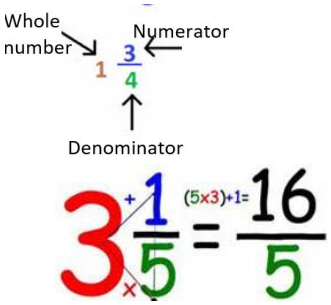
### Questions

Two fair, six-sided dice are rolled.
Their outcomes are multiplied together.
Complete the table.

| Dice 2 \ Dice 1 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

P(Prime)
P(Square)
P(Cube number)
P( less than 9)

**Topic 3 – Operations with Mixed Numbers**

| Comparing fractions and using Improper Fractions and Mixed Numbers | Adding or subtracting mixed numbers | Multiplying or Dividing Mixed Numbers |
|---|---|---|
| Which is bigger $\frac{3}{4}$ or $\frac{4}{5}$<br>You must show your working<br><br>Make common denominator or convert to decimal or percentage.<br><br>$\frac{3}{4}=\frac{15}{20}$ $\frac{4}{5}=\frac{16}{20}$<br><br>$\frac{15}{20}<\frac{16}{20}$<br><br>$\frac{7}{4}$ means $7 \div 4 \rightarrow$<br>  <br> | You can convert to improper fractions first but this can result in large numbers to simplify.<br>$3\frac{1}{2}+5\frac{1}{3}$<br>Add the integers: 3 + 5 = 8<br><br>Add the fractions:<br><br>$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$<br><br>Put together $8\frac{5}{6}$<br><br>***Careful with subtracting***<br>$56\frac{2}{5}-40\frac{3}{7}$<br>$=56\frac{14}{35}-40\frac{15}{35}$<br>$=55\frac{49}{35}-40\frac{15}{35}$<br>$=15\frac{34}{35}$ | Convert mixed numbers to improper fractions first when multiplying or dividing.<br>$5\frac{1}{3}\times 2\frac{1}{4}$<br>$\frac{16}{3}\times\frac{9}{4}$<br>$\frac{4\times 3}{1\times 1}=12$<br><br>$3\frac{3}{4}\div 2\frac{2}{8}$<br>When we divide we multiply by the reciprocal<br><br>K. F. C<br>Keep Flip, Change to X<br>1st, 2nd<br>$\frac{15}{4}\div\frac{18}{8}$<br>Look for Cross-Cancels!<br>$\frac{5}{4}\times\frac{8}{6}$<br>$\frac{5\times 2}{1\times 6}=\frac{10}{6}=1\frac{4}{6}=1\frac{2}{3}$ |
| **Questions**<br>Convert to mixed numbers<br>$\frac{13}{5}$ $\frac{13}{3}$ $\frac{13}{4}$<br><br>Convert to improper fractions<br>$2\frac{1}{5}$ $5\frac{2}{3}$ $6\frac{3}{4}$<br><br>Give your answers as mixed numbers<br>$\frac{5}{8}+\frac{3}{4}=$ $\frac{7}{8}-\frac{4}{5}=$<br><br>Work out the total of<br>$2\frac{5}{6}$ and $5\frac{1}{3}=$<br><br>Work out the difference between<br>$3\frac{1}{4}$ and $7\frac{1}{3}=$ | **Questions**<br>$1\frac{5}{12}\div 2\frac{2}{11}$<br><br>$2\frac{3}{7}\div\frac{1}{2}$<br><br>A sandwich shop sells ham, cheese and tuna sandwiches. ¼ of the sandwiches are ham, 2/5 of the sandwiches are cheese and the rest are tuna. What fraction of the sandwiches are tuna? | **Questions**<br><br>¾ of an orchestra own an instrument. Of those that own an instrument, 1/8 own a violin. What fraction of the orchestra owns a violin?<br><br>A dog eats $2\frac{3}{4}$ kg of food each month. How much would two dogs eat in one year?<br><br>A motor-racing track has $2\frac{3}{4}$ miles of corners and $6\frac{1}{5}$ miles of straights. How long is one lap of the racetrack? |

**Topic 4: Working with Decimals**

## Multiplying and Dividing by decimals

**Multiplying Decimals**

1 d.p. + 2 d.p. = 3 d.p.

$3.2 \times 0.04 = 0.128$

Ignore decimal points

32 x 4 = 128

**Dividing Decimals**

$1.47 \div 0.3$

$= 14.7 \div 3$ — Multiply each number by 10 until we're dividing by a whole number.

$3 \overline{)14.7}$ $\quad 04.9$ — Ensure decimal point goes in same place in result.

$2.53 \div 0.011$

$= 2530 \div 11$

$= 230$

## Rounding to significant figures

Applies to integers and decimals.

The 1st significant figure is the first digit that is not a zero. Ignore any zeros at the beginning of a number

Any zero digits that come after the 1st non-zero are counted.

Example:

| | |
|---|---|
| 13.45 | to 1 significant figure is 10 |
| 136.45 | to 1 significant figure is 100 |
| 2521 | to 1 significant figure is 3000 |
| 0.0562 | to 1 significant figure is 0.06 |
| 0.0506 | to 2 significant figures is 0.051 |

## Approximation

Round all numbers to 1 significant figure.

Do the calculation with the new numbers

$$\frac{312 \times 5.94}{2.03} \approx$$

Round each number to 1sf first

$\frac{300 \times 6}{2}$

$\frac{1800}{2} \approx 900$

**Questions**

45.3 x 1.2

174.9 ÷ 0.03

6.055 ÷ 0.7

**Questions**

Round to 1 significant figure:

119.28 0.068

Round to 2 significant figures

4562 0.4051

**Questions**

Approximate

0.54 x 215.23

Estimate:

$$\frac{241 \times 15.89}{0.1903}$$

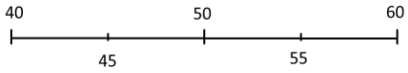
Approximate

$\sqrt{17}$ + 21 x 0.18

## Error intervals

Example

A shelf (x) measures 50cm to the nearest 10 cm.



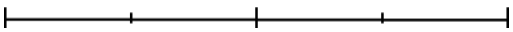

The error interval is 45 ≤ x < 55

The largest ***integer*** length is 54

The smallest ***integer*** length is 45

**Question**

A shelf (x) measures 14m to the nearest metre.



The error interval is ____ ≤ x <____

**Topic 5: Ratio**

## One part known

In a bag there are 2 red counters for every 5 yellow counters. There are 20 red counters in the bag. How many counters were in the bag in total?

| | R | Y | T |
|---|---|---|---|
| R | 2 | 5 | 7 |
| Mx | x10 | x10 | x10 |
| A | 20 | 50 | 70 |

**Multiplier:** $\frac{20}{2}$ **= 10**

**Total is 70 counters**

## Total known

There are 3 circles for every 2 diamonds. If there are 25 shapes in total, how many are circles?

| | C | D | T |
|---|---|---|---|
| R | 3 | 2 | 5 |
| Mx | x5 | x5 | x5 |
| A | 15 | 10 | 25 |

**Multiplier:** $\frac{25}{5}$ **= 5**

**Circles = 15**

## Fractions and Ratios

At a concert, the ratio of adults to children is 3:4.
What proportion of the people are adults?
What fraction are children?

| | Adult | Child | Total |
|---|---|---|---|
| Ratio | 3 | 4 | 7 |

*Note: no amount or multiplier*

Adult Fraction = $\frac{3}{7}$

Child Fraction = $\frac{4}{7}$

Note: The words fraction and proportion are interchangeable.

## Maximising Ratio

Purple paint is made from red paint and blue paint in the ratio 2 : 5. I have 10 litres of red and 15 litres of blue. What's the most purple paint I can make? You must show your working.

1) Max out the red

| | R | B | T |
|---|---|---|---|
| R | 2 | 5 | 7 |
| Mx | X5 | x5 | x5 |
| A | 10 | 25 | 35 |

Not enough blue (I have 15 litres and need 25)

1) Max out the red

| | R | B | T |
|---|---|---|---|
| R | 2 | 5 | 7 |
| Mx | x3 | x3 | x3 |
| A | 6 | 15 | 21 |

I do have enough red
So the max purple is 21 litres

**Questions**

Two numbers are in the ratio 3:7

One of the numbers is 42.

Work out the two possible totals of the numbers. You must show your working.

**Questions**

Share 48kg in the ratio
1 : 3 : 4

| | A | B | C | T |
|---|---|---|---|---|
| R | | | | |
| M | | | | |
| A | | | | |

**Questions**

A farmer keeps cows and sheep on his farm. The ratio of cows to sheep is 2:3
a) What proportion of the animals on the farm are sheep?
b) What fraction of the animals on the farm are cows?

**Topic 6: Speed**

## Distance

Amina is driving at 45mph. How far would she drive in 20mins?

| Distance | Time |
|---|---|
| 45 miles | 60 mins |
| ÷ 3 | ÷ 3 |
| 15miles | 20mins |

Chelsea is driving at 45mph. How far would she drive in 36mins?

| Distance | Time |
|---|---|
| 45 miles | 60 mins |
| ÷ 5 | ÷ 5 |
| 9miles | 12mins |
| x3 | x3 |
| 27miles | 36mins |

**Questions**

Work out the distance travelled when I travel at:

- 12mph for 10 minutes
- 12mph for 40 minutes
- 48mph for 36 minutes

## Speed

Dawud travels 240km in 3 hours. What is his speed in km per hour?

| Distance | Time |
|---|---|
| 240 km | 3 hours |
| ÷ 3 | ÷3 |
| 80 km | 1 hour |

80km in 1 hour
80km/h

Hannah travels 10 miles in 20 minutes hours. What is her speed in mph?

| Distance | Time |
|---|---|
| 10 miles | 20 mins |
| x3 | x3 |
| 30 miles | 60mins |

30miles in 1 hour
30mph

**Questions**

Work out the speed if I travel

- 20miles in 4 hours
- 8 miles in 12 minutes
- 1.6 miles in 24 minutes

## Scaling up (changing units)

Fred flies a distance of 2km in 10 seconds. What is his speed in kph?

| Distance | Time |
|---|---|
| 2km | 10 secs |
| x 6 | x 6 |
| 12km | 1 minute |
| x 60 | x 60 |
| 720 km | 60 mins |

Speed = 720 kph

**Questions**

Fred flies a distance of 4 km in 15 seconds. What is his speed in kph?

**Topic 7: Percentages**

| Percentage of an amount with a calculator (RMS) | Increase and Decrease by % using a calculator | Reverse percentage |
|---|---|---|
| The fastest way to work out percentages is using a multiplier.<br><br>**Rate (%) :**<br>**Multiplier:**<br>**Sum :**<br><br>Find 18% of £365<br><br>**Amount :** 365<br>**Rate (%) :** 18%<br>**Multiplier:** 18/100 = 0.18<br>**Sum :** A x M<br>365 x 0.18 = £65.7**0**<br><br>(£ is always given to 2dp) | Q) Increase £425 by 14.5%<br><br>[increase means we **add** the % rate to 100%]<br><br>**Amount :** 425<br>**Rate (%) :** 100 + 14.5 = 114.5%<br>**Multiplier :** 114.5/100 = 1.145<br>**Sum :** 425 x 1.145<br><br>486.625 = £486.63<br><br>Q) Decrease £425 by 12%<br><br>[decrease means we **subtract** the % rate from 100%]<br><br>**Amount :** 425<br>**Rate (%) :** 100 - 12 = 88%<br>**Multiplier :** 88/100 = 0.88<br>**Sum :** 425 x 0.88<br><br>486.625 = £374 | A train ticket has increased by 20%. The new price is £14.50.<br>What was the original price?<br><br>**Original :** ?<br>**Rate (%):** 100 + 20 = 120%<br>**Multiplier :** 120/100 = 1.20<br>**Sum :** ? x 1.20 = £14.50<br><br>Notice we know the "answer" this time.<br><br>To work backwards we have to divide<br><br>14.50 ÷ 1.20 = 12.0833..<br>= £12.08<br><br>A sofa has decreased in price by 20%. The new price is £340.<br>What was the original price?<br><br>**Original :** ?<br>**Rate (%) :** 100 - 20 = 80%<br>**Multiplier :** 80/100 = 0.80<br>**Sum :** ? x 0.80 = £340<br><br>To work backwards we have to divide<br><br>£340 ÷ 0.80 = £425 |

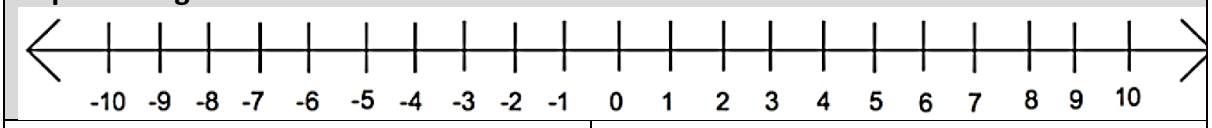
**Questions**

- Find 24% of £360
- Find 112% of £480
- Find 0.6% of £340
- Increase £540 by 6%
- Decrease £540 by 6%

- A house increases in value by 12.5%. It was originally worth £210,000. What is the new value?
- A car costs £10,500. I pay a deposit of 20% then pay the rest in equal monthly instalments for 4 years. How much do I pay each year?
- A house value has increased by 22%. The new value is £240,000.
  What was the original value?
- There are two special offers on coffee.
  Offer A: A 300g jar, with "1/4 off" its normal price of £5.00.
  Offer B**:** A 250g jar with "20% off" its normal price of £4.50.
  Which jar is better value?

# Year 8 Maths - Spring Term

## Topic 8 - Negative Numbers




### Comparing negatives

The number line is symmetrical.
The bigger the value next to the negative sign, the smaller it is.
For example, -25 is smaller than -5

Ascending Order – Smallest to Biggest

Descending Order – Biggest to Smallest

Write in **ascending** order:

9 , -3 , 7 , 0 , -1

−3, −1, 0, 7, 9

Write in **descending** order: down

-2 , 7 , -5 , -15 , 5

7, 5, -2, -5, -15

### Multiplying negatives

**Rules for multiplying with negatives:**

(-) × (-) = (+)
(-) × (+) = (-)
(+) × (-) = (-)

a) $4\times(-3)$
b) $(-15)\div(-5)$
c) $6\times(-4)$
d) $20\div-10$
e) $(-5)\times 7$

### Dividing negatives

**Rules for dividing with negatives:**

(-) ÷ (-) = (+)
(-) ÷ (+) = (-)
(+) ÷ (-) = (-)

a) $(-1)\times 2\times(-3)\times 4$
b) $(-2)^2$
c) $-5(-2)^2$
d) $-40\div\ ?=-10$
e) $-100\div\ ?=20$

### Adding and Subtracting Negative Numbers

If we have two signs **next** to each other change the signs as follows:

++ = +  - - = +
+ - = -  - + = +

Examples: 5 (+ -) 7 = 5 – 7 = -2
5 (- -) 7 = 5 + 7 = 12

Examples

a) 8 + - 3 — 8 – 3 = 5
b) 9 - + 5 — 9 – 5 = 4
c) 2 - - 9 — 2 + 9 = 11
d) 3 + - 4 — 3 – 4 = -1
e) 10 + - 16 — 10 – 16 = -6
f) -4 + - 7 — -4 – 7 = -11
g) -1 - - 6 — -1 + 6 = 5
h) -3 + 9 — 9 – 3 = 6
i) -7 – 4 — -(7 + 4) = -11
j) -12 + - 11 — -(12 + 11) = -23

In these calculations, where the signs are NEXT to each other, the rule above applies, when the signs are NOT next to each other, we just move up / down the number line

Examples:

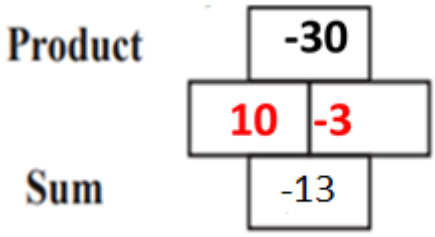
- 5 – 7 = – 12
- 5 + 7 = 2

(a) $-4+6$  (b) $-5+8$
(d) $-4+7$  (e) $2+(-3)$
(g) $-2+(-3)$  (h) $-6+6$
(j) $-6+2$  (k) $-7+2$

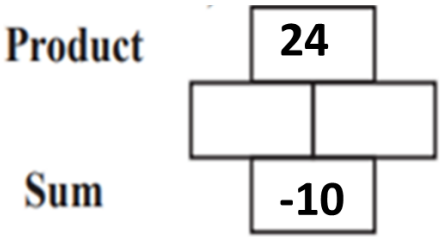
## Product and sum

**Example**

Product means multiply, sum means add




Question




## Topic 9: Identities – Expanding, Simplifying and Factorising

1. Multiply the term outside the bracket by the first term inside.

2. Multiply the term outside the bracket by the second term inside.

3. Collect like terms if possible.

$$2(x+3) \equiv 2x+6$$

$$6(x+6) \equiv 6x+36$$

$$2(3x-10) \equiv 6x-20$$

$$2(3a+6) = 6a+12$$

**Examples with powers**

$x(x+7)$ $\quad x^2+7x$

$x(4x+5)$ $\quad 4x^2+5x$

$3x(2x+5)$ $\quad 6x^2+15x$

**Questions to try**

$x(x-6)$

$x(3-x)$

$3x(2x+5)$

$4x(6x+9)$

$2x(4-2x)$

Expand and Simplify

E.g.1 $3(2x-5)-8(x-2)$

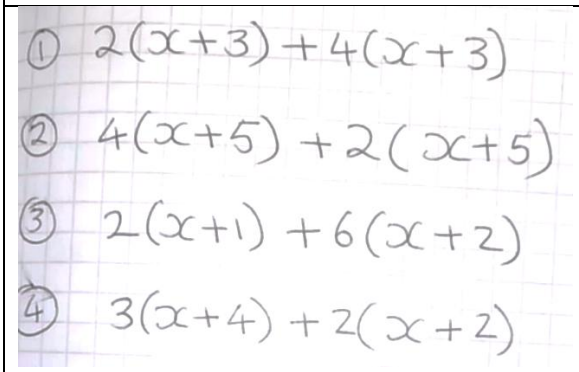
$= 6x-15-8x+16$

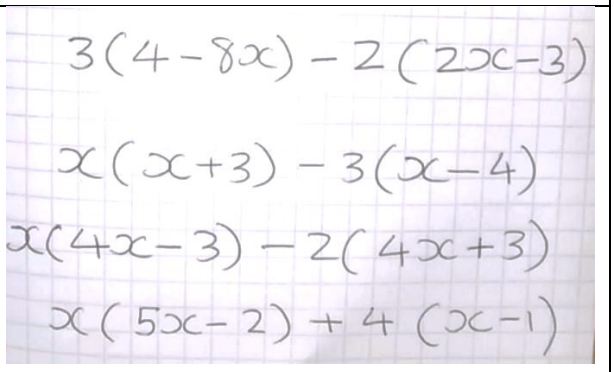
$= -2x+1$

E.g.2 $4(2x+3)+2(3x-4)$

$= 8x+12+6x-8$

$= 14x+4$



## Factorising – The opposite of expanding brackets

Factorise $10x + 15$.

Find the **HCF** of the numbers. → $10x + 15$ ← **Divide each term by the HCF and close the bracket.**

**HCF** = $= 5(2x + 3)$

Write the **HCF** and **'open' the brackets.**

Factorise,

**[a]** $2x + 6$

**[b]** $3t + 9$

**[c]** $5a - 15$

Factorise $12a - 8$.

Find the **HCF** of the numbers. → $12a - 8$ ← **Divide each term by the HCF and close the bracket.**

**HCF** = $= 4(3a - 2)$

Write the **HCF** and **'open' the brackets.**

**[d]** $3d + 12$

**[e]** $6h - 9$

**[f]** $12e - 18$

**[g]** $14r + 35$

Factorise $2ab + 4b$.

Find the **HCF** of the **variables.** → $2ab + 4b$ ← **Divide each term by the HCFs and close the bracket.**

**HCF** =

**HCF** = $= 2b(a + 2$

Only **'open the brackets'** once **all HCFs are found.**

Factorise,

**[a]** $6gh + 12h$

**[b]** $2ab + ad$

**[c]** $9rt - 12r$

**[d]** $jk + k$

**[e]** $2x + 6xy$

**[f]** $14us - 21ut$

**Example: Factorising letters**

$x^2 + 5x = x(x + 5)$

HCF in this case is $x$ so $x$ is outside the bracket

$6x^2 - 9x = 3x(2x - 3)$

HCF in this case is $3x$ so so $3x$ is outside the bracket

**Questions to try**

Factorise these completely

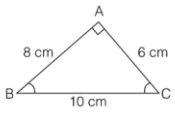
$3x^2 + 15x =$

$12x^2 - 9x =$

$10x^2y + 5yx =$

# Topic 10: Enlargement and Similarity (Scale Factors)

## Example: Labelling sides



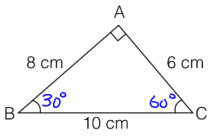

We use two capital letters to show the distance between two points.
In a shape this is the length of one side.

AB = 8cm

BC = 10cm

AC = 6cm

## Example: Labelling angles



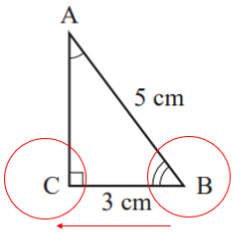

We use three capital letters to show an angle. This is the turn from facing one point to another.

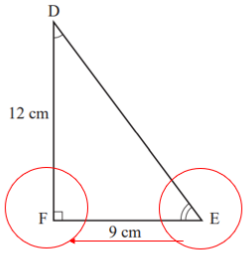
$A\hat{B}C = 30$

$A\hat{C}B = 60$

$B\hat{A}C = 90$

## Shapes ABC and DEF are similar




AC = ☐ cm

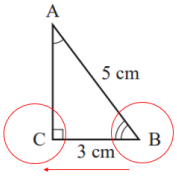
DE = ☐ cm

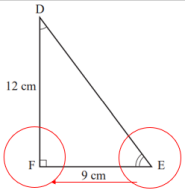
Side BC (3cm) is between the right angle and the angle marked with two curves

Side EF (9cm) is between the right angle and the angle marked with two curves

**This makes them the corresponding sides**

Shapes ABC and DEF are similar

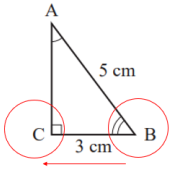


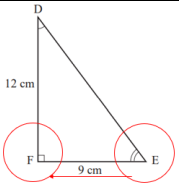




$\frac{EF}{BC} = \frac{9}{3}$

Scale factor from ABC to DEF is x 3

Shapes ABC and DEF are similar




$\frac{BC}{EF} = \frac{3}{9}$

Scale factor from DEF to ABC is x $\frac{1}{3}$

Remember this is the same as ÷ 3

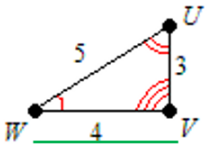
## Finding scale factors and using it to find missing sides

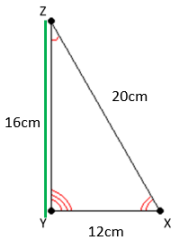
1) Angles are shown to be equal so mathematically similar
2) Highlight one pair of corresponding sides
3) Work out scale factor

$\frac{YZ}{VW} = \frac{16}{4} = 4$

4) Check at least one other pair

$\frac{UW}{XZ} = \frac{20}{5} = 4$



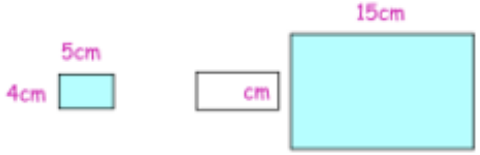

The scale factor from UVW to XYZ is x 4

The scale factor from XYZ to UVW is x $\frac{1}{4}$

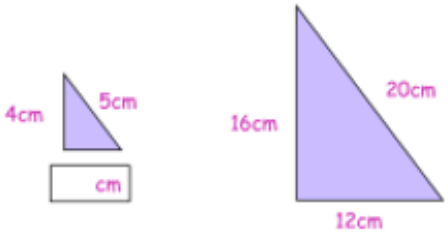
**Question 1:** Below are pairs of similar shapes.
Find the missing lengths.

(a)

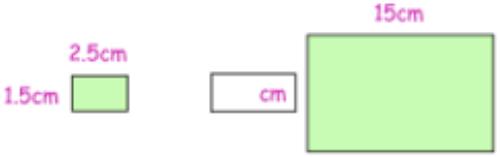

(b)




(c)

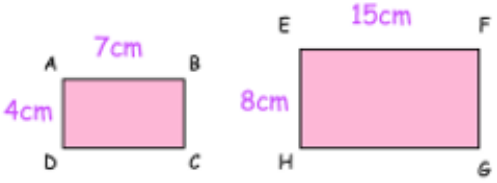



(d)

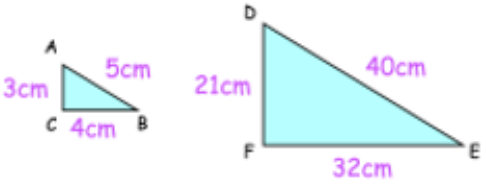



**Question 2:** These pairs of shapes are **not** similar.
Explain why.

(a)

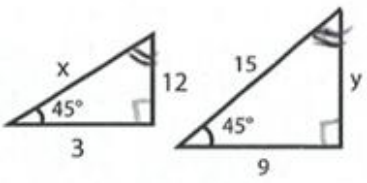



(b)



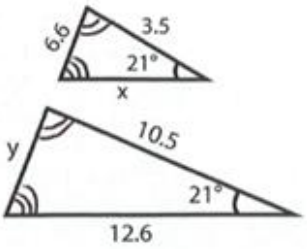

Practice Questions – use the scale factor to find missing lengths with some different orientations

1



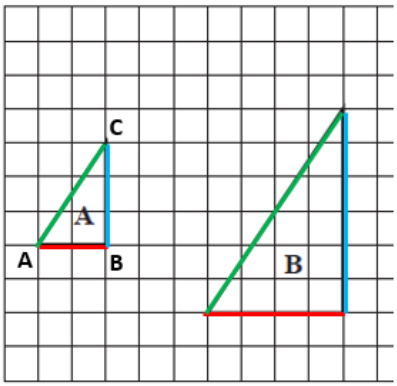

x = _____ , y = _____

2



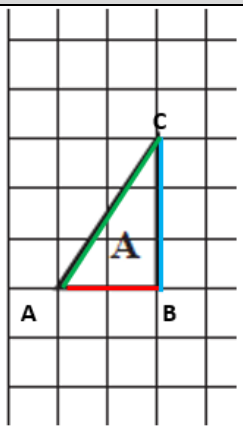

x = _____ , y = _____

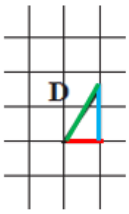
## Enlargement




B is an enlargement as all the lengths of A have been doubled.

The scale factor of the enlargement from A to B is 2



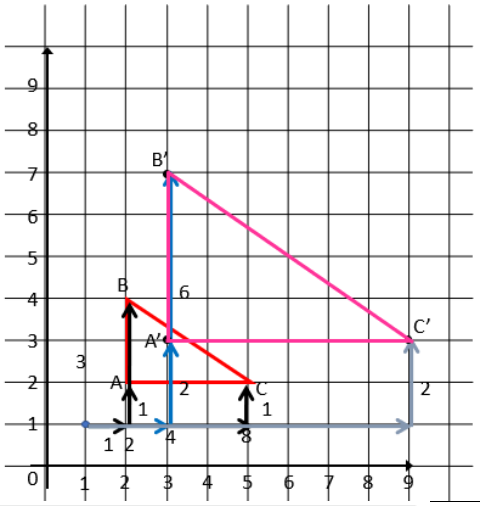

D is an enlargement

All of the lengths have been halved.

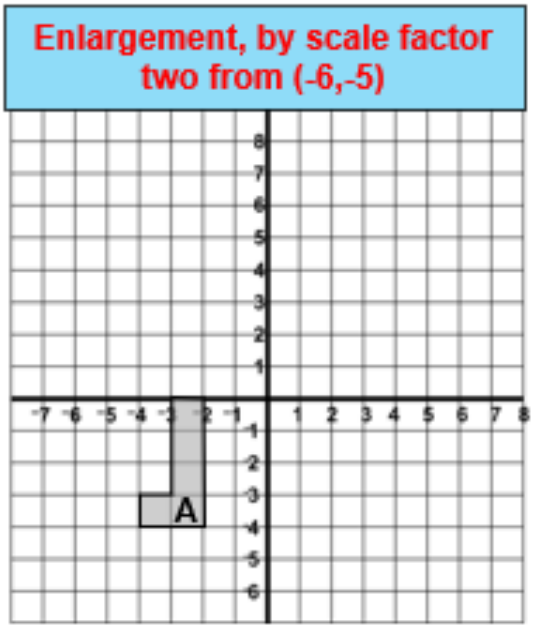
The scale factor of the enlargement from A to D is ½

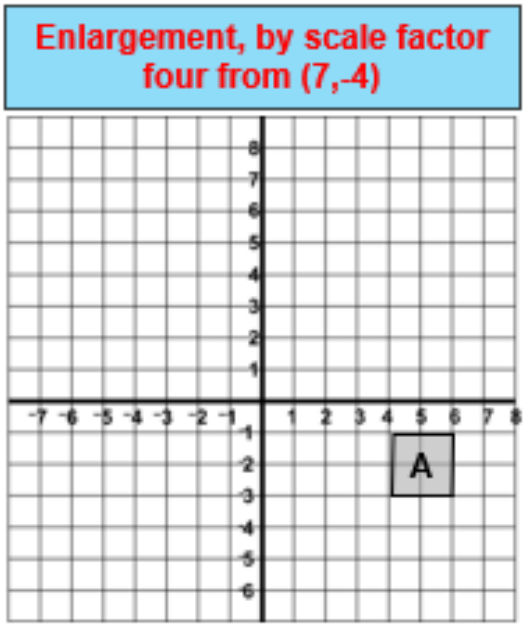
## Using a "Centre of enlargement"

Enlarge triangle ABC using a scale factor of 2, centre of enlargement (1, 1).




A scale factor of 2 means the length of the shape is multiplied by 2 **and** the distance from the centre of enlargement to each **vertex** is multiplied by 2.



**Topic 11: Solving Equations**

Solve by using inverse operation

$x + 3 = 6$

$-3 \quad -3$

$x = 3$

$\frac{x}{2} = 3$

$\times 2 \quad \times 2$

$x = 6$

$4x = 16$

$\div 4 \quad \div 4$

$x = \frac{16}{4}$

$x = 4$

$x - 5 = 16$

$+5 \quad +5$

$x = 21$

**Try these**

| | |
|---|---|
| x + 5 = 2 | x – 6 = -4 |
| x + 8 = -2 | x – 4 = - 10 |
| 4x = 12 | 12x = 4 |
| 3x = -12 | -12x = 6 |
| $\frac{x}{3} = 12$ | $-8 = \frac{x}{4}$ |

Guided Examples: The balance method

Q) $2x + 3 = 13$

B) $-3 \quad -3$

A) $\frac{2x}{2} = \frac{10}{2}$

B) $\div 2 \quad \div 2$

A) $x = 5$

Checked ✓ $2(5) + 3$

Q) $4x - 3 = 9$

B) $+3 \quad +3$

A) $4x = 12$

B) $\div 4 \quad \div 4$

A) $x = 3$

Checked ✓

**Try to Solve these**

i) $2x - 3 = 11$

*ii*) $4x + 9 = 25$

iii) 7y + 2 = 30

iv) 67 = 9w – 5

v) 8m – 4 = 20

vi) 8x + 2 = 6

**Equations with brackets**

$2(x + 3) = 12$

$2x + 6 = 12$

$-6 \quad -6$

$2x = 6$

$\div 2 \quad \div 2$

$x = 3$

$2(3+3) = 12$

$2(6) = 12$ ✓

EQUATIONS WITH

$9 = 4(x + 3)$

$9 = 4x + 12$

$-12 \quad -12$

$-3 = 4x$

$\div 4 \quad \div 4$

$\frac{-3}{4} = x$

OR

$-0.75 = x$

**Try these**

a) 2 (4x + 8) = 32

b) 5 (7x- 4) = 15

c) 5 (5x- 3) = 35

d) 4 (6x + 3) = 36

Equations with fractions

$\frac{x-5}{2} = 9$ | $\frac{x}{2} - 5 = 9$

Compare these

$\frac{3x}{2} - 12 = 9$ | $\frac{3x-12}{2} = 9$

$\frac{4y}{3} + 3 = 9$ | $\frac{4y+3}{3} = 9$

Equations with two fractions (cross multiplying)

$\frac{14}{2x} = \frac{7}{3}$

Butterfly Method

$42 = 14x$

$3 = x$

$\frac{2}{x} = 6$

Integer into fraction "Butterfly"

$\frac{2}{6} = \frac{6x}{6}$

$\frac{2}{6} = x$

$\frac{1}{3} = x$

$\frac{2x}{7} = \frac{3}{2}$

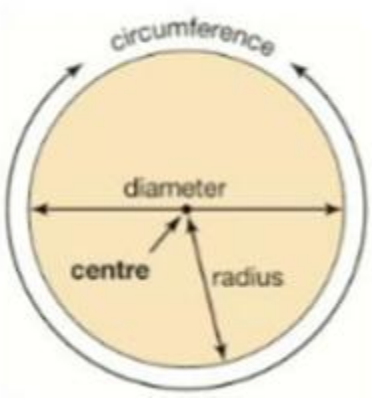
$\frac{2x}{7} = \frac{3}{14}$

$\frac{2x}{14} = \frac{3}{14}$

**Topic 12: Circles** **Area and Circumference**

Parts of a circle

| Part | Definition |
|---|---|
| Circumference | The perimeter of a circle (the distance around the outside) |
| Diameter | A straight line that passes through the centre of a circle and touches two points on the circumference. |
| Radius | A straight line from the centre to the circumference of a circle. Stays the same all the way around (like the hand of a clock) |
| Arc | Part of the circumference |




**Circumference**

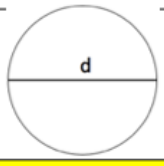
The circumference of a circle is its perimeter.
To calculate the circumference of a circle we use the formula:

$$Circumference = \pi \times diameter$$

We can shorten this to:

$$C = \pi \times d$$

where d is the diameter.




Remember: **C**herry **pi**(e) is **d**elicious

Circumference is a length, so the units are mm, cm, m, km, inches, feet, yards etc.

**Area**

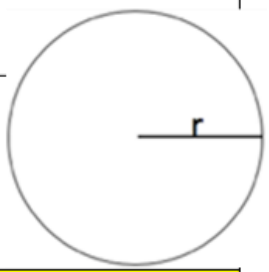
The area of a circle is the number of square units inside that circle. To calculate the area of a circle we use the formula:

$$Area = \pi \times radius^2$$

We can shorten this to:

$$A = \pi \times r^2$$

where r is the radius.

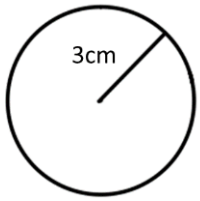
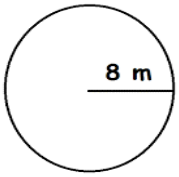
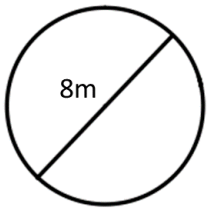
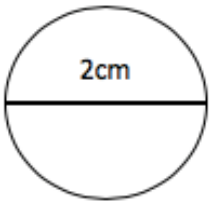
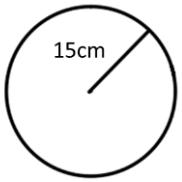



Remember: **A**pple **pi**(es) are **t**oo

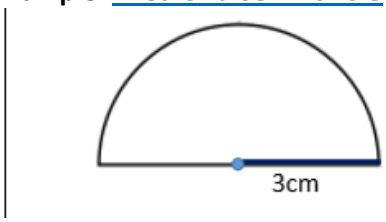
The units of area are squared, so $mm^2$, $cm^2$, $m^2$, $km^2$, square feet, etc.

$\pi$ has the value of approximately $\pi \approx 3.142$

**Try these:** Work out the area and circumference of each of these circles

| a | b | c | d | e |
|---|---|---|---|---|
|  |  |  |  |  |

**Example:** Area of a semi-circle

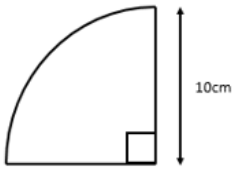



Area = ½ x $\pi r^2$

Area = $\frac{\pi x 3^2}{2}$ = 14.139

Area = 14.1cm$^2$ to 1 decimal place

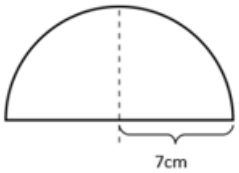
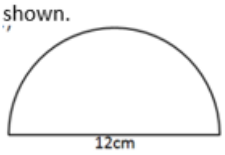
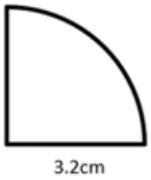
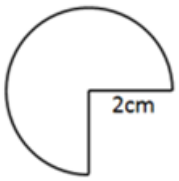
**Example:** Area of a quarter-circle



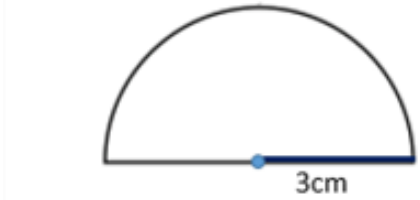

Area = ¼ x $\pi r^2$

Area = $\frac{\pi x 10^2}{4}$ = $\frac{100\pi}{4}$ = 25 $\pi$

Area = 78.6cm$^2$ to 1 decimal place

| a | b | c |
|---|---|---|
| Find the area of the semi-circle  | Find the area of a semi-circle with diameter 9cm. | Find the area of semi-circle shown.  |
| **d** | **e** | **f*** |
| Find the area of the quarter-circle  | Find the area of a quarter-circle with radius 9cm. | Find the area of 3 quarters of a circle  |

Perimeter of a semi-circle




r = 3

d = 6

C = $\pi d$  Arc of semi circle = $\frac{\pi d}{2}$

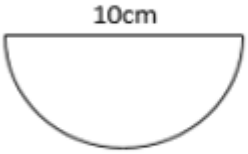
Arc = $\frac{\pi \times 6}{2}$ = $3\pi$

Perimeter = Diameter + Arc

P = 6 + $3\pi$

Your turn:

Find perimeter of these sectors

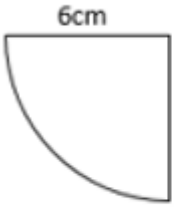



r =

d =

Arc =

P = d + arc =



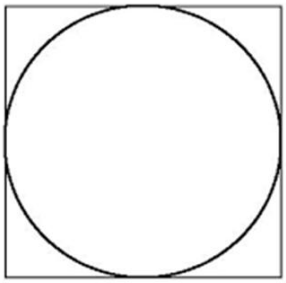

r =

d =

Arc =

P = d + arc =

**Example: Find the area of circle which fits exactly inside a square with area 144cm$^2$**



Width of square = $\sqrt{144}$

Width of square = 12cm

Width of square = Diameter of Circle

Radius = Diameter ÷ 2

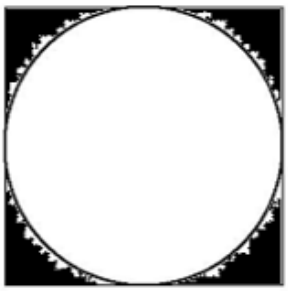
Radius = 6 cm

Area of Circle, $A = \pi r^2$

$A = \pi\ 6^2$

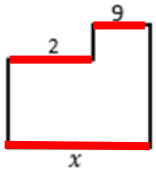
The circle below which fits exactly inside a square with area 64cm$^2$.

Find the shaded area.

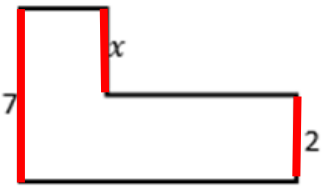


## Topic 13: Area of Trapezia and Compound Shapes

Missing Lengths –
Long line missing, add $x = 2 + 9$

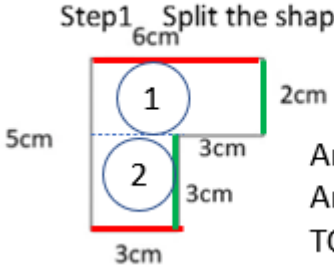



Missing Lengths –
Short line missing, subtract $x = 7 - 2$



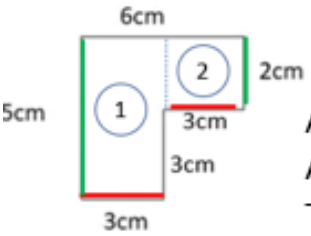

Compound Area – Find any missing sides, split the shape, work out separate areas and add

Step1 Split the shape




Area 1 = 6x2=12
Area 2 = 3x3=9
TOTAL = 21

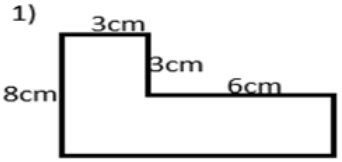
Step1 Split the shape



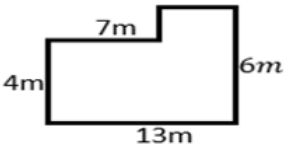

Area 1 = 5x3=15
Area 2 = 3x2=6
TOTAL = 21

Questions to try

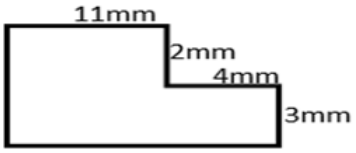
1)



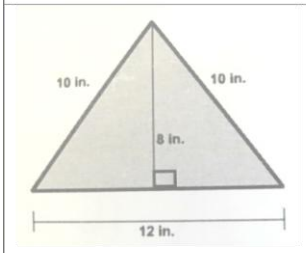
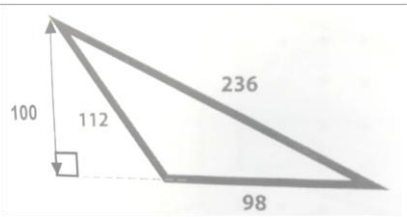
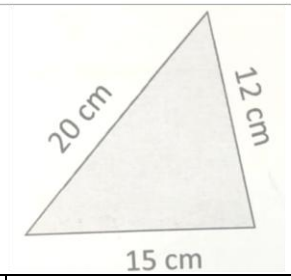

2)




3)




Area of Triangle = Base x Perpendicular Height (at 90 degrees to the base) ÷ 2 $A = \frac{b \times pH}{2}$

| Perpendicular height inside | Perpendicular height given outside of scalene | No perpendicular height |
|---|---|---|
|  |  |  |
| Area = 12 x 8 ÷ 2 = 48 in$^2$ | Area = 98 x 100 ÷ 2 = 4900 | No perpendicular height.<br>Not possible with this formula. |

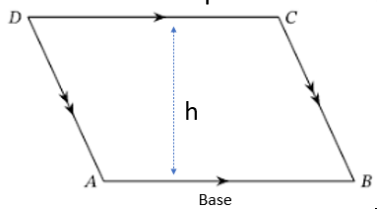






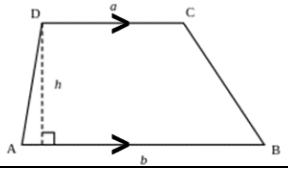
### Area of Parallelogram

Area = Base x Perpendicular Height




### Area of Trapezium

a and b are the parallel sides
h is the perpendicular height

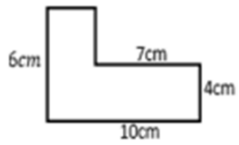



**Area = $\frac{a+b}{2}$ x height**

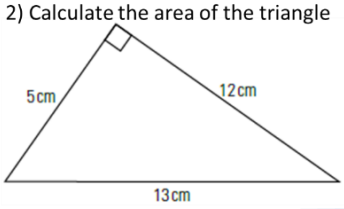
## Questions to try

1) Calculate the area of the compound shape

**Answer in cm²**

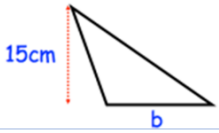



2) Calculate the area of the triangle **Answer in cm²**

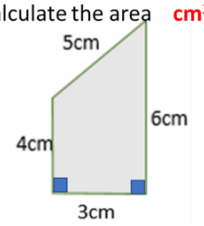



3) What is the base of the triangle

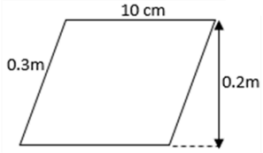
Area = 165cm²



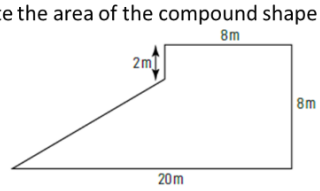

4) – Calculate the area **cm² And mm²**



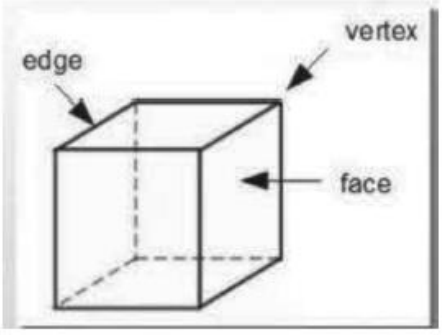

Q5) – Calculate the area **cm²**




6) Calculate the area of the compound shape **Answer in m²**




## Topic 14: 3D Shapes and Volume of Prisms

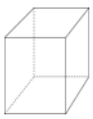
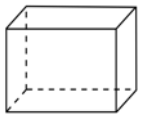
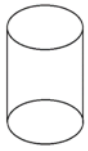
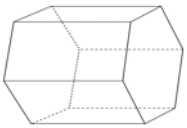
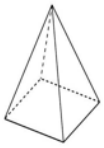
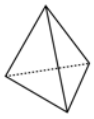
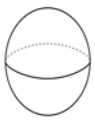



For prisms with flat faces (so NOT cylinders)

F + V – E = 2 (Eulers Law)

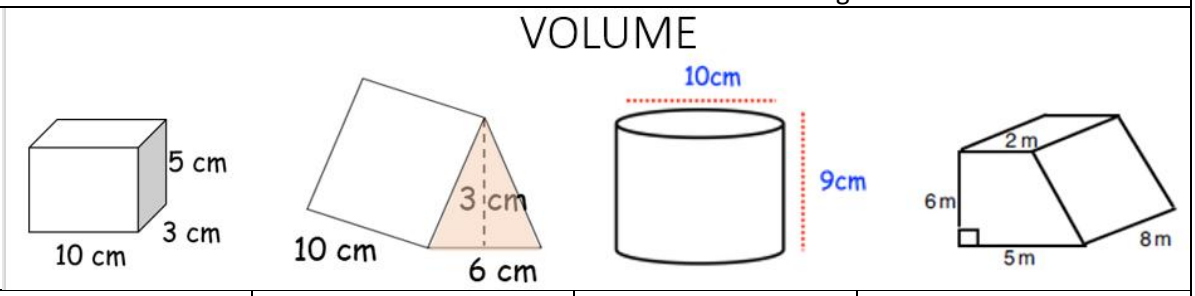
Faces + Vertices – Edges = 2

A prism is a 3D shape with at least one pair of congruent (identical) faces. For example: Cubes, Cuboids, Cylinders, Triangular Prisms..

| Prisms | Non Prisms |
|---|---|
|  Cube  Cuboid  Triangular prism  Cylinder  Hexagonal prism |  Square-based pyramid  Triangular-based Pyramid / Tetrahedron  Cone  Sphere |

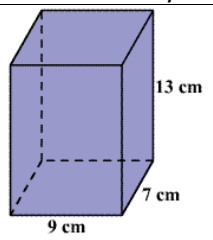
## Volume

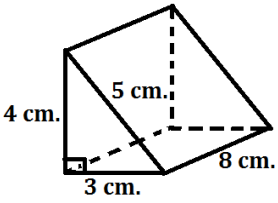
Volume of Prism = Area of Cross Section x Length

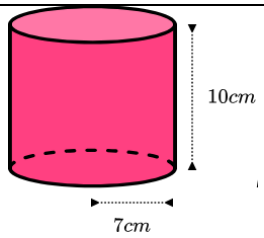


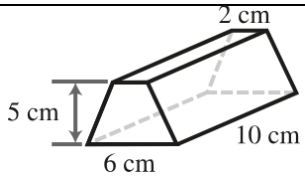

| Cuboid | Triangular prism | Cylinder | Trapezoidal prism |
|---|---|---|---|
| AREA = 3 x 5 = 15<br>VOLUME =<br>15 x 10 = 150cm$^3$ | AREA = $\frac{6 \times 3}{2}$ = 9<br>VOLUME =<br>9 x 10 = 90cm$^3$ | AREA =$\pi \times 5^2$ = 25π<br>VOLUME = 25π x 9<br>=225π = 706.9cm$^3$ | AREA = $\frac{2+5}{2} \times 6$ = 21<br>VOLUME =<br>21 x 8 = 168cm$^3$ |

Questions to try

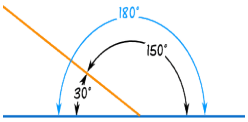
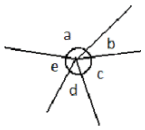
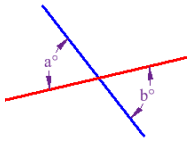
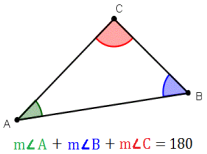
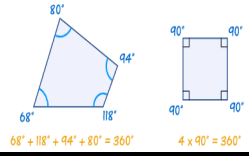


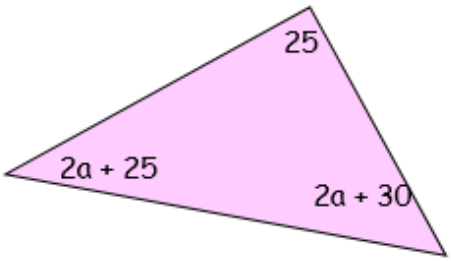




## Topic 15: Angles and Properties of Triangles and Quadrilaterals

| | | |
|---|---|---|
| Angles on a straight line | Add up to 180° |  |
| Touching angles that add up to 180° | Are on a straight line. | |
| Exterior and Interior Angles | Are on a straight line so add to 180° |  |
| Angles around a point | Add up to 360° |  |
| Vertically opposite angles | Are equal (created by a pair of straight lines that cross) |  |
| Angles in a triangle | Add up to 180° |  |
| Angles inside a polygon add to 180° | Must be a triangle | |
| Angles in a quadrilateral | Add up to 360° |  |

**Angles with Algebra:**




**Form and solve equation**

Interior angles of a triangle add up to 180°

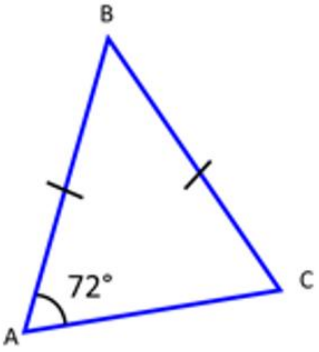
$2a + 25 + 25 + 2a + 30 = 180$

$4a + 80 = 180$

$4a = 100$

$a = 25$

**Isosceles triangles**




Work out angle $A\hat{B}C$

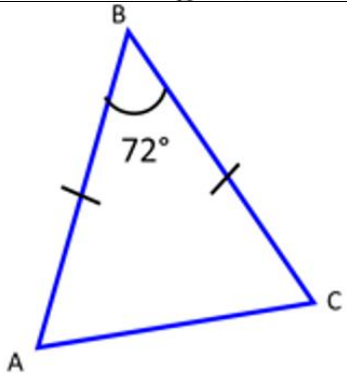
Isosceles Triangle – base angles are equal. Total angle sum = 180

Angle BAC = ACB = 72°

Angle ABC = 180 – (2 x 72)

Angle ABC = 36°

CHECK: 72 + 72 + 36 = 180




Work out angle $B\hat{A}C$

Angle Sum = 180

180 – 72 = 108

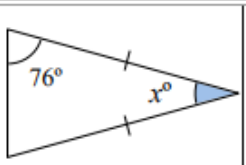
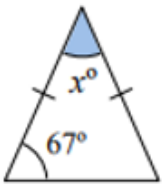
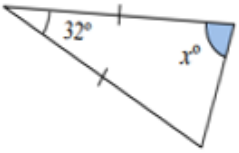
Angle BAC = Angle BCA

108 ÷ 2 = 54

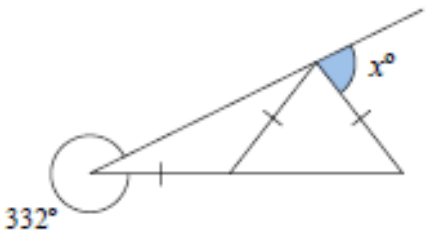
BAC = BCA = 54

(Check: 54 + 54 + 72 = 180)

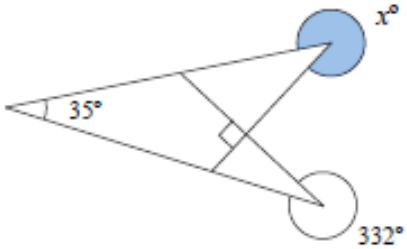
Questions to Try

| 1) | 2) | 3) | 4) |
|---|---|---|---|
|   |   |   |   |

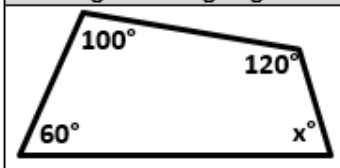
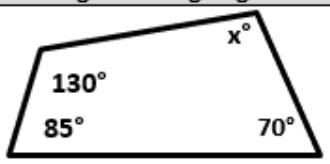
**C2** Find the value of $x$



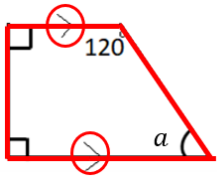

**A3** Find the value of $x$



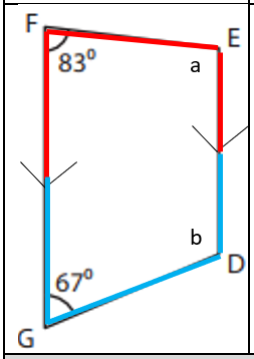
## Angles in any quadrilateral sum to 360

| Finding a missing angle: Example | Finding a missing angle: Your turn |
|---|---|
|  |  |
| Add up the angles we know:<br><br>Subtract from 360° | Add up the angles we know:<br><br>Subtract from 360° |

## Angles in a trapezium

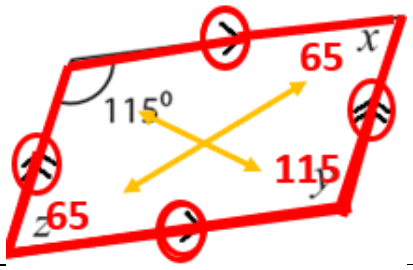



1. Connect the arrows to make a "capital C"
2. The two angles are called Co-interior angles. they add up to 180° (90 and 90 make 180)
3. This works if the capital C is reversed
4. 180° – 120° = 60°




1. 180 – 83 = 97
2. 180 – 67 = 113

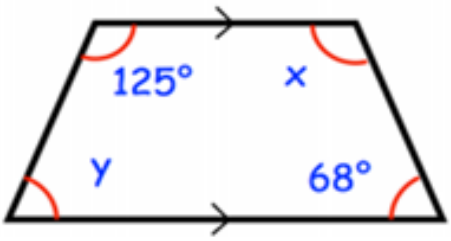
## Angles in a Parallelogram

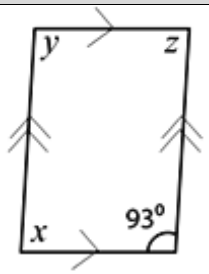



1. Connect the arrows to make a "capital C"
2. The two angles are called Co-interior angles. They add up to 180°. 180° – 115° = 65°
3. This works if the capital C is rotated or reflected
4. 180° – 65° = 115°
5. 180° – 115° = 65°

Notice that opposite angles in parallelogram are equal!

## Questions to try




$x =$ _____ ; $y =$ _____

$z =$ _____

## Topic 17: Averages and Frequency Tables

| | |
|---|---|
| Range = Largest Value – Smallest Value | Mode = Most Common Value (might not be one, could be more than one) |
| Median = Middle Value, after data placed in ascending order | Mean = $\frac{Total\ of\ all\ values}{How\ many\ values}$ |

**5, 8, 5, 7, 9, 10, 5, 2, 3, 6, 3**

| | |
|---|---|
| Range = 10 – 2 = 8 | Mode = 5 |
| Median – 2, 3, 3, 5, 5, 5, 7, 8, 9, 10<br>Median = 5 | Mean = $\frac{5+8+5+7+9+10+5+2+3+6+3}{11} = 5.\dot{7}\dot{2}$ |

### Missing Value Problems

Four numbers have a mean of 10. We know three of the numbers. Work out the fourth number

| | |
|---|---|
| The total must be 40 (4 x 10).<br><br>40 – (5+1+10) = 16<br><br>40 – 16 = 24 | Check:<br><br>$\frac{5+1+10+24}{4} = 10$ |

### Mean, mode and range from Frequency Table

A factory tested how many sweets were in each bag it produced

| Number of Sweets | Frequency | N x F |
|---|---|---|
| 23 | 1 | 23 |
| 24 | 4 | 96 |
| 25 | 9 | 225 |
| 26 | 3 | 78 |
| 27 | 3 | 81 |
| | 20 | 503 |

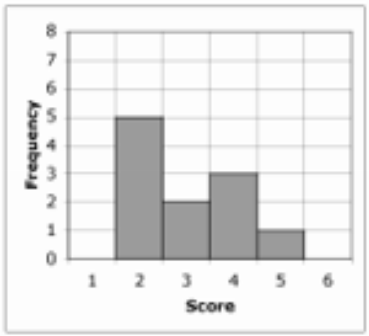
The factory tested 20 bags
Altogether there were 503 sweets

MEAN = $\frac{503}{20} = 25.15\ sweets\ per\ bag$

MODE = 25 sweets per bag (Highest frequency)

RANGE = 27 – 23 = 4 (highest number of sweets – lowest)

### Mean from a bar chart




| Goals Scored | Frequency | N x F |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 5 | 10 |
| 3 | 2 | 6 |
| 4 | 3 | 12 |
| 5 | 1 | 5 |
| 6 | 0 | 0 |
| | **11** | **33** |

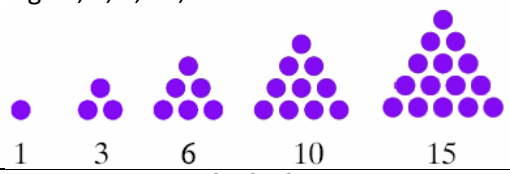
The team played 11 games
Altogether they scored 33 goals

MEAN = $\frac{33}{11} = 3$
$3\ goals\ per\ game$

MODE = 2 goals (Highest frequency)

RANGE = 5 – 2 = 3 (most number of goals scored in a game – lowest number)

## Topic 18: Sequences

| | |
|---|---|
| Arithmetic/Linear<br>Add or subtract the same amount each time | e.g. 3, 7, 11, 15… (add 4) |
| Geometric<br>Multiply or divide by the same amount each time | e.g. 3, 6, 12, 24… (multiply by 2) |
| Fibonacci – next term is found by adding two previous terms | e.g. 3, 6, 9, 15… (3+6 = 9 6+9 =15) |
| Triangular numbers – create a pyramid with one extra in each row. | e.g. 1, 3, 6, 10, 15<br>  <br>1 3 6 10 15 |
| Square numbers: $1^2$ $2^2$ $3^2$ = 1, 4, 9… | Cube numbers: $1^3$ $2^3$ $3^3$= 1, 8, 27… |
| Nth term of a linear sequence<br><br>1st 2nd 3rd 4th<br>e.g. 3, 7, 11, 15<br><br>Use method called Di n 0<br>Di (fference) = +4<br>N n<br>0 (0'th term) would have been -1<br>Nth term = 4n -1 | 100'th number in sequence would be 4(100) – 1<br>= 399<br><br>Is 275 in the sequence ?<br>Solve : 4n – 1 = 275<br>4n = 276<br>n = 69<br>(integer answer, so 275 is the 69th number in the sequence) |

**Questions to try:**

Find the nth term of a) 7, 12, 17, 22 b) 22, 18, 14, 10

Is 428 in the sequence that starts 3, 7, 11, 15?

Work out the 100th term of the sequence that starts 4, 7, 10, 13

## Plotting Linear Sequences (Completing tables and plotting graphs)

Q) Plot the graph of y = 3x – 1

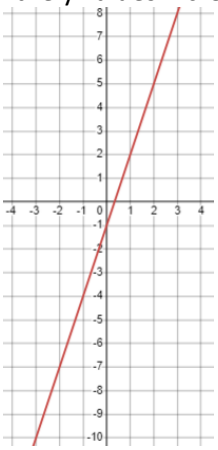
| X | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| y | -10 | -7 | -4 | -1 | 2 | 5 | 8 |

Coordinates

(x,y)

(-3, -10) (-2,-7) (-1, -4) (0, -1) (1, 2) (2, 5) (3, 8)

Note how the y values increase by 3.



Your Turn: Plot the graph of y = 3x + 1

| X | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |