

Topic 1 - Factors and Multiples

Definition

A **factor** is an integer that divides exactly into a whole number leaving no remainder.

The highest common factor (HCF) is the highest factors shared by 2 numbers.

Examples

- 1. List the factors of 18**
- 1×18
 2×9
 3×6

The factors of 18 are 1, 2, 3, 6, 9, 18

- 2. List the factors of 24**
- 1×24
 2×12
 3×8
 4×6

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

3. What is the HCF of 18 and 24?

Factors of 18: 1, 2, 3, 6, 9, 18

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

The HCF of 18 and 24 is 6

Definition

A **multiple** is the result of multiplying a number by an integer.

The lowest common multiple (LCM) is the lowest multiple shared by 2 numbers.

Examples

- 1. List the first five multiples of 3**
- 3, 6, 9, 12, 15

- 2. List the first five multiples of 4**
- 4, 8, 12, 16, 20

3. What is the LCM of 3 and 4?

Multiples of 3: 3, 6, 9, 12, 15...

Multiples of 4: 4, 8, 12, 16, 20...

The LCM of 3 and 4 is 12

Questions

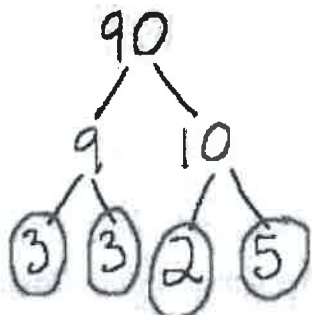
List the factors of 16	List the factors of 36	What is the HCF of 16 and 36
List the factors of 32	List the factors of 48	What is the HCF of 32 and 48
List the first six multiples of 5	List the first six multiples of 6	What is the LCM of 5 and 6
List the first six multiples of 7	List the first six multiples of 8	What is the LCM of 7 and 8

Prime Numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

A **factor Tree** is a method used to write a number as a product of prime factors

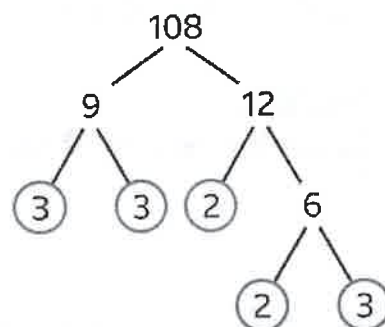
Examples

1. Write 90 as a product of prime factors in index form



Answer = $2 \times 3^2 \times 5$

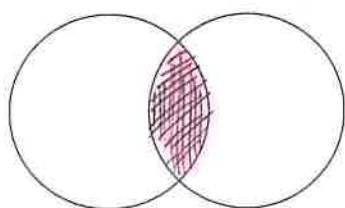
2. Write 108 as a product of prime factors in index form



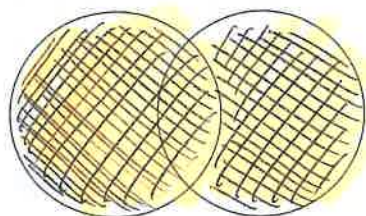
Answer = $2^2 \times 3^3$

We can use a **venn diagram** to find the HCF and LCM of a pair of numbers once we have wrote them as a product of prime factors

HCF – multiple the numbers in the middle



LCM – multiple ALL of the numbers

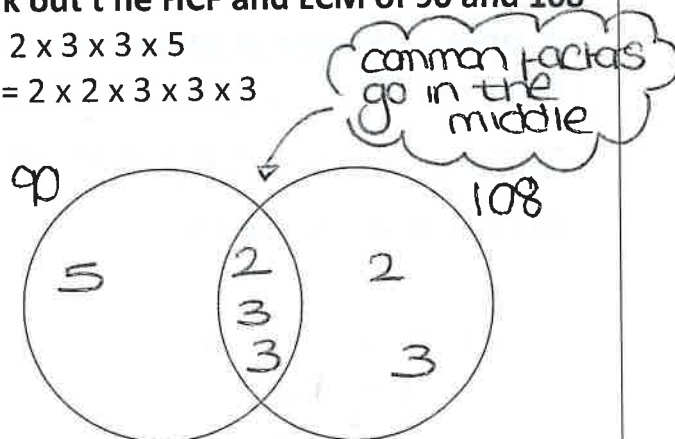


Example

Work out the HCF and LCM of 90 and 108

$$90 = 2 \times 3 \times 3 \times 5$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$



$$\text{HCF} = 2 \times 3 \times 3 = 18$$

$$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$$

Questions (use examples to help you)

List the first 10 prime numbers	Write 60 as a product of prime factors in index form	Write 72 as a product of prime factors in index form
Work out the HCF and LCM of 60 and 72	Work out the HCF and LCM of 60 and 108	Work out the HCF and LCM of 72 and 108

Topic 2- Powers and Roots

Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

The **power** of a number tells you how many times to multiply the number by itself.

Square root is the inverse of power of 2
Cube root is the inverse if power of 3

Examples

1. $4^2 = 4 \times 4 = 16$
2. $5^3 = 5 \times 5 \times 5 = 125$
3. $3^2 + 6^3 = 9 + 216 = 225$

Examples

1. $\sqrt{25} = 5$
2. $\sqrt[3]{512} = 8$
3. $\sqrt{16} + \sqrt[3]{125} = 4 + 5 = 9$

Questions

11^2	9^3	$5^2 + 3^2$	$7^3 - 5^3$
$12^2 + 3^3$	$\sqrt{36} + \sqrt{25}$	$\sqrt[3]{729} - \sqrt{81}$	$5^3 + \sqrt{144}$

Topic 3 – Rounding and Estimation

Rounding to the nearest integer

Rounding line goes after the unit/ones column

Examples

Round to the nearest integer

- a) $15.\overset{\text{S}}{\underset{\text{decider}}{\text{3}}} \approx 15$
- b) $140.\overset{\text{S}}{\underset{\text{decider}}{\text{73}}} \approx 141$
- c) $300.\overset{\text{S}}{\underset{\text{decider}}{\text{5}}} \approx 301$

Rounding to the nearest 10, 100, 1000

Rounding line goes **after** the place value

Examples

9847

1. Round to the nearest 10

$984\overset{\text{S}}{\underset{\text{decider}}{\text{7}}} \approx 9850$

2. Round to the nearest 100

$98\overset{\text{S}}{\underset{\text{decider}}{\text{47}}} \approx 9800$

3. Round to the nearest 1000

$9\overset{\text{S}}{\underset{\text{decider}}{\text{847}}} \approx 10000$

Questions

Round to the nearest integer:	Round to the nearest 10:	Round to the nearest 100:	Round to the nearest 1000:
a) 21.8	a) 89	a) 289	a) 5891
b) 84.51	b) 375	b) 8915	b) 9751
c) 125.29	c) 2841	c) 2973	c) 12583

Round to decimal places

Decimal places are the digits after the decimal point

1 decimal place = nearest tenth

2 decimal places = nearest hundredth

3 decimal places = nearest thousandth

Examples

153.2843

1. Round to 1 decimal place

153.2843 $\approx$ 153.3
↑
decider

2. Round to 2 decimal place

153.2843 $\approx$ 153.28
↑
decider

3. Round to 3 decimal places

153.2843 $\approx$ 153.284
↑
decider

Round to significant figures

We start counting significant from the left from the first non-zero digit.

Examples

4981

1. Round to 1 significant figure

4981 $\approx$ 5000
↑
decider

2. Round to 2 significant figures

4981 $\approx$ 5000
↑
decider

3. Round to 3 significant figures

4981 $\approx$ 4980
↑
decider

Examples

0.5681

1. Round to 1 significant figure

0.5681 $\approx$ 0.6
↑
decider

2. Round to 2 significant figures

0.5681 $\approx$ 0.57
↑
decider

3. Round to 3 significant figures

0.5681 $\approx$ 0.568
↑
decider

Questions

Round to 1 decimal place:	Round to 2 decimal places:	Round to 3 decimal places:
a) 51.97 b) 108.542 c) 0.8681	a) 0.4956 b) 138.1571 c) 57.1346	a) 183.1697 b) 48.2438 c) 0.05931
Round to 1 significant figure:	Round to 2 significant figures:	Round to 3 significant figures:
a) 4210 b) 0.891 c) 12.861	a) 123 b) 2.493 c) 0.1056	a) 3450 b) 0.86152 c) 25.8961

Estimation/Approximation

When we estimate we round each number to 1 significant figure first, then calculate.

Example

Estimate the answer to

$$48.7 \times 61.2$$

Step 1 - Round each number to 1 significant figure

$$48.7 \approx 50$$

↓
↑ decider

$$61.2 \approx 60$$

↓
↑ decider

Step 2 - Rewrite and calculate

$$48.7 \times 61.2 \approx 50 \times 60 = 300$$

Answer = 300

Example

$$\frac{8.3 \times 18.7}{0.52}$$

Estimate the answer to

Step 1 - Round each number to 1 significant figure

$$8.3 \approx 8$$

↓
↑ decider

$$18.7 \approx 20$$

↓
↑ decider

$$0.52 \approx 0.5$$

↓
↑ decider

Step 2 - Rewrite and calculate

$$\frac{8.3 \times 18.7}{0.52} \approx \frac{8 \times 20}{0.5} = \frac{160}{0.5} = \frac{1600}{5} = 320$$

Answer = 320

When estimating with a square root, we round to the nearest square number first

Examples

$$1. \sqrt{19} \approx \sqrt{16} = 4$$

$$2. \sqrt{38} \approx \sqrt{36} = 6$$

$$3. \sqrt{98} \approx \sqrt{100} = 10$$

Example

$$\text{Estimate the answer to } \frac{2.3^2 \times \sqrt{99}}{4.962}$$

Step 1 - Round each number

$$2.3 \approx 2$$

↓
↑ decider

$$4.962 \approx 5$$

↓
↑ decider

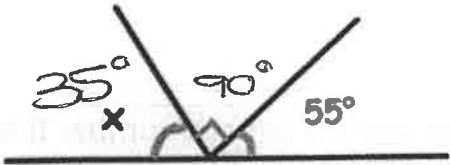
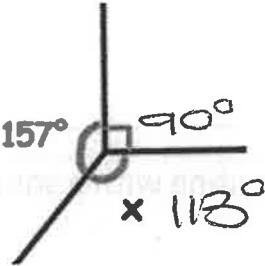
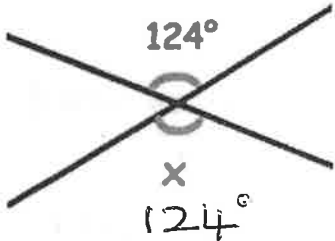
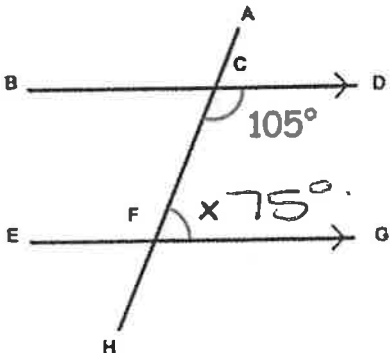
$$99 \approx 100 \text{ (nearest square number)}$$

Step 2 - Rewrite and calculate

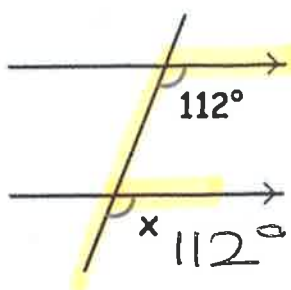
$$\frac{2.3^2 \times \sqrt{99}}{4.962} \approx \frac{2^2 \times \sqrt{100}}{5} = \frac{4 \times 10}{5} = \frac{40}{5} = 8$$

Answer = 8

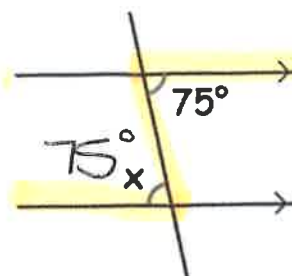
Questions		
Estimate $\frac{28.4 \times 21.05}{5.9}$	Approximate $\frac{41.2 \times 19.8}{0.49}$	Estimate $\frac{21.75 + \sqrt{98.1}}{0.192}$
Estimate 5.9^2	Estimate $\sqrt{120}$	Approximate $\sqrt{83}$
Approximate $9.04 + 19.85 \times 2.99 - 5.03$	Estimate $\frac{431.1}{14.3 + 3.8^2}$	Estimate $\frac{804 \times 2.86}{0.513}$

Topic 4 – Basic Angles and Parallel Lines	
<u>Angles on a straight line at a point sum to 180°</u> Find x:  $ \begin{array}{rcl} x + 90 + 55 & = & 180 \\ x + 145 & = & 180 \\ -145 & & -145 \\ x & = & 35^\circ \end{array} $	<u>Angles around a point sum to 360°</u> Find x:  $ \begin{array}{rcl} x + 90 + 157 & = & 360 \\ x + 247 & = & 360 \\ -247 & & -247 \\ x & = & 113 \end{array} $
<u>Vertically opposite angles are equal</u>  $ \begin{array}{rcl} x & = & 124^\circ \end{array} $	<u>Co-interior angles sum to 180°</u>  $ \begin{array}{rcl} x + 105 & = & 180 \\ -105 & & -105 \\ x & = & 75^\circ \end{array} $

Corresponding angles are equal

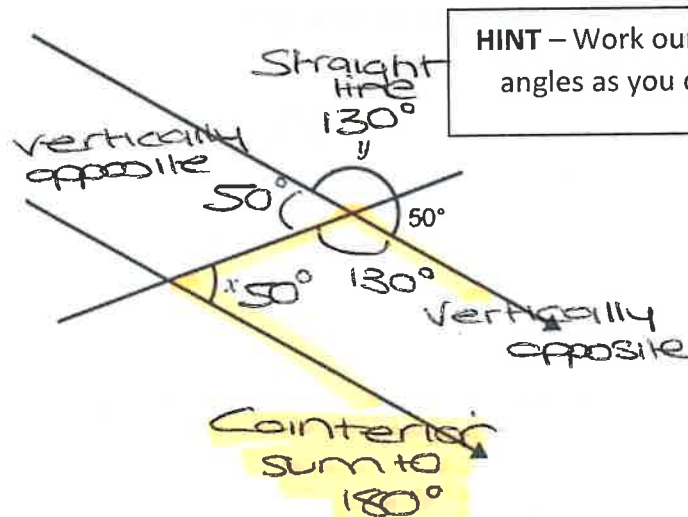


Alternate angles are equal



Examples

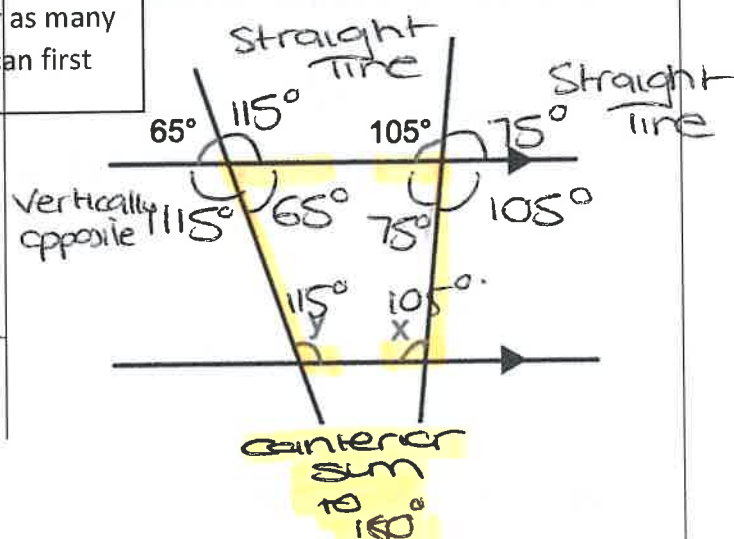
Calculate the size of angles x and y



HINT – Work out as many angles as you can first

$$\begin{array}{r} x + 130 = 180 \\ -130 \quad -130 \\ \hline x = 50^\circ \end{array}$$

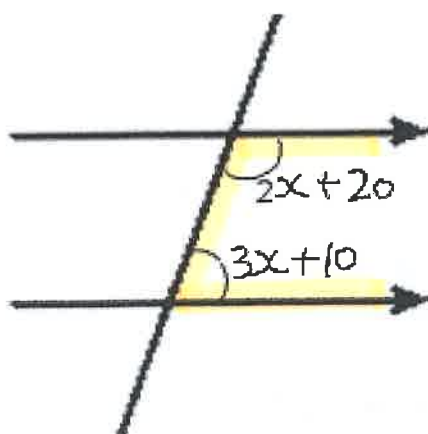
Calculate the sizes of angles x and y



$$\begin{array}{r} y + 65 = 180 \\ -65 \quad -65 \\ \hline y = 115^\circ \end{array}$$

$$\begin{array}{r} x + 75 = 180 \\ -75 \quad -75 \\ \hline x = 105^\circ \end{array}$$

Work out the value of x



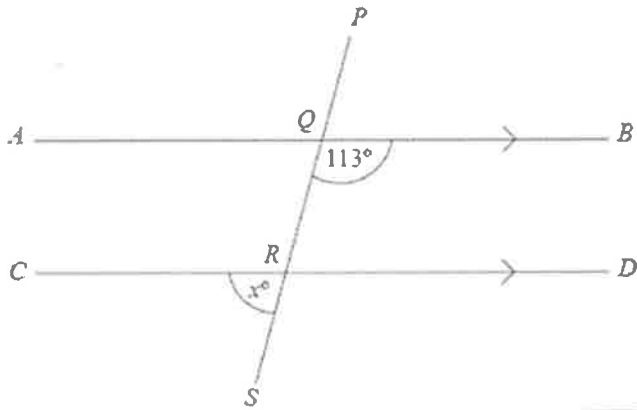
co-interior angles sum to 180°

1. Form an equation

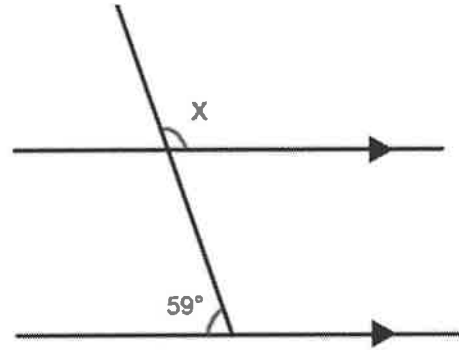
$$\begin{array}{r} 2x + 20 + 3x + 10 = 180 \\ 5x + 30 = 180 \\ -30 \quad -30 \\ \hline 5x = 150 \\ \hline x = 30 \end{array}$$

Questions

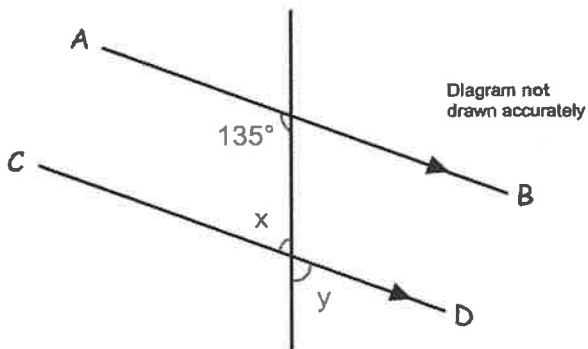
Work out the size of angles x



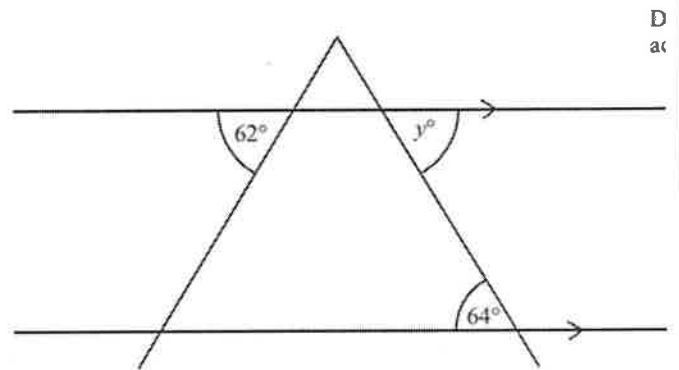
Work out the size of angle x



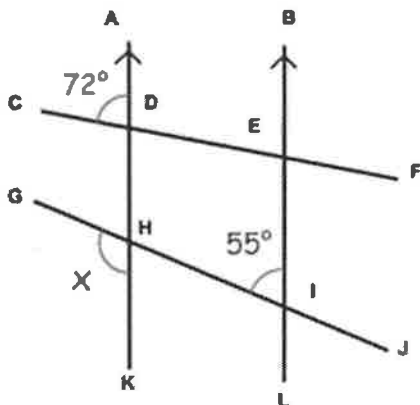
Work out the size of angles x and y



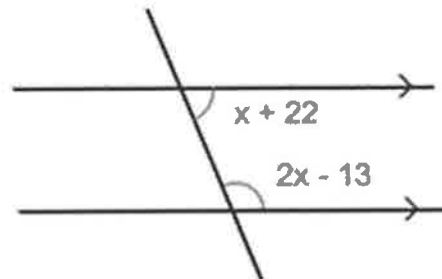
Work out the size of angle y



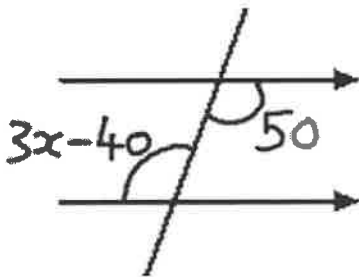
Work out the size of angle x



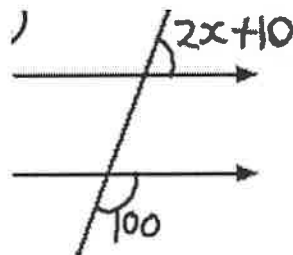
Work out x



Work out x



Work out x

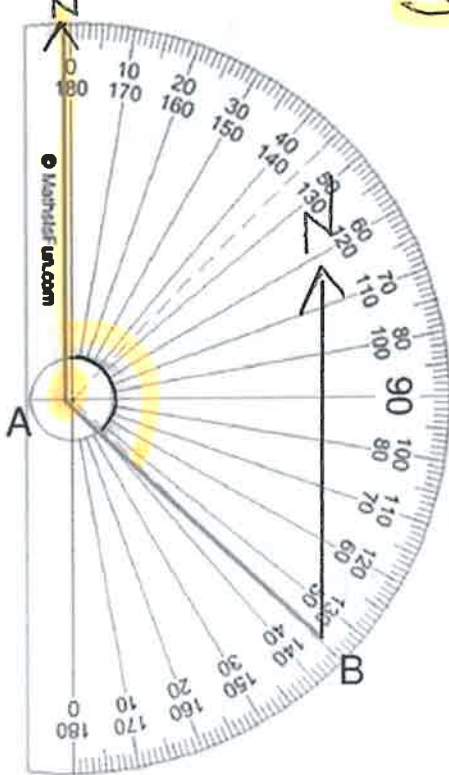


Bearings follow 3 rules:

1. Measure from North
2. Measure clockwise
3. Must be 3 figures

Example

Measure the bearing of B from A

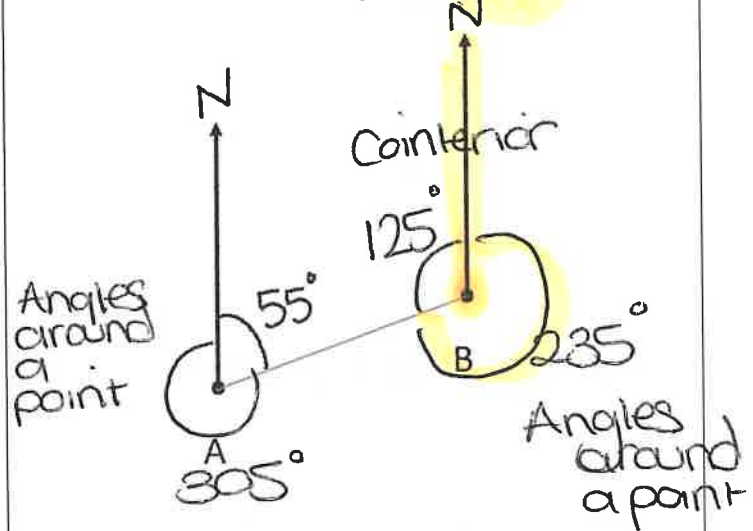


Answer = 135°

HINT – Work out as many angles as you can first

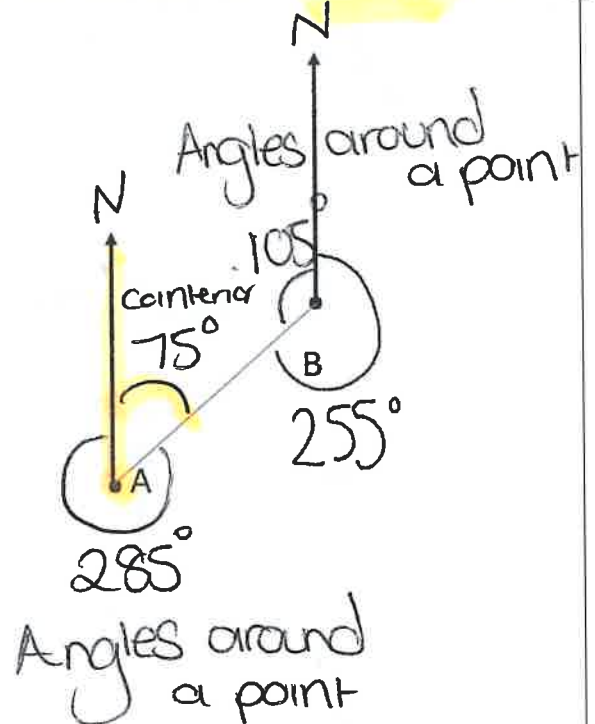
Example

Work out the bearing of A from B = 235°



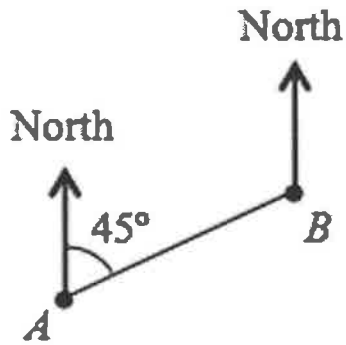
Example

Work out the bearing of B from A = 075°

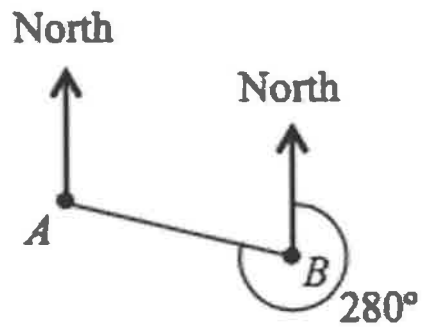


Questions

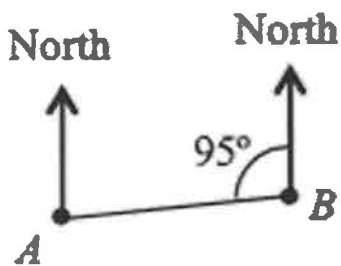
Find the bearing of B from A



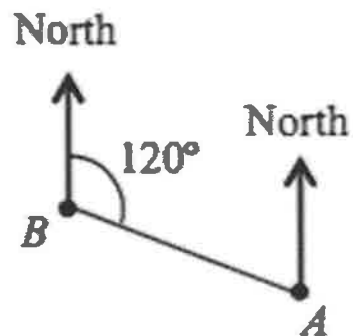
Find the bearing of A from B



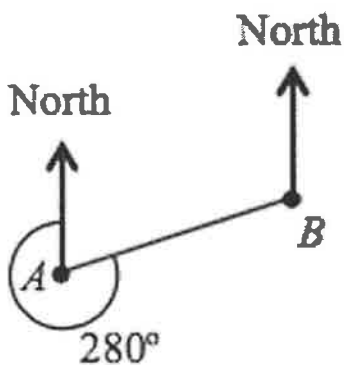
Find the bearing of B from A



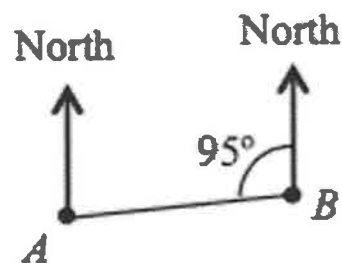
Find the bearing of B from A



Find the bearing of A from B



Find the bearing of A from B



A **scale** is the ratio that defines the relation between the actual figure and its model

Example

A map has a scale of 1cm : 3 miles.

a) On the map, the distance between two towns is 7cm. What is the actual distance between the two towns?

$$\frac{7}{1} = 7 \times 7 \left(\begin{array}{l} 1\text{cm} : 3\text{ miles} \\ 7\text{cm} : 21\text{ miles} \end{array} \right) \times 7$$

b) The actual distance between another two towns is 90 miles. What is the distance between the towns on the map?

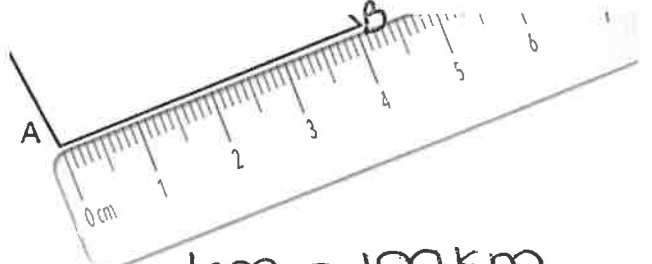
$$\frac{30}{1} = 30 \times 30 \left(\begin{array}{l} 1\text{cm} : 3\text{ miles} \\ 30\text{cm} : 90\text{ miles} \end{array} \right) \times 30$$

Example

The diagram shows a scale drawing.

Scale: 1cm = 100km

Calculate the actual distance between A and B



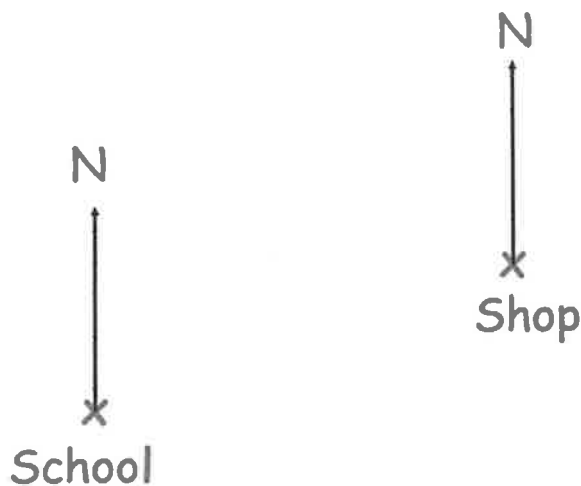
$$\begin{aligned} &1\text{cm} = 100\text{ km} \\ &\times 4.1 \left(\begin{array}{l} 1\text{cm} = 100\text{ km} \\ 4.1\text{cm} = 410\text{ km} \end{array} \right) \times 4.1 \\ &\frac{4.1}{1} = 4.1 \end{aligned}$$

Questions

A map has a scale of 1cm : 4 kilometres. The actual distance between two cities is 52 kilometres. What is the distance between the cities on the map?

A map has a scale of 1cm : 2.5 miles. On the map, the distance between two cities is 8cm. What is the actual distance between the two cities?

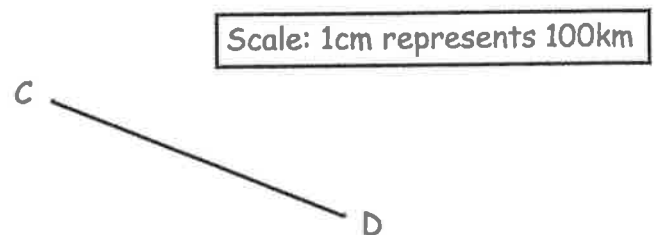
The diagram shows part of a map. It shows the position of a school and a shop.



The scale of the map is 1cm = 100 metres.

Work out the real distance between the school and the shop. Give your answer in metres.

The diagram shows a scale drawing.



(a) Use the diagram to calculate the actual distance from C to D.

Topic 6 - Indices

The rules below can only be used if the base is the same

$3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5
 3 is the base
 5 is the power
 We read this as "3 to the power of 5"

Anything to the power of zero is 1

Examples

$$3^0 = 1$$

$$5^0 = 1$$

$$a^0 = 1$$

$$x^0 = 1$$

Anything to the power of 1 is itself

Examples

$$3^1 = 3$$

$$5^1 = 5$$

$$a^1 = a$$

$$x^1 = x$$

When we multiply we add the powers

Examples

$$\bullet \quad 3^2 \times 3^5 = 3^{2+5} = 3^7$$

$$\bullet \quad x^6 \times x^2 = x^{6+2} = x^8$$

$$\bullet \quad n^5 \times n^{-2} = n^{5+(-2)} = n^{5-2} = n^3$$

$$\bullet \quad a^3 b^4 \times a^{-5} b^2 = a^{3+(-5)} b^{4+2} = a^{-2} b^6$$

When there are coefficients, we multiply them first and then work out the power

$$\bullet \quad 4a^3 \times 6a^5 = 24a^{3+5} = 24a^8$$

When we divide we subtract the powers

Examples

$$\bullet \quad 3^7 \div 3^3 = 3^{7-3} = 3^4$$

$$\bullet \quad \frac{x^3}{x^7} = x^{3-7} = x^{-4}$$

$$\bullet \quad \frac{n^4}{n^{-5}} = n^{4-(-5)} = n^9$$

$$\bullet \quad \frac{x^5 y^2}{x^2 y^6} = x^{5-2} y^{2-6} = x^3 y^{-4}$$

When there are coefficients, we divide them first and then work out the power

$$\bullet \quad \frac{30a^7}{5a^4} = 6a^{7-4} = 6a^3$$

When we have brackets we multiply the powers

Examples

$$\bullet \quad (3^2)^5 = 3^{2 \times 5} = 3^{10}$$

$$\bullet \quad (x^4)^3 = x^{4 \times 3} = x^{12}$$

$$\bullet \quad (n^{-3})^6 = n^{-3 \times 6} = n^{-18}$$

When there are coefficients, we also need to put the coefficient to the power

$$\bullet \quad (2x^4)^3 = 2x^4 \times 2x^4 \times 2x^4 = 8x^{12}$$

(We can do this quickly by doing 2^3)

Sometimes one questions may contain several rules

Examples

$$\frac{a^3 \times a^4}{a^5} = \frac{a^7}{a^5} = a^2$$

$$\frac{(b^2)^4 \times b^3}{b^2 \times b^5} = \frac{b^8 \times b^3}{b^7} = \frac{b^{11}}{b^7} = b^4$$

$$\frac{6x^2 y \times 5x^3 y^2}{3xy^5} = \frac{30x^5 y^3}{3xy^5} = 10x^4 y^{-2}$$

Questions

$7^2 \times 7^6$	$10^5 \div 10^3$	$(4^5)^4$	$a^4 \times a^{-9}$	$x^8 \div x^3$
$(b^3)^{-5}$	$2x^3 \times 8x^5$	$\frac{m^6}{m^{-2}}$	$(5a^6)^2$	$2x^4$
$m^3n^2 \times m^{-4}n^5$		$\frac{a^3b^5}{a^7b^2}$		$5x^4y^2 \times 8x^{-3}y^2$
$\frac{24a^3b^5}{6ab^9}$		$\frac{x^6 \times x^{-2}}{x^4}$		$\frac{(p^4)^{-3} \times p^5}{p^2 \times p^3}$
$\frac{4x^3y \times 9x^{-2}y^3}{2x^2y^6}$			$\frac{8a^5b^3 \times 5a^4b^9}{4ab^7}$	

Sometimes you are required to change the base.

"Write as a single power of 2" means that you want the base to be 2 ie 2

"Write as a single power of 3" means that you want the base to be 3 ie 3

Examples

- Write 9 as a single power of 3

$$9 = 3 \times 3 = 3^2$$

- Write 8^4 as a single power of 2

$$8 = 2 \times 2 \times 2 = 2^3$$

We then replace 8 with 2^3

$$(2^3)^4 = 2^{12} \text{ (brackets = multiply)}$$

- Write 16 as a single power of 2

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

- Write 25^3 as a single power of 5

$$25 = 5 \times 5 = 5^2$$

We then replace 25 with 5^2

$$(5^2)^3 = 5^6 \text{ (brackets = multiply)}$$

Questions

Write 27 as a single power of 3	Write 4 as a single power of 2
Write 81 as a single power of 3	Write 125 as a single power of 5
Write 9^5 as a single power of 3	Write 16^3 as a single power of 2
Write 8^5 as a single power of 2	Write 32^4 as a single power of 2
Write 27^9 as a single power of 3	Write 25^7 as a single power of 5

Topic 7 – Expand and factorise single brackets

Expand means to multiply out the brackets. You multiply the term outside the brackets but each term inside the bracket. You should always check to see if you can simplify your answer.

Examples

- Expand $3(x - 4) \equiv 3x - 12$
- Expand $-(x - 5) \equiv -x + 5$
invisible 1
- Expand $-2(4x + 3) \equiv -8x - 6$
- Expand $4a(5b + 8c) \equiv 20ab + 32ac$
- Expand $x(3 - 2x) \equiv 3x - 2x^2$
- Expand $4x^2(3x - 10) \equiv 12x^3 - 40x^2$
- Expand and simplify $5x + 2(3x - 4) \equiv 5x + 6x - 8 = 11x - 8$
- Expand and simplify $2x(3x + 5) - 7x \equiv 6x^2 + 10x - 7x \equiv 6x^2 + 3x$
- Expand and simplify $4(2x + 5) - 2(3x - 9) \equiv 8x + 20 - 6x + 18 \equiv 2x + 38$

Remember:

$$- \times - = +$$

$$+ \times - = -$$

Questions

A1 Expand: $7(m+3)$	A2 Expand: $4(m-4)$	B1 Expand: $2(3p+7)$	B2 Expand: $3(p+2q)$
C1 Expand: $8(rs-7)$	C2 Expand: $7(2rs+3)$	D1 Expand and simplify: $3w+4(w+5)$	D2 Expand and simplify: $8w+2(3w-7)$
E1 Expand and simplify: $4(x+1)+7$	E2 Expand and simplify: $6(x+3)+2x$	D3 Expand: $3x^2(6x-5)$	D4 Expand: $4ax(3x+2a)$
E3 Expand: $3a^2(5a+4b-2ab)$	E4 Expand: $5a^2b(2a-ab+3b)$	B4 Expand: $-5(-3x+y-4)$	C4 Expand: $-x(6-y)$
E2 Expand: $8x-4(7-3x)$	E3 Expand: $9x-x(x+5)$	D1 Expand and simplify: $2(3b-1)+(b+9)$	D2 Expand and simplify: $5(k-3)-4(3k+2)$

Factorising is the opposite of expand. You put the highest common factor of all terms outside the bracket.

Examples

- Factorise $5x + 10 \equiv 5(x + 2)$
- Factorise $12x - 18 \equiv 6(2x - 3)$
- Factorise $24 - 9x \equiv 3(8 - 3x)$
- Factorise $4ab + 5ac \equiv a(4b + 5c)$
- Factorise $x^2 + 6x \equiv x(x + 6)$
- Factorise $6x^2 - 15x \equiv 3x(2x - 5)$
- Factorise $12x^2y + 8xy^2 \equiv 4xy(3x + 2y)$

Questions

12x + 16	$x^2 + 5x$	$4xy + 8y$	A4 Factorise fully: $9x^3 - 36x^2$
16x - 12	$x^2 - x$	$10xy - 5x^2y$	B4 Factorise fully: $9x^3 - 12x^2$
16y - 12x	$4a - 7a^2$	$x^2y + 4xy^2$	C4 Factorise fully: $6w^2xy + 2wx^2y$
10x + 5y - 30	$3y + \underline{xy}$	$9x^3y^2 - 6x^2y^5$	D4 Factorise fully: $7x^2y^3 + x^4y$
	$5x^2 + \underline{xy} - 3x$	$12x^5y - 6x^4y^2 + 9x^3y^3$	

Topic 8 – FDP and Operations with fractions

Fraction	Decimal	Percentage
$\frac{0.6}{1} \times 10 = \frac{6}{10} \div 2 \leftarrow$ $= \frac{3}{5}$ 1. put over 1 2. multiply 3. simplify	0.6	0.6×100 $= 60\%$
$\frac{2}{5}$ $\rightarrow$ 1. write over 100	0.4	$\frac{2}{5} = \frac{40}{100} = 40\%$ $\times 20$
$\frac{1.42}{1} \times 100 = \frac{142}{100} \div 2$ $= \frac{71}{50}$	$142 \div 100$ $= 1.42$	142%

Questions

1. Convert these decimals into percentages

- | | | | |
|-----------|-----------|------------|------------|
| (a) 0.25 | (b) 0.75 | (c) 0.13 | (d) 0.88 |
| (e) 0.075 | (f) 0.021 | (g) 0.1425 | (h) 0.9682 |
| (a) 1.63 | (b) 1.25 | (c) 1.8 | (d) 1.01 |

2. Convert these percentages into decimal

- | | | | |
|----------|----------|-----------|----------|
| (a) 53% | (b) 19% | (c) 25% | (d) 74% |
| (e) 1.7% | (f) 6.8% | (g) 8.15% | (h) 0.5% |
| (a) 162% | (b) 190% | (c) 115% | (d) 144% |

3. Convert these fractions into percentages

- | | | | | | | |
|-------------------|-------------------|-------------------|--------------------|--------------------|-------------------|-------------------|
| (a) $\frac{1}{2}$ | (b) $\frac{1}{4}$ | (c) $\frac{3}{4}$ | (d) $\frac{3}{20}$ | (e) $\frac{3}{25}$ | (f) $\frac{4}{5}$ | (g) $\frac{5}{8}$ |
|-------------------|-------------------|-------------------|--------------------|--------------------|-------------------|-------------------|

4. Convert these percentages into fractions

- | | | | |
|---------|---------|---------|---------|
| (i) 15% | (j) 29% | (k) 16% | (l) 44% |
| (m) 99% | (n) 85% | (o) 52% | (p) 80% |

Topic 8 – Four Rules with Fractions

Addition and Subtraction

In order to add fractions then denominator must be the same. We can make it the same by using the lowest common multiple.

Example

- $\frac{4}{5} + \frac{3}{4}$ The LCM of 4 and 5 is 20:

$$\frac{4}{5} + \frac{3}{4} = \frac{16}{20} + \frac{15}{20} = \frac{31}{20} = 1\frac{11}{20}$$

- $\frac{4}{7} - \frac{2}{5}$ The LCM of 5 and 7 is 35:

$$\frac{4}{7} - \frac{2}{5} = \frac{20}{35} - \frac{14}{35} = \frac{6}{35}$$

- $3\frac{2}{5} + 2\frac{1}{6}$ First we must convert the mixed numbers into improper fractions.
The LCM of 5 and 6 is 30:

$$3\frac{2}{5} + 2\frac{1}{6} = \frac{17}{5} + \frac{13}{6} = \frac{102}{30} + \frac{65}{30} = \frac{167}{30} = 5\frac{17}{30}$$

Questions

(i) $\frac{7}{15} - \frac{1}{5}$	(j) $\frac{3}{4} - \frac{2}{5}$	(k) $\frac{3}{10} + \frac{3}{8}$	(l) $\frac{2}{5} + \frac{4}{7}$
----------------------------------	---------------------------------	----------------------------------	---------------------------------

(e) $\frac{19}{20} + \frac{4}{5}$	(f) $\frac{5}{9} + \frac{13}{18}$	(g) $\frac{5}{12} + \frac{9}{10}$	(h) $\frac{4}{7} + \frac{7}{8}$
-----------------------------------	-----------------------------------	-----------------------------------	---------------------------------

(e) $2\frac{1}{2} + 1\frac{1}{3}$	(f) $2\frac{2}{9} - 1\frac{1}{3}$	(g) $2\frac{2}{9} + \frac{5}{6}$	(h) $1\frac{5}{12} + 1\frac{5}{8}$
-----------------------------------	-----------------------------------	----------------------------------	------------------------------------

Multiply

To multiply fractions we multiply the numerators together and multiply denominators together. We should check to see if we can cross cancel first (diagonals only). You should also simplify your answer where possible.

Example

- We can cross cancel the 3 and 6 by dividing both by 3, we can also cross cancel the 4 and the 8 by dividing by 4:

$$\frac{3}{8} \times \frac{4}{6} = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

- We can cross cancel the 10 and 15 by dividing both by 5:

$$\frac{10}{12} \times \frac{7}{15} = \frac{2}{12} \times \frac{7}{3} = \frac{2 \times 7}{12 \times 3} = \frac{14}{36} = \frac{7}{18}$$

- Turn 3 into a fraction over 1

$$\frac{1}{8} \times 3 = \frac{1}{8} \times \frac{3}{1} = \frac{1 \times 3}{8 \times 1} = \frac{3}{8}$$

- Convert mixed numbers into improper fractions:

$$2\frac{3}{4} \times 1\frac{2}{7} = \frac{15}{4} \times \frac{9}{7} = \frac{15 \times 9}{4 \times 7} = \frac{135}{28} = 4\frac{23}{28}$$

Questions

(a) $\frac{6}{7} \times \frac{2}{9}$

(r) $\frac{3}{10} \times \frac{5}{6}$

(s) $\frac{6}{15} \times \frac{3}{4}$

(t) $\frac{3}{5} \times \frac{11}{15}$

(e) $8 \times \frac{3}{4}$

(f) $\frac{2}{3} \times 12$

(g) $5 \times \frac{1}{3}$

(h) $8 \times \frac{2}{5}$

(e) $\frac{1}{4} \times 3\frac{1}{3}$

(f) $1\frac{2}{3} \times 1\frac{1}{4}$

(g) $4\frac{3}{5} \times 1\frac{2}{3}$

(h) $1\frac{2}{11} \times \frac{8}{9}$

Divide

To divide fractions we use KFC; **K**eeP the first fractions, **F**lip the second fraction, **C**hange the divide into a multiply. Then the multiply the fractions, checking to cross cancel first.

Example

- KFC:

$$\frac{1}{4} \div \frac{3}{5} = \frac{1}{4} \times \frac{5}{3} = \frac{1 \times 5}{4 \times 3} = \frac{5}{12}$$

- KFC. Cross cancel 2 and 8 by dividing both by 2:

$$\frac{2}{3} \div \frac{8}{10} = \frac{2 \div 2}{3} \times \frac{10}{8 \div 2} = \frac{1}{3} \times \frac{10}{4} = \frac{1 \times 10}{3 \times 4} = \frac{10}{12} = \frac{5}{6}$$

- Write 4 over 1. KFC.

$$\frac{5}{6} \div 4 = \frac{5}{6} \div \frac{4}{1} = \frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \frac{5}{24}$$

- Convert into improper fractions. KFC.

$$2\frac{3}{4} \div 1\frac{1}{3} = \frac{11}{4} \div \frac{4}{3} = \frac{11}{4} \times \frac{3}{4} = \frac{11 \times 3}{4 \times 4} = \frac{33}{16} = 2\frac{1}{16}$$

Questions

(i) $\frac{3}{5} \div \frac{1}{2}$

(j) $\frac{7}{9} \div \frac{2}{3}$

(k) $\frac{8}{15} \div \frac{7}{10}$

(l) $\frac{9}{10} \div \frac{1}{3}$

(a) $\frac{3}{4} \div 2$

(b) $\frac{4}{7} \div 8$

(c) $\frac{11}{20} \div 3$

(d) $\frac{9}{40} \div 5$

(a) $\frac{2}{3} \div 1\frac{4}{5}$

(b) $1\frac{1}{2} \div 1\frac{9}{10}$

(c) $2\frac{3}{7} \div \frac{1}{2}$

(d) $2\frac{1}{3} \div 5\frac{1}{2}$

Example

Find $\frac{2}{5}$ of 35

$$\frac{2}{5} \times \frac{35}{1} = \frac{2 \div 5}{1} \times \frac{7}{1} = \frac{14}{1} = 14 \checkmark$$

Example

$\frac{3}{7}$ of a number is 9. What is the number?

$$\square \times \frac{3}{7} = 9$$

$$\div \frac{3}{7}$$

$$\square = 9$$

$$= \frac{9 \div 3}{1} \div \frac{3}{7} = \frac{3}{1} \times \frac{7}{3} = 7$$

$$= \frac{3 \times 7}{1} = 21 \checkmark$$

Questions

(a) $\frac{1}{2}$ of 10

(b) $\frac{1}{3}$ of 18

(c) $\frac{1}{5}$ of 20

(d) $\frac{1}{4}$ of 24

(a) $\frac{2}{3}$ of 15

(b) $\frac{7}{10}$ of 20

(c) $\frac{2}{5}$ of 30

(d) $\frac{3}{4}$ of 32

(e) $\frac{3}{5}$ of 45

(f) $\frac{2}{7}$ of 28

(g) $\frac{3}{8}$ of 88

(h) $\frac{3}{10}$ of 120

(a) $\frac{1}{2}$ of a number is 7, what is the number? (b) $\frac{1}{3}$ of a number is 4, what is the number?

(c) $\frac{1}{4}$ of a number is 8, what is the number? (d) $\frac{1}{5}$ of a number is 9, what is the number?

(a) $\frac{2}{3}$ of a number is 12, what is the number? (b) $\frac{2}{5}$ of a number is 10, what is the number?

(c) $\frac{2}{7}$ of a number is 6, what is the number? (d) $\frac{3}{10}$ of a number is 60, what is the number?

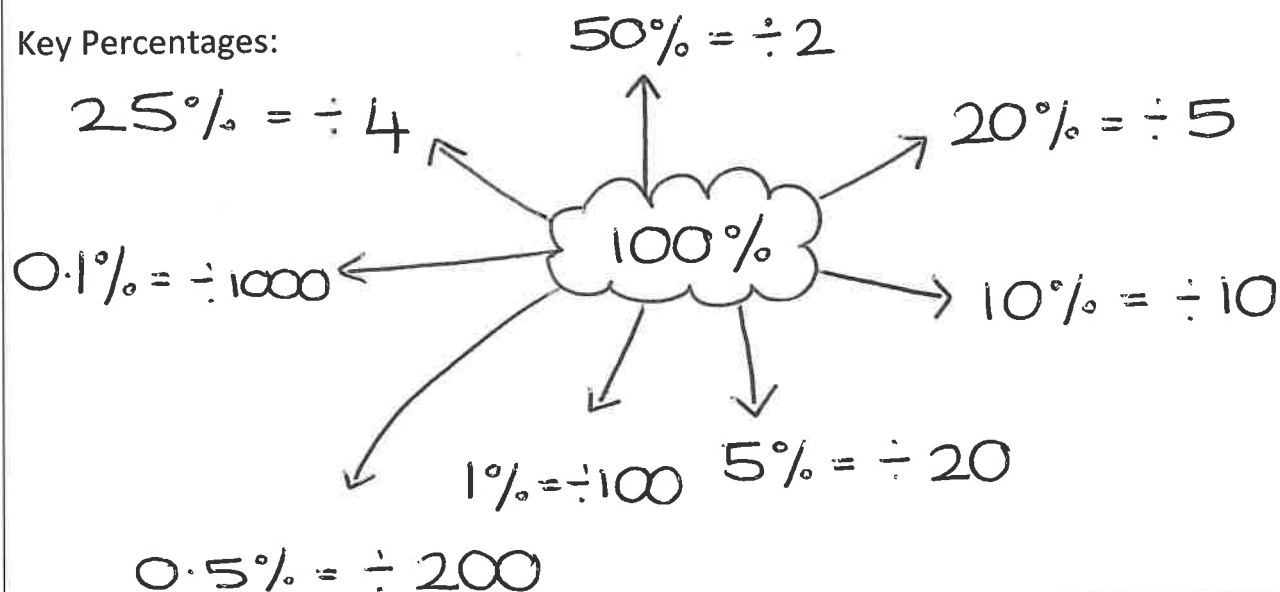
(e) $\frac{4}{9}$ of a number is 12, what is the number? (f) $\frac{2}{3}$ of a number is 3, what is the number?

Topic 9- Percentages

NON-CALCULATOR

To calculate a percentage without a calculator we use the build-up method. This is where we use key percentages and add them together.

Key Percentages:



Questions

- | | | | |
|-----------------|-----------------------|--------------------|-------------------|
| (a) 10% of 70m | (b) 25% of 16 seconds | (c) 10% of 400kg | (d) 50% of 26g |
| (e) 75% of 40ml | (f) 1% of £300 | (g) 25% of 36 days | (h) 50% of 9 days |
| (i) 75% of 24p | (j) 25% of £18 | (k) 1% of \$6300 | (l) 10% of £7 |
| (m) 1% of 60m | (n) 75% of 8 miles | (o) 1% of 80kg | (p) 50% of 1.6km |

Example

Find 43% of £420.

1. Find key percentages

$$\begin{array}{l}
 100\% = \text{£}420 \\
 \swarrow \div 10 \quad \searrow \div 10 \\
 10\% = \text{£}42 \\
 \swarrow \div 10 \quad \searrow \div 10 \\
 1\% = \text{£}4.20
 \end{array}$$

2. Build up

$$\begin{array}{l}
 10\% = \text{£}42 \\
 \swarrow \times 4 \quad \searrow \times 4 \\
 40\% = \text{£}168
 \end{array}$$

3. Add percentages together

$$40\% = \text{£}168.00$$

$$+ 3\% = \text{£}12.60$$

$$43\% = \text{£}180.60$$

$$\begin{array}{l}
 1\% = \text{£}4.20 \\
 \swarrow \times 3 \quad \searrow \times 3 \\
 3\% = \text{£}12.60
 \end{array}$$

Example

Find 17.5% of £160.

1. Find key percentages

$$\begin{array}{l}
 100\% = \text{£}160 \\
 \swarrow \div 10 \quad \searrow \div 10 \\
 10\% = \text{£}16 \\
 \swarrow \div 10 \quad \searrow \div 10 \\
 1\% = \text{£}1.60 \\
 \swarrow \div 2 \quad \searrow \div 2 \\
 0.5\% = \text{£}0.80
 \end{array}$$

2. Build up

$$\begin{array}{l}
 1\% = \text{£}1.60 \\
 \swarrow \times 7 \quad \searrow \times 7 \\
 7\% = \text{£}11.20
 \end{array}$$

3. Add percentages together

$$10\% = \text{£}160.00$$

$$7\% = \text{£}11.20$$

$$+ 0.5\% = \text{£}0.80$$

$$17.5\% = \text{£}172.00$$

Questions

- | | | | |
|-------------------|--------------------|---------------------|-----------------------|
| (a) 35% of £800 | (b) 6% of 160g | (c) 23% of 330cm | (d) 52% of 910m |
| (e) 61% of 1400 | (f) 7% of 640GB | (g) 45% of 350g | (h) 80% of 450 people |
| (i) 90% of 1250ml | (j) 76% of £80,000 | (k) 85% of 90 hours | (l) 12% of £6 |

Percentage Increase

Percentage increase means that we **add** the percent onto the original amount

Example

Increase £280 by 22%.

1. Find key percentages

$$100\% = £280$$

$$\begin{array}{l} \div 10 \downarrow \\ 10\% = £28 \end{array}$$

$$\begin{array}{l} \div 10 \downarrow \\ 1\% = £2.80 \end{array}$$

2. Build up together

$$10\% = £28$$

$$1\% = £2.80$$

$$\begin{array}{l} \times 2 \downarrow \\ 20\% = £56 \end{array}$$

$$2\% = £5.60$$

3. Add percentages together

$$20\% = £56.00$$

$$+ 2\% = £5.60$$

$$22\% = £61.60$$

4. Add onto the original amount

$$100\% = £280.00$$

$$+ 22\% = £61.60$$

$$122\% = £341.60$$

Percentage Decrease

Percentage decrease means that we **subtract** the percent from the original amount

Example

Decrease £130 by 31%.

1. Find key percentages

$$100\% = £130$$

$$\begin{array}{l} \div 10 \downarrow \\ 10\% = £13 \end{array}$$

$$\begin{array}{l} \div 10 \downarrow \\ 1\% = £1.30 \end{array}$$

2. Build up together

$$10\% = £13$$

$$1\% = £1.30$$

$$\begin{array}{l} \times 3 \downarrow \\ 30\% = £39 \end{array}$$

3. Add percentages together

$$30\% = £39.00$$

$$+ 1\% = £1.30$$

$$33\% = £40.30$$

4. Subtract from the original amount

$$100\% = £280.00$$

$$- 33\% = £40.30$$

$$67\% = £239.70$$

Questions

(a) Increase 20 by 50%

(b) Increase 60p by 10%

(c) Increase 12g by 25%

(d) Increase 400 litres by 20%

(e) Increase 32ml by 75%

(f) Increase 70m by 40%

(a) Decrease 40 by 10%

(b) Decrease 30 hours by 50%

(c) Decrease 8kg by 25%

(d) Decrease 55cm by 40%

(e) Decrease 64 by 75%

(f) Decrease £3 by 10%

CALCULATOR

To calculate a percentage with a calculator we use the multiplier method. To find the multiplier you divide the percentage by 100.

Examples – Find the multiplier

- $45\% = \frac{45}{100} = 0.45$
- $63\% = \frac{63}{100} = 0.63$
- $192\% = \frac{192}{100} = 1.92$

Questions

- | | | | |
|---------|----------|----------|----------|
| (a) 50% | (b) 80% | (c) 10% | (d) 25% |
| (e) 45% | (f) 95% | (g) 5% | (h) 3% |
| (i) 7% | (j) 36% | (k) 71% | (l) 44% |
| (m) 0% | (n) 175% | (o) 104% | (p) 160% |

Example

Find 12% of 132kg.

Amount: 132

Rate: 12%

Multiplier: $\frac{12}{100} = 0.12$

Sum: $132 \times 0.12 = 15.84\text{kg}$

Find 9.7% of 40m.

Amount: 40

Rate = 9.7%

Multiplier: $\frac{9.7}{100} = 0.097$

Sum: $40 \times 0.097 = 3.88\text{m}$

Find 154% of 14

Amount: 14

Rate: 154%

Multiplier: $\frac{154}{100} = 1.54$

Sum: $14 \times 1.54 = 21.56$

Questions

- (a) 15% of 80ml (b) 9% of 205kg (c) 45% of £135 (d) 17% of 540km
 (e) 53% of 700g (f) 14% of 12 hours (g) 31% of 280kg (h) 6% of 4GB
- (a) 2.5% of 60cm (b) 7.2% of 104ml (c) 24.5% of 30m (d) 47.9% of £3200
- (a) 109% of 30m (b) 124% of 38 seconds (c) 186% of £40 (d) 196% of 20 miles

Percentage increase

Percentage increase means that we **add** the percent onto 100%

Example

Increase £425 by 78%

Amount: £425

Rate: $100\% + 78\% = 178\%$

Multiplier: $\frac{178}{100} = 1.78$

Sum: $425 \times 1.78 = £756.50$

Percentage decrease

Percentage decrease means that we **subtract** the percent from 100%

Example

Decrease £390 by 27%

Amount: £390

Rate: $100\% - 27\% = 73\%$

Multiplier: $\frac{73}{100} = 0.73$

Sum: $390 \times 0.73 = £284.70$

Questions

- (a) Increase 80ml by 9% (b) Increase 420g by 70% (c) Decrease 8 by 12%
 (d) Decrease £1250 by 38% (e) Increase 6000km by 23% (f) Decrease 48GB by 6%
 (g) Increase 204 by 98% (h) Decrease 149mm by 91% (i) Increase 88 by 185%

Topic 10 - Substitution

Example 1

Work out the value of $2a^2 + b^3$
when $a = 5$ and $b = -3$

Example 2

This formula converts degrees Celsius (C) to degrees Fahrenheit (F).

$$F = \frac{9}{5}C + 32$$

Use the formula to convert 28°C to $^{\circ}\text{F}$
Give your answer to the nearest whole number.

Questions

Q1) If $a = 7$ $b = 10$ $c = 3$ $d = 8$ and $e = 15$. Find the value of each expression.

- | | | | | |
|--------------|--------------|--------------|-------------------|---------------|
| (a) $a + 5$ | (b) $e - d$ | (c) $5c$ | (d) $\frac{d}{2}$ | (e) a^2 |
| (f) $2a + 1$ | (g) $3b - 7$ | (h) $4d - b$ | (i) $e - 4c$ | (j) $ab - cd$ |

Q2) $P = 2L + 2W$, work out P if $L = 8$ and $W = 3$

Q3) $C = 15h + 30$, work out C if $h = 6$

Q4) If $a = -2$ $b = 5$ $c = -6$ $d = 10$ and $e = 9$. Find the value of each expression.

- | | | | | |
|---------------|-------------|--------------|----------|-----------|
| (a) $a + 4$ | (b) $a - d$ | (c) $d - c$ | (d) $2c$ | (e) $-7b$ |
| (f) $6e + 3a$ | (g) ac | (h) $bd + a$ | (i) ve | (j) c^2 |

Q5) If $a = 1.5$ $b = 4$ $c = 6$ $d = 0.5$ and $e = -3$. Find the value of each expression.

- | | | | |
|-----------------------|------------------------|--------------------------|----------------|
| (a) $4(a + d)$ | (b) $5(c + b)$ | (c) $3(10 - e)$ | (d) abc |
| (e) e^3 | (f) d^2 | (g) $5b^2$ | (h) $8e^2 + 3$ |
| (i) $\frac{b + 2}{3}$ | (j) $\frac{2c - e}{4}$ | (k) $\frac{10d + 4b}{7}$ | |

Example 1

$$y = 2x + 1$$

x	-1	0	1	2	3
y	-1	1	3	5	7

$$2(3) + 1 = 7$$

$$2(2) + 1 = 5$$

$$2(1) + 1 = 3$$

$$2(0) + 1 = 1$$

$$2(-1) + 1 = -1$$

Example 2

$$x + 2y = -2$$

x	-1	0	1	2	3
y	-0.5	-1	-1.5	-2	-2.5

$$-1 - \frac{1}{2}(3) = -2.5 \quad \text{Re-arrange to make y the subject}$$

$$-1 - \frac{1}{2}(2) = -2$$

$$-1 - \frac{1}{2}(1) = -1.5$$

$$-1 - \frac{1}{2}(0) = -1$$

$$-1 - \frac{1}{2}(-1) = -0.5$$

$$x + 2y = -2$$

$$(-x) \quad (-x)$$

$$\frac{2y}{2} = \frac{-2 - x}{2}$$

$$y = -1 - \frac{1}{2}x$$

Example 3

$$y = x^2 - 3x$$

x	-2	-1	0	1	2	3	4
y	10	4	0	-2	-2	0	4

$$(4)^2 - 3(4) = 16 - 12 = 4$$

$$(2)^2 - 3(2) = 4 - 6 = -2$$

$$(-1)^2 - 3(-1) = 1 - -3 = 4$$

Example 4

$$y = x^2 + 2x + 1$$

x	-3	-2	-1	0	1	2	3
y	4	1	0	1	4	9	16

$$(3)^2 + 2(3) + 1 = 9 + 6 + 1 = 16$$

$$(2)^2 + 2(2) + 1 = 4 + 4 + 1 = 9$$

$$(1)^2 + 2(1) + 1 = 1 + 2 + 1 = 4$$

$$(0)^2 + 2(0) + 1 = 0 + 0 + 1 = 1$$

$$(-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$$

$$(-2)^2 + 2(-2) + 1 = 4 - 4 + 1 = 1$$

$$(-3)^2 + 2(-3) + 1 = 9 - 6 + 1 = 4$$

Questions

Q1) Complete the table of values for $y = 2x + 4$.

x	-1	0	1	2	3
y		4			10

Q2) 5. (a) Complete the table of values for $y = -2x + 11$.

x	0	1	2	3	4	5
y			7	5		1

Q3) (a) Complete the table of values for $y = \frac{1}{2}x + 1$

x	0	1	2	3	4
y					

Q4) Complete the table of values for $y = 6 - 4x$.

x	-1	0	1	2	3
y			2		-6

Q5) $2x - y = 4$

x	-1	0	1	2	3
y					

Q6) $2x + y = 4$

x	-1	0	1	2	3
y					

Q7) Draw and complete a table of values for $y = 3x - 1$ for the values of x from 0 to 5

Q8) Draw and complete a table of values for $y = 10 - 2x$ for the values of x from -1 to 5

Q9) 1. (a) Complete the table of values for $y = x^2 - 1$

x	-2	-1	0	1	2	3
y	3		-1		3	

Q10) 2. (a) Complete the table of values for $y = x^2 + x$

x	-3	-2	-1	0	1	2	3
y	6		0		2	6	

Q11) 5. (a) Complete the table of values for $y = 3x^2 + 1$

x	-2	-1	0	1	2
y	13		1	4	

Q12) 6. (a) Complete the table for the graph $y = 4 - x^2$

x	-3	-2	-1	0	1	2
y	-5		3	4	3	

Q13) Complete the table of values for $y = x^2 + x - 4$

x	-2	-1	0	1	2	3
y						

Q14) Complete the table of values for $y = x^2 - 3x + 2$

x	-2	-1	0	1	2	3
y	12				0	

Topic 11 – Solving Linear equations

Examples Solve

$$x + 4 = 20$$

$$x + 4 = 20$$

$$4x = 20$$

$$\frac{x}{4} = 20$$

Questions

Solve the following equations

(a) $w + 5 = 7$ (b) $c + 2 = 10$ (c) $a - 1 = 6$ (d) $x - 4 = 5$

(e) $x + 4 = 13$ (f) $3w = 12$ (g) $2x = 18$ (h) $\frac{w}{2} = 6$

(i) $\frac{x}{4} = 7$ (j) $5y = 30$ (k) $x + 10 = 40$ (l) $2x = 34$

Examples

Solve

$$3x - 7 = 20$$

$$2x + 8 = 15$$

$$3x + 15 = 3$$

Questions

Solve the following equations

(a) $2x + 3 = 9$

(b) $3w - 1 = 14$

(c) $7y + 2 = 30$

(d) $5x + 20 = 35$

(e) $6c - 12 = 48$

(f) $8m - 4 = 20$

(g) $7w + 13 = 90$

(h) $12p - 18 = 30$

(i) $9w - 5 = 67$

(d) $5w + 11 = 19$

(e) $8x + 2 = 30$

(f) $4x + 11 = 3$

(g) $6w + 20 = 2$

(h) $2w - 9 = -6$

(i) $3c + 8 = -13$

Examples

Solve

$$\frac{2x}{5} = 3$$

Add a one underneath to make it a fraction

$$\frac{2x}{5} \times \frac{1}{1}$$

Fraction=fraction
Use scissors

$$2x(1) = 3(5)$$

$$\frac{2x}{2} = \frac{15}{2}$$

$$x = \frac{15}{2} = 7\frac{1}{2}$$

Solve

$$\frac{3w+4}{5} \times \frac{2}{7}$$

Fraction=fraction
Use scissors

$$7(3w+4) = 5(2)$$

$$21w + 28 = 10$$

$$(-28) \quad (-28)$$

$$\frac{21w}{21} = \frac{-18}{21}$$

$$w = \frac{-18}{21}$$

Questions

Solve the following equations

(a) $\frac{x+1}{2} = 9$

(b) $\frac{x-3}{4} = 8$

(c) $\frac{7x-5}{10} = 10$

(i) $\frac{10m+20}{15} = 6$

(d) $\frac{2x}{3} = 6$

(e) $\frac{3x}{5} = 30$

(f) $\frac{5x}{4} = 20$

(k) $\frac{2x+5}{3} = 7$

Examples

Solve

$$\frac{3x}{5} + 2 = 7$$

(-2) (-2)

$$\frac{3x}{5} = 5$$

$$\frac{3x}{5} \times \frac{5}{1}$$

$$1(3x) = 5(5)$$

$$\frac{3x}{3} = \frac{25}{3}$$

$$x = \frac{25}{3}$$

Add a one underneath to make it a fraction

Fraction=fraction
Use scissors

Solve

$$\frac{4}{5w} + 1 = 12$$

(-1) (-1)

$$\frac{4}{5w} = 11$$

$$\frac{4}{5w} \times \frac{1}{1}$$

$$4(1) = 5w(11)$$

$$\frac{4}{5} = \frac{55w}{5}$$

$$w = \frac{4}{55}$$

Add a one underneath to make it a fraction

Fraction=fraction
Use scissors

Questions

$$\frac{x}{2} + 1 = 3$$

$$\frac{a}{3} + 4 = 9$$

$$\frac{2r}{5} - 3 = 2$$

$$\frac{3}{c} + 8 = 2$$

$$\frac{3x}{4} + 1 = 12$$

$$\frac{4}{p} + 3 = 5$$

Examples

Solve

$$4(3x - 5) = 76$$

Solve

$$3(2x + 4) + 8 = 50$$

Solve $4(2c + 3) - (5c - 1) = 28$

CHECK:

Questions

$$3(x - 1) = 6$$

$$2(a + 3) = 16$$

$$2(2b - 3) = 6$$

$$3(2r + 1) = 21$$

$$100 = 5(2 + 3t)$$

$$84 = 7(5p - 3)$$

$$100 = 5(2 + 3t)$$

$$84 = 7(5p - 3)$$

$$3(2x + 1) + 2(4x + 1) = 75$$

$$7(2x + 4) + 3(2x - 5) = 53$$

$$2(7x - 4) + 3(8 - 2x) = 36$$

$$4(6 - 3x) + 8(2x - 1) = 52$$

Examples

Solve

$$6x + 15 = 8x + 45$$

CHECK:

Solve

$$4(3x - 2) = 20 - 2x$$

CHECK:

CHECK:

Solve the Equation :

$$\frac{8h + 7}{2} = 3h - 4$$

Solve $\frac{3x-1}{3} = \frac{2x+3}{4}$

CHECK:

Questions

(a) $4x + 1 = 2x + 7$

(b) $5x + 4 = 3x + 16$

(c) $2x + 8 = x + 12$

(d) $7x + 1 = 2x + 46$

(e) $6x - 3 = 2x + 13$

(f) $9x - 10 = 7x + 24$

(a) $3x + 3 = x + 8$

(b) $9x + 10 = 7x + 39$

(c) $3x + 1 = 7x - 17$

(a) $4x + 15 = x + 3$

(b) $8x + 40 = 3x + 5$

(c) $9x + 7 = 11x + 20$

(a) $5(x + 3) = 3(x + 9)$

(b) $8(x - 1) = 4(x + 3)$

(c) $3(x + 13) = 10(x - 1)$

(d) $2(4x - 3) = 5(2x - 5)$

(e) $9(2x - 5) = 3(4x + 7)$

(f) $2(9 - x) = 3(x + 16)$

$$\frac{11 - w}{5} = 3 + w$$

$$\frac{9(4x - 1)}{2x} = 15$$

$$\frac{x}{2x - 1} = 5$$

$$\frac{2(3x - 6)}{5} = \frac{3x}{2}$$

$$\frac{5x}{5} = 3x - 8$$

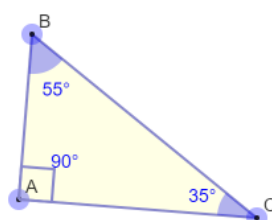
$$\frac{x + 15}{3} = 2(x + 10)$$

Topic 12 – Angles in triangles and quadrilaterals

Angles in triangles add to 180°

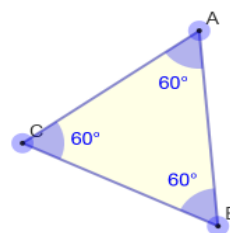
Right Triangle

Has a right angle (90°)



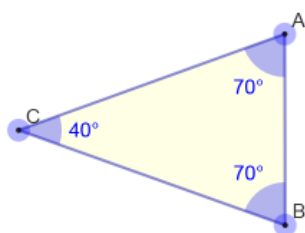
Equilateral Triangle

Three equal sides and three equal angles of 60° each



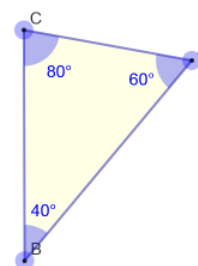
Isosceles Triangle

Two equal sides and two equal angles



Scalene Triangle

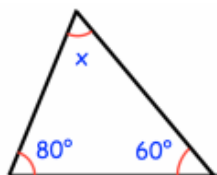
No equal angles and no equal sides



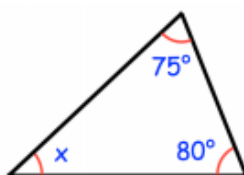
Note: can have a right angle

Questions

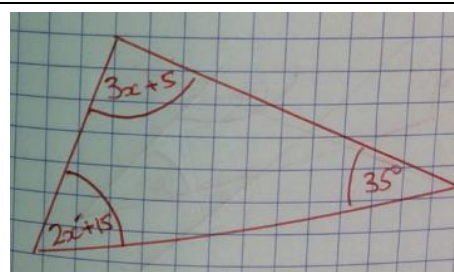
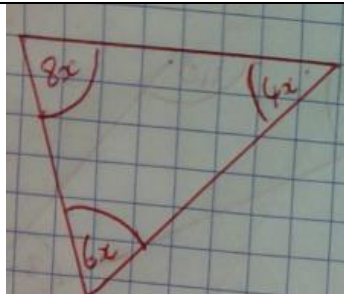
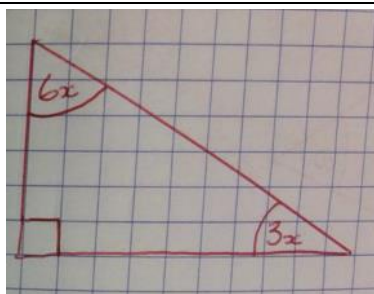
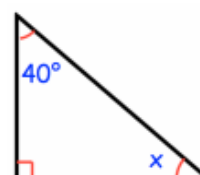
(a)



(b)

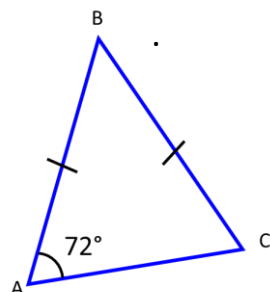


(c)

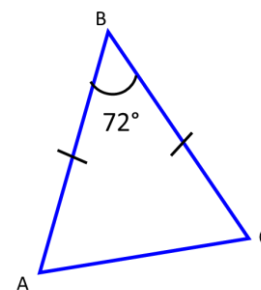


Examples

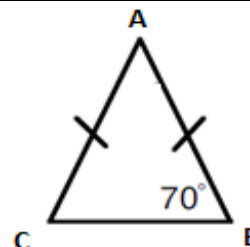
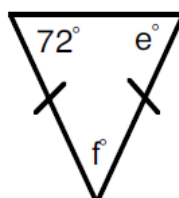
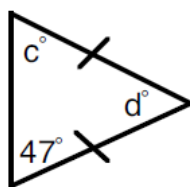
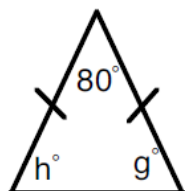
Work out angle $\hat{A}BC$



Work out angle $\hat{B}AC$

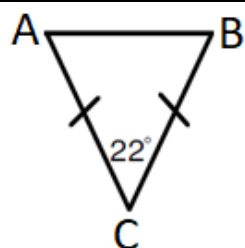


Questions



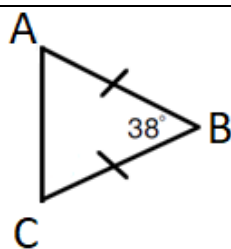
Work out angle $\hat{B}CA$

Work out angle $\hat{C}AB$



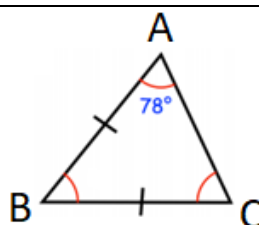
Work out angle $\hat{C}AB$

Work out angle $\hat{A}BC$



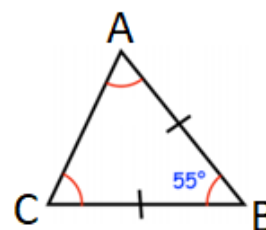
Work out angle $\hat{C}AB$

Work out angle $\hat{A}CB$



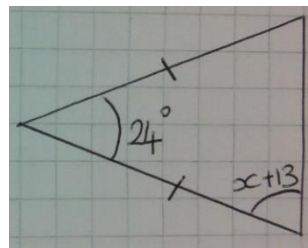
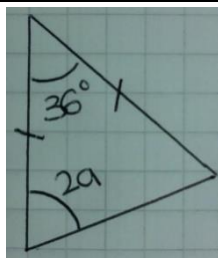
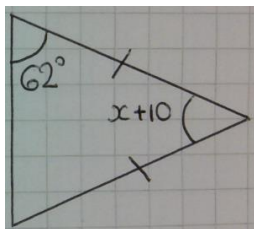
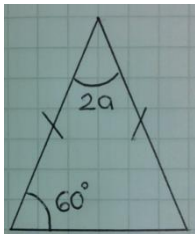
Work out angle $\hat{A}CB$

Work out angle $\hat{A}BC$



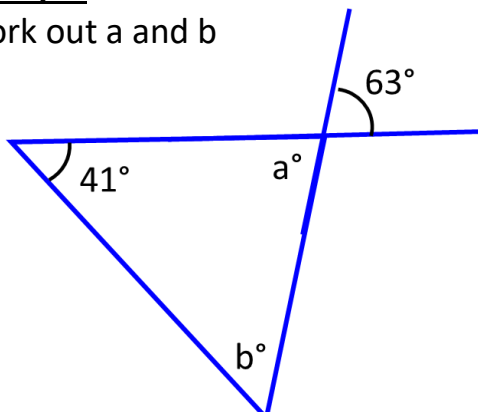
Work out angle $\hat{B}CA$

Work out angle $\hat{C}AB$



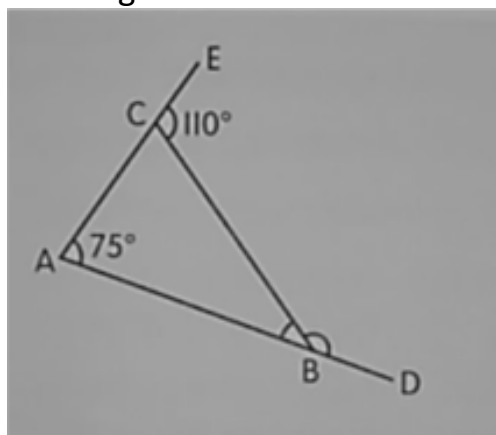
Example

Work out a and b

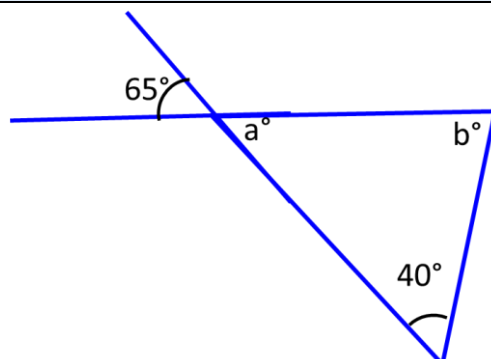
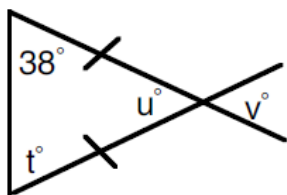


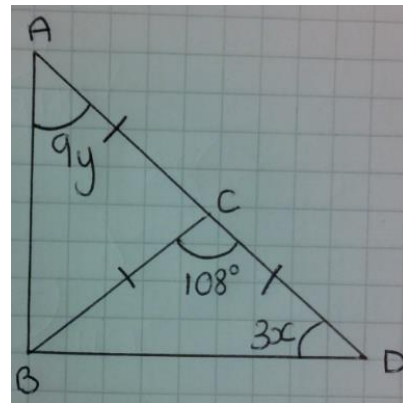
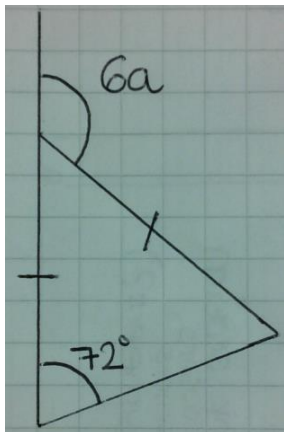
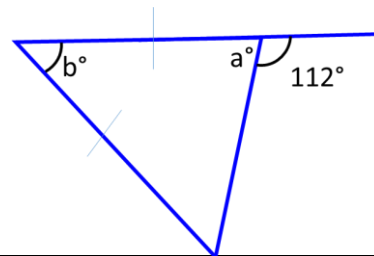
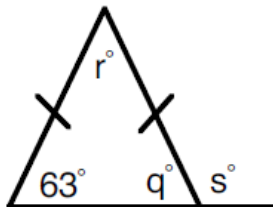
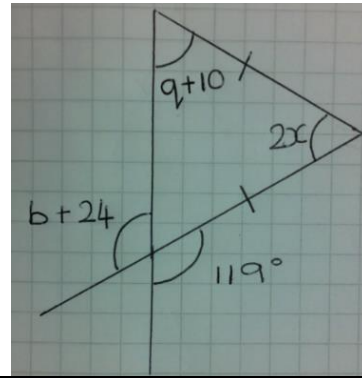
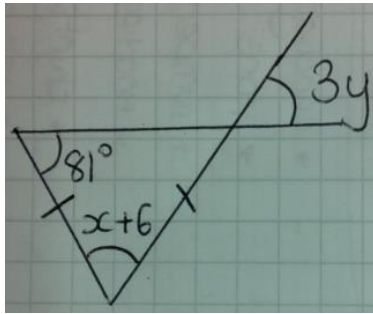
Example

Find angle $\hat{C}BD$



Questions

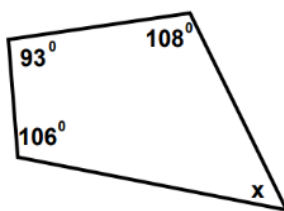




Angles in quadrilaterals add to 360°

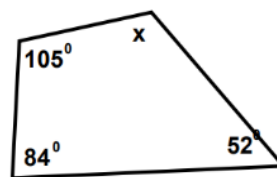
Questions

1)



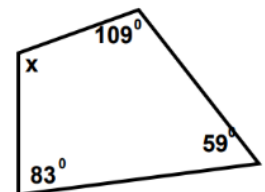
Solve for x _____

2)

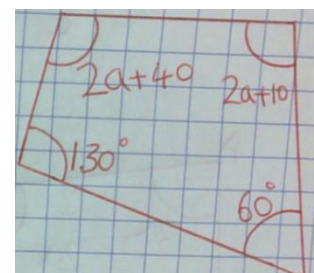
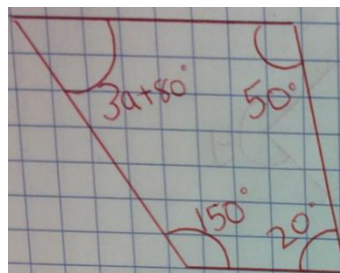
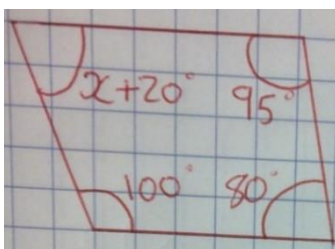


Solve for x _____

3)

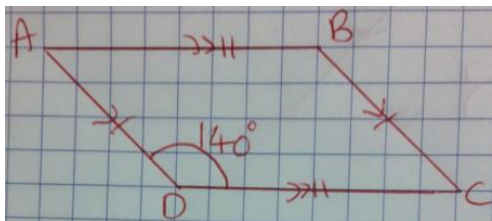


Solve for x _____



Parallelogram

Find the missing angles

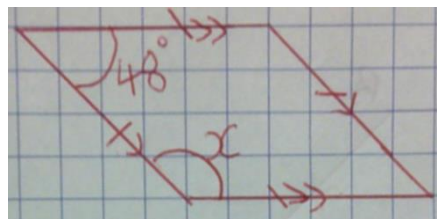


1. Diagonally opposite angles are equal

2. Co-interior angles sum to 180°

Rhombus

Find the missing angles

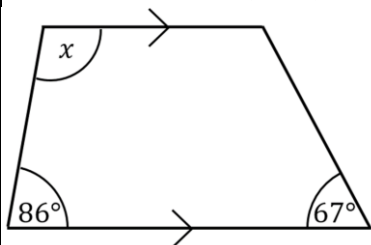


1. Diagonally opposite angles are equal

2. Co-interior angles sum to 180°

Trapezium

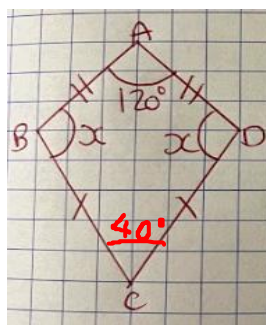
Find the missing angles



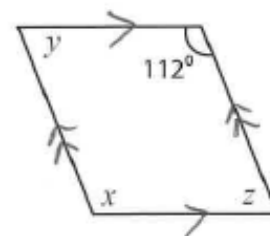
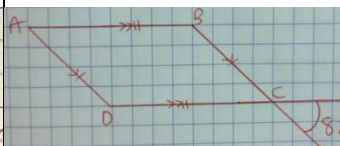
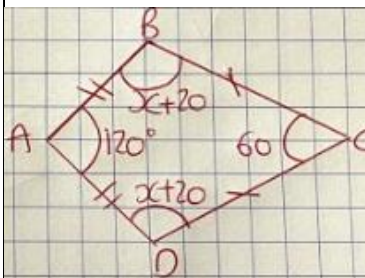
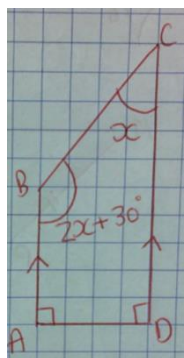
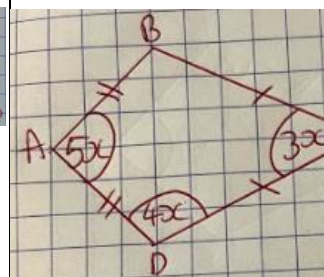
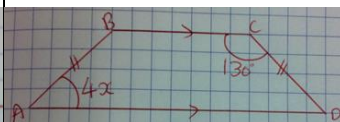
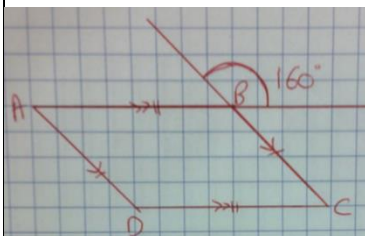
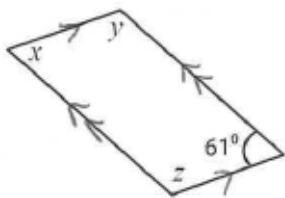
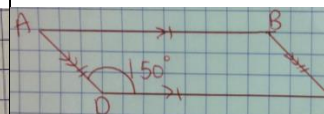
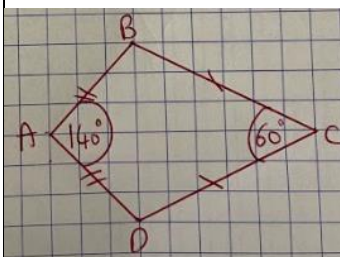
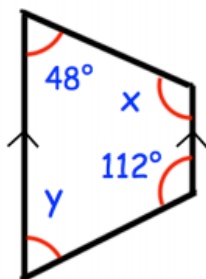
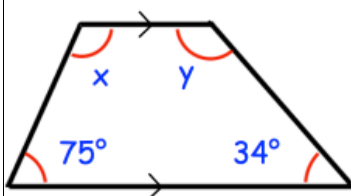
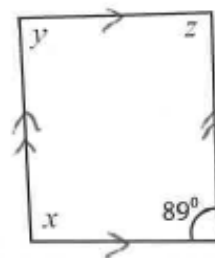
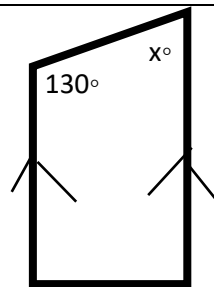
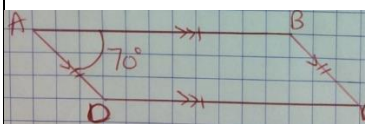
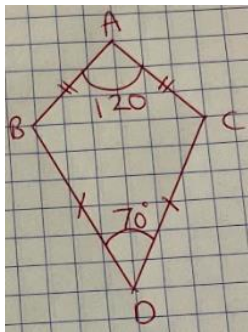
Co-interior angles sum to 180°

Kite

Find the missing angle



Find all of the missing angles in each shape



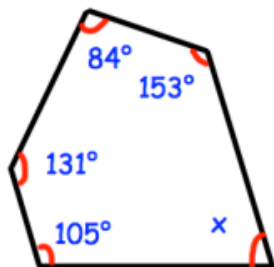
Topic 13 – Angles in polygons

Facts

$\frac{360}{\text{number of sides}} = \text{exterior angle}$	$\frac{360}{\text{exterior angles}} = \text{number of sides}$
$\text{Interior} + \text{Exterior} = 180^\circ$	$(n - 2) \times 180 = \text{angle sum}$

Example 1

Work out angle x

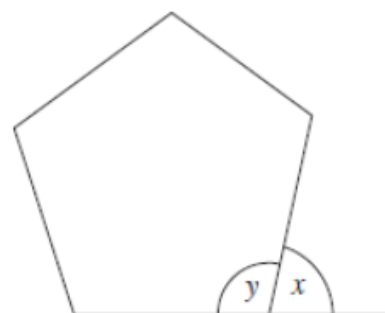


$$\begin{aligned}
 \text{Angle Sum} &= (5-2) \times 180 \\
 &= 3 \times 180 \\
 &= 540^\circ \\
 540 - (84 + 153 + 131 + 105) \\
 &= 67^\circ
 \end{aligned}$$

Example 2

Shown is a regular pentagon.

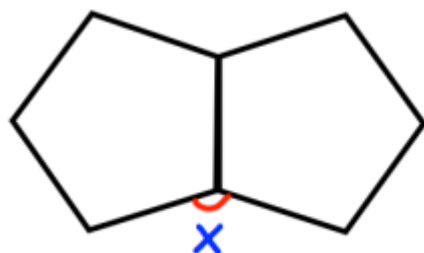
Work out angle x and y



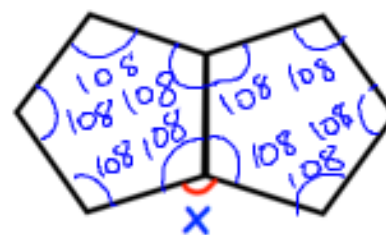
$$\begin{aligned}
 \text{Angle Sum} &= (5-2) \times 180 \\
 &= 3 \times 180 \\
 &= 540 \\
 540 \div 5 &= 108 \quad \leftarrow \text{Interior } y = 108 \\
 \text{Interior} + \text{exterior} &= 180^\circ \\
 180 - 108 &= 72^\circ \\
 x &= 72^\circ
 \end{aligned}$$

Example 3

Work out the size of angle x

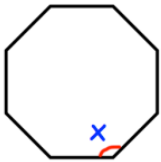


$$\begin{aligned}
 \text{Angle Sum} &= (5-2) \times 180 \\
 &= 3 \times 180 \\
 &= 540 \\
 540 \div 5 &= 108^\circ
 \end{aligned}$$

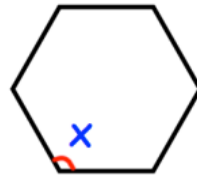


$$\begin{aligned}
 108 + 108 &= 216 \\
 360 - 216 \\
 &= 144^\circ = x
 \end{aligned}$$

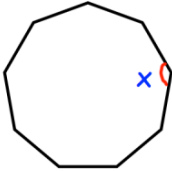
Questions



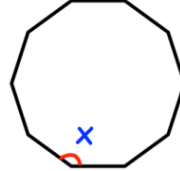
regular octagon



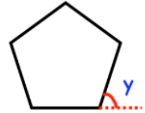
regular hexagon



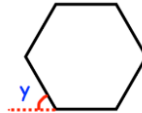
regular nonagon



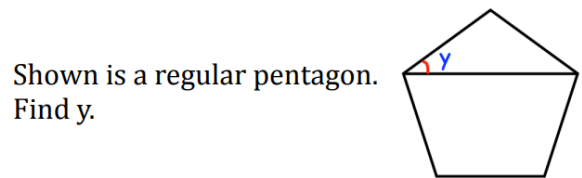
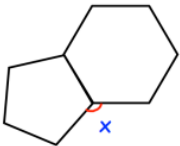
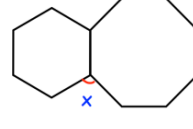
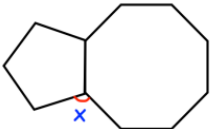
regular decagon



regular pentagon



regular hexagon

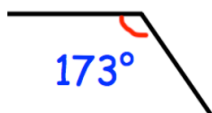


Shown is a regular pentagon.
Find y .

A regular polygon has 18 sides.
Calculate the size of each interior angle.

A regular polygon has 30 sides.
Calculate the size of each interior angle.

Explain why this cannot be an interior angle from regular polygons.



Part of a regular polygon with 15 sides is shown.



Work out the size of an **interior** angle.

Topic 14 – Pie charts

Pie charts show proportions and not amounts

Angles in a pie chart sum to 360°

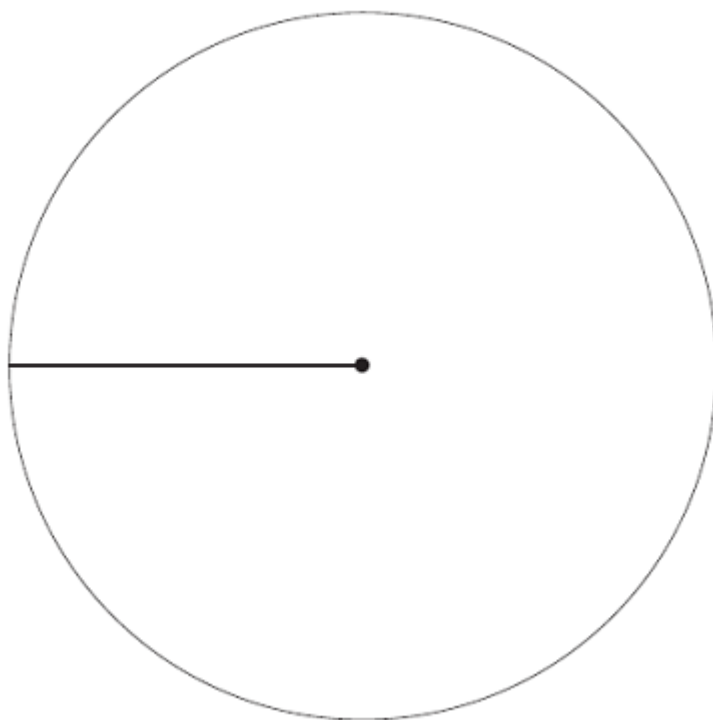
$$\text{Angle} = \frac{\text{Frequency}}{\text{Total}} \times 360$$

Example – Finding missing angles

The table shows where 72 people went on holiday.

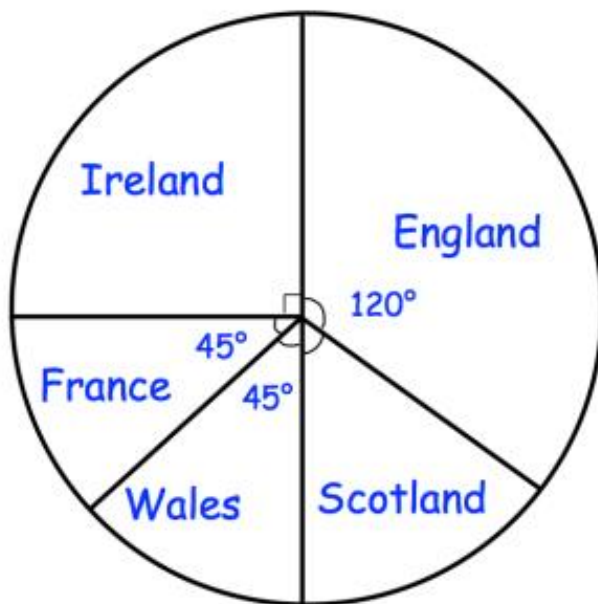
Spain	36	
Portugal	18	
Turkey	6	
Other	12	
Total = 72		

Draw a fully labelled pie chart for the data.



Example – Finding missing frequencies

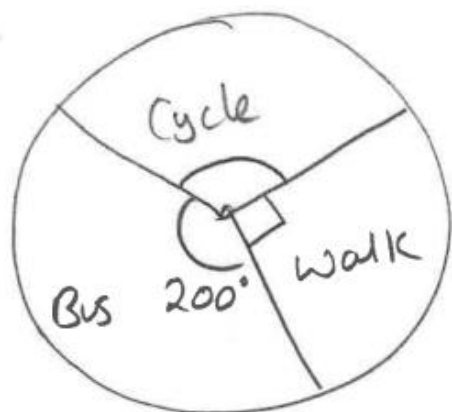
A group of rugby fans were asked who they supported.
The pie chart and table show information about who they support.



Use the pie chart to complete the table.

Team	Angle of sector	Number of fans
England	120°	
Scotland		
Wales	45°	
France	45°	
Ireland	90°	12

Example



a) What fraction use the BUS?

b) What fraction walk to School?

c) 2700 walk to school, how many cycle to school?

Questions

Complete the table to draw the following pie charts

Method of Transport	Frequency
Car	8
Bus	11
Walk	12
Cycle	5

Colour	Frequency
Blue	25
Green	14
Red	21

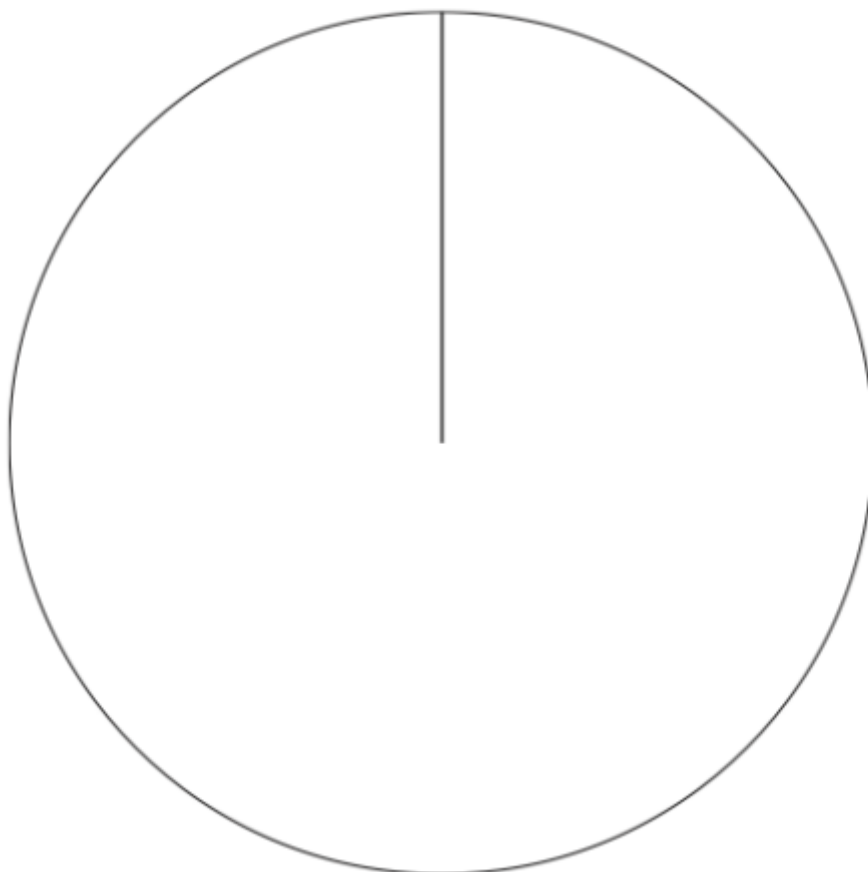
Make	Frequency
Ford	8
Mazda	14
Volkswagen	21
Fiat	20
Honda	9

1. The table gives information about the holiday destination of 18 students in a class.



Country	Frequency	
France	3	
Wales	4	
England	11	

Draw an accurate pie chart to show this information.

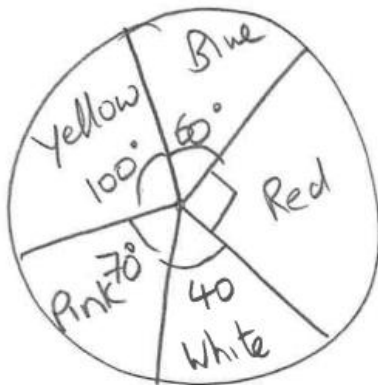
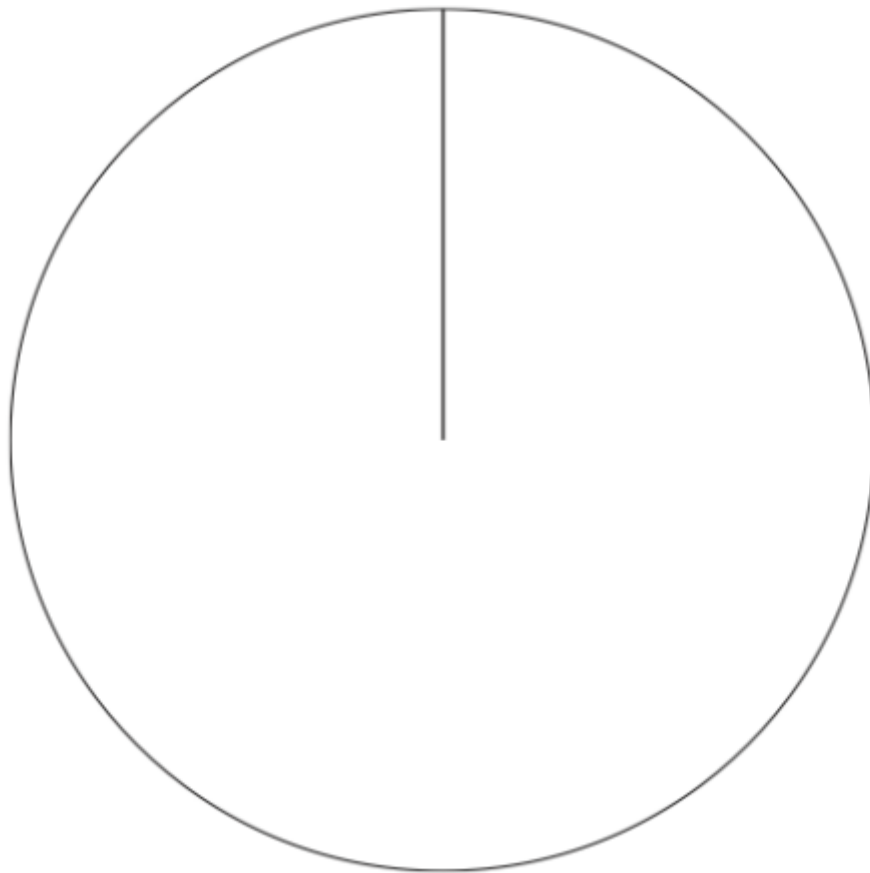


3. The table gives information about students staying after school to play sport.

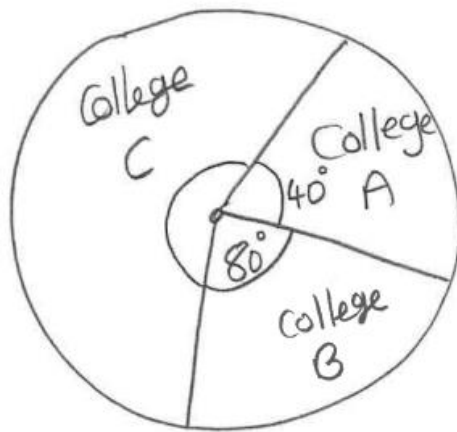


Sport	Frequency
Netball	15
Hockey	10
Rugby	26
Football	9

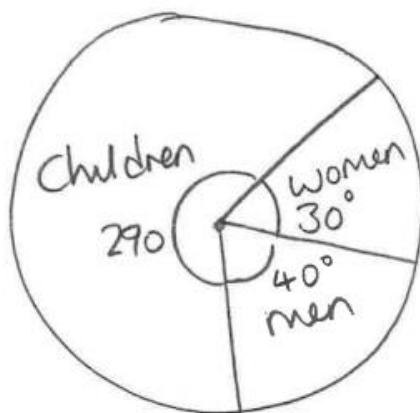
Draw an accurate pie chart to show this information.



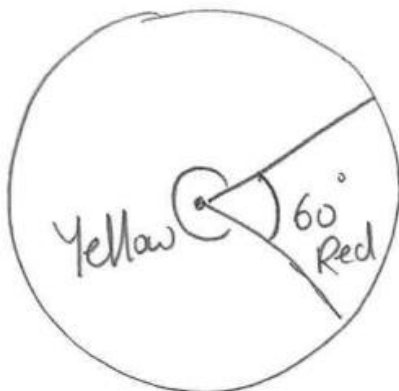
There are 60 more
Yellows than pinks.
How many reds are
there?
Work out the number
of each colour.



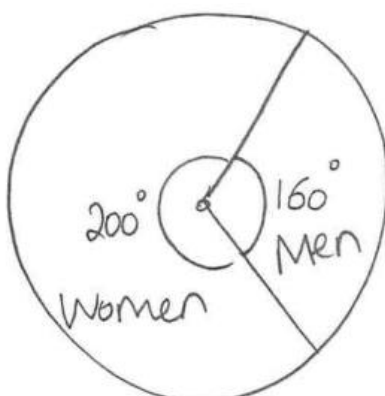
There are 2400 students in college B. How many students in college A?



There are 50 more men than women. How many children were there?



There are 150 yellow pens. How many pens are there altogether?



There are 1600 more women than men. How many men are there?

Topic 15 – Mean, Median, Mode and Range

To find the **mean** we:

1. Find the total frequency (add)
2. Divide by how many there are (divide)

Examples

Work out the mean 5, 7, 1, 9, 8

1. $5 + 7 + 1 + 9 + 8 = 30$
2. $30 \div 5 = 6$

Mean = 6

Work out the mean -3, 7, -9, 11, 4, 8

1. $-3 + 7 + -9 + 11 + 4 + 8 = 18$
2. $18 \div 6 = 3$

Mean = 3

To find the **median** we:

1. Write the numbers in ascending order
2. Find the middle number
If there is an even number of numbers, we add the two middle numbers and divide by 2

Examples

Work out the median 5, 8, 2, 6, 1

1. 1, 2, 5, 6, 8

Median = 5

Work out the median -1, 5, 12, -4, 8, 20

1. -4, -1, 5, 8, 12, 20

$$\text{Median} = \frac{5+8}{2} = \frac{13}{2} = 6.5$$

The **mode** is the most common

Examples

Work out the mode 1.2, 8, 3, 3.4, 4, 8, 5.1, 8

Mode = 8

Range = largest value – smallest value

Examples

Work out the range 15, 7, 2, 11, 9

$$\text{Mode} = 15 - 2 = 13$$

Work out the range -2, -6, 9, 3, 19, 11

$$\text{Mode} = 19 - -6 = 25$$

Questions

Question 1: Find the mean for each of the sets of data below

- | | | |
|----------------------------|---------------------------------------|---|
| (a) 4, 9, 7, 10, 5 | (b) 2, 8, 6, 3, 12, 7, 4 | (c) 3, 2, 1, 3, 2, 2, 1, 3, 1, 2, 3, 2, 1 |
| (d) 1, 8, 7, 5, 6, 4, 7, 6 | (e) 20, 30, 24, 32 | (f) 12, 8, 14, 5, 1, 3, 0, 8, 10, 11 |
| (g) 9, -3, -6, 5, 0 | (h) 1.4, 2.8, 2.4, 2.5, 2.8, 3.1, 1.1 | |

Question 1: Work out the median for the each of the following

- (a) 5, 1, 4, 6, 8 (b) 9, 1, 3, 6, 7, 8, 9 (c) 6, 4, 7, 1, 3, 8, 1, 10
(d) 7, 3, 8, 9, 6, 5 (e) 9, 8, 6, 6, 6, 7, 1, 2, 6, 8 (f) -4, 5, -7, -1, 2, 0, 9
(g) 20, 30, 10, 20, 40, 50, 60, 10, 80, 30 (h) 49, 34, 12, 10, 53, 20, 65, 34, 90, 100, 33
(i) 6.2, 6.8, 6.6, 7.2, 6.4, 7.4, 5.8 (j) 124, 53, 39, 230, 155, 180

Question 1: Work out the mode for the each of the following

- (a) 5, 6, 6, 7, 8, 10 (b) 1, 1, 1, 4, 6, 8, 12 (c) 5, 5, 7, 7, 7, 8, 8, 9
(d) 5, 7, 3, 5, 8, 9, 10, 2 (e) 8, 3, 3, 4, 6, 8, 13, 3, 18 (f) 12, 14, 15, 17, 15
(g) 2.3, 2.6, 2.8, 2.7, 2.8, 2.7, 2.4, 2.3, 2.1, 2.3 (h) -2, -1, 5, 8, -2, 2, -1, 9, -1, 1, 2, -1

Question 1: Find the range for each of the following

- (a) 5, 9, 1, 5, 7, 4, 3 (b) 6, 7, 10, 8, 9, 9 (c) 21, 15, 19, 24, 30, 26
(d) 210, 250, 260, 180, 240 (e) 6.2, 7.3, 8.8, 1.5, 4.1 (f) 3, 1, 2, 1, 3, 4, 5, 0, 1
(g) -5, 1, 3, 6, -8, 1 (h) -6, -10, -2, -9 (i) 0, 7, 9, -21, 10, -4
(j) 7, 9, -2, 13, 9, 8, 20, -8, 1 (k) -10, -6, -15, -9, -8, -7, 8, -3

Question 2: A basketball team plays 8 matches.

The number of points they score in each match are:

62, 68, 67, 79, 82, 50, 74, 62

- (a) Work out the mean number of points scored
(b) Write down the modal number of points scored
(c) Write down the median number of points scored



Question 3: The height of some footballers are listed below:

1.81m, 1.78m, 1.88m, 1.79m, 1.86m, 1.85m, 1.78m, 1.93m

(a) Work out the median height

(b) What is the modal height?

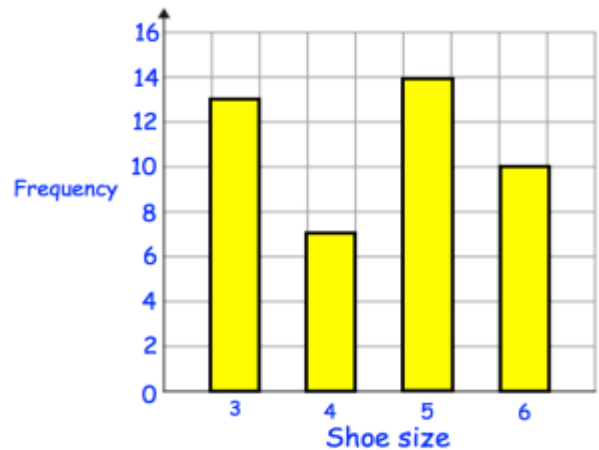
The bar chart shows the shoe sizes of a group of students.

(a) How many students in total are there?

(b) What is the modal shoe size?

(c) What is the mean shoe size

(d) What is the median shoe size?



Example

The mean of 5 numbers is 12.

Four of the numbers are 9, 7, 15, 21.

What is the fifth number?

Questions

The mean of four numbers is 10. Three of the numbers are 9, 11 and 7.
Work out the fourth number.

The mean of six numbers is 5. Five of the numbers are 6, 6, 5, 3 and 1.
Work out the sixth number.

The mean of five numbers is 8.2. Four of the numbers are 8, 10, 12 and 10.
Work out the fifth number.

Example

The table shows the ages of different students at a summer club.
Using the table work out the mean, median, mode and range.

Age	Frequency
5	2
6	2
7	5
8	1

For this type of table, because there isn't a lot of data we can take of the data and list it

5, 5, 6, 6, 7, 7, 7, 7, 7, 8

$$\text{Mean} = \frac{5+5+6+6+7+7+7+7+7+8}{10} = \frac{65}{10} = 6.5 \text{ years old}$$

Median = 5, 5, 6, 6, 7, 7, 7, 7, 7, 8 = 7 years old

Mode = 7 years old

Range = 8 – 5 = 3 years old

Example

The table shows the number of bedrooms in houses for sale.
Work out the mean.

Number of bedrooms	Frequency
1	34
2	275
3	512
4	179

Questions

For each table find the mean, median, mode and range:

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

Level	Frequency
3	1
4	9
5	7
6	2
7	1

For each table find the mean:

Money Withdrawn	Frequency
£10	16
£20	19
£30	4
£40	3
£50	6
£60	2

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

Age	Frequency
16	28
17	7
18	3
19	2

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

Times visited	Frequency
0	131
1	873
2	599
3	205

Siblings	Frequency
0	71
1	25
2	14

Number of bedrooms	Frequency
1	34
2	275
3	512
4	179

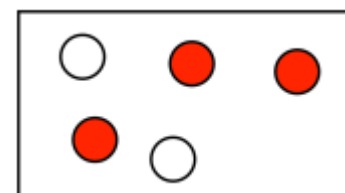
Star rating	Frequency
0	9
1	12
2	17
3	19
4	21
5	8

Topic 16 - Probability

- Probability is out of 1
- Relative frequency is written as fraction, decimal or percentage
- **AND** means multiply
- **OR** means add

Example

Theo has 3 red sweets and 2 white sweets.
He picks a sweet at random.



(a) Write down the probability that Theo picks a red sweet

(b) Write down the probability that Theo picks a white sweet

Example

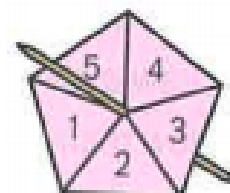
The probability of it raining tomorrow is 0.3.
What is the probability of it not raining?

Example

Here is a spinner with some numbers on it.
The spinner is spun.

What is the probability of it landing on:

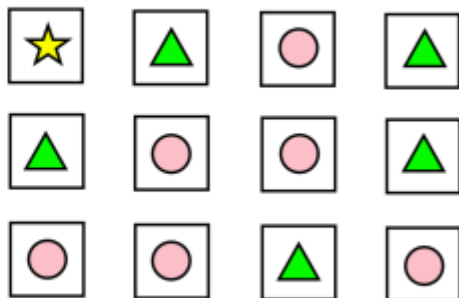
- a 2?
- an even number?
- a prime number?
- a square number?



Questions

Question 2: Leah has 12 cards, each with a shape on it.
She takes a card at random.

- (a) What is the probability that Leah takes a card with a star on it?
- (b) What is the probability that Leah takes a card with a triangle on it?
- (c) What is the probability that Leah takes a card with a circle on it?



Question 3: Ralph has 9 cards, each with a number on it.



He picks a card at random.

Write down the probability that the chosen card is

- (a) the number 8 (b) an even number (c) a number less than 7
- (d) a multiple of 4 (e) a square number (f) a prime number

Question 4: There are 12 red roses, 5 yellow roses and 3 white roses in a vase.
Felix takes a rose, at random, from the vase.

- (a) Write down the probability that he takes a white rose.
- (b) Write down the probability that he takes a red **or** a white rose.
- (c) Write down the probability that Felix takes a rose that is **not** red.

Leon throws a biased coin.

The probability of getting tails is 0.4

Work out the probability of getting heads.



Edith plants a daffodil bulb.

The probability that the bulb will grow is 0.8

What is the probability that the bulb will **not** grow?

Wycombe Wanderers play a match of football.

The probability that they win the match is 0.28

The probability that they draw the match is 0.55

Work out the probability that they lose the match.

Evelyn has 80 pens in a drawer.

15 pens are black and the other pens are blue.

Evelyn picks a pen at random from the drawer.

- (a) What is the probability that Evelyn picks a black pen?
- (b) What is the probability that Evelyn picks a blue pen?



Expected outcome = probability x number of trials
--

Example

A biased coin is tossed 300 times.

The probability of it landing on heads is 0.2.

How many times would you **expect** to get a head?

How many times would you **expect** to get a tail?

Topic 15 – Mean, Median, Mode and Range

To find the **mean** we:

1. Find the total frequency (add)
2. Divide by how many there are (divide)

Examples

Work out the mean 5, 7, 1, 9, 8

1. $5 + 7 + 1 + 9 + 8 = 30$
2. $30 \div 5 = 6$

Mean = 6

Work out the mean -3, 7, -9, 11, 4, 8

1. $-3 + 7 + -9 + 11 + 4 + 8 = 18$
2. $18 \div 6 = 3$

Mean = 3

To find the **median** we:

1. Write the numbers in ascending order
2. Find the middle number
If there is an even number of numbers, we add the two middle numbers and divide by 2

Examples

Work out the median 5, 8, 2, 6, 1

1. 1, 2, 5, 6, 8

Median = 5

Work out the median -1, 5, 12, -4, 8, 20

1. -4, -1, 5, 8, 12, 20

$$\text{Median} = \frac{5+8}{2} = \frac{13}{2} = 6.5$$

The **mode** is the most common

Examples

Work out the mode 1.2, 8, 3, 3.4, 4, 8, 5.1, 8

Mode = 8

Range = largest value – smallest value

Examples

Work out the range 15, 7, 2, 11, 9

$$\text{Mode} = 15 - 2 = 13$$

Work out the range -2, -6, 9, 3, 19, 11

$$\text{Mode} = 19 - -6 = 25$$

Questions

Question 1: Find the mean for each of the sets of data below

- | | | |
|----------------------------|---------------------------------------|---|
| (a) 4, 9, 7, 10, 5 | (b) 2, 8, 6, 3, 12, 7, 4 | (c) 3, 2, 1, 3, 2, 2, 1, 3, 1, 2, 3, 2, 1 |
| (d) 1, 8, 7, 5, 6, 4, 7, 6 | (e) 20, 30, 24, 32 | (f) 12, 8, 14, 5, 1, 3, 0, 8, 10, 11 |
| (g) 9, -3, -6, 5, 0 | (h) 1.4, 2.8, 2.4, 2.5, 2.8, 3.1, 1.1 | |

Question 1: Work out the median for the each of the following

- (a) 5, 1, 4, 6, 8 (b) 9, 1, 3, 6, 7, 8, 9 (c) 6, 4, 7, 1, 3, 8, 1, 10
(d) 7, 3, 8, 9, 6, 5 (e) 9, 8, 6, 6, 6, 7, 1, 2, 6, 8 (f) -4, 5, -7, -1, 2, 0, 9
(g) 20, 30, 10, 20, 40, 50, 60, 10, 80, 30 (h) 49, 34, 12, 10, 53, 20, 65, 34, 90, 100, 33
(i) 6.2, 6.8, 6.6, 7.2, 6.4, 7.4, 5.8 (j) 124, 53, 39, 230, 155, 180

Question 1: Work out the mode for the each of the following

- (a) 5, 6, 6, 7, 8, 10 (b) 1, 1, 1, 4, 6, 8, 12 (c) 5, 5, 7, 7, 7, 8, 8, 9
(d) 5, 7, 3, 5, 8, 9, 10, 2 (e) 8, 3, 3, 4, 6, 8, 13, 3, 18 (f) 12, 14, 15, 17, 15
(g) 2.3, 2.6, 2.8, 2.7, 2.8, 2.7, 2.4, 2.3, 2.1, 2.3 (h) -2, -1, 5, 8, -2, 2, -1, 9, -1, 1, 2, -1

Question 1: Find the range for each of the following

- (a) 5, 9, 1, 5, 7, 4, 3 (b) 6, 7, 10, 8, 9, 9 (c) 21, 15, 19, 24, 30, 26
(d) 210, 250, 260, 180, 240 (e) 6.2, 7.3, 8.8, 1.5, 4.1 (f) 3, 1, 2, 1, 3, 4, 5, 0, 1
(g) -5, 1, 3, 6, -8, 1 (h) -6, -10, -2, -9 (i) 0, 7, 9, -21, 10, -4
(j) 7, 9, -2, 13, 9, 8, 20, -8, 1 (k) -10, -6, -15, -9, -8, -7, 8, -3

**Question 2: A basketball team plays 8 matches.
The number of points they score in each match are:**

62, 68, 67, 79, 82, 50, 74, 62

- (a) Work out the mean number of points scored
(b) Write down the modal number of points scored
(c) Write down the median number of points scored



Question 3: The height of some footballers are listed below:

1.81m, 1.78m, 1.88m, 1.79m, 1.86m, 1.85m, 1.78m, 1.93m

(a) Work out the median height

(b) What is the modal height?

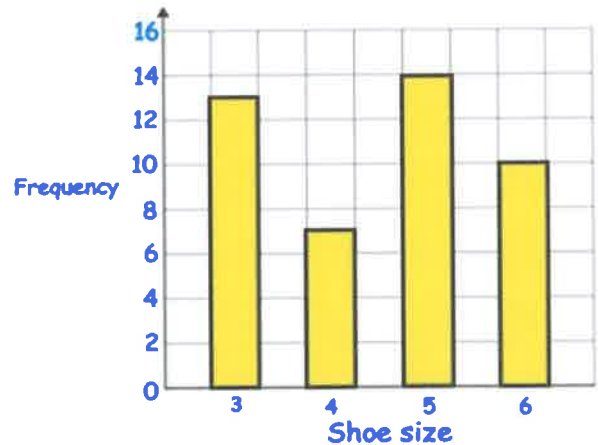
The bar chart shows the shoe sizes of a group of students.

(a) How many students in total are there?

(b) What is the modal shoe size?

(c) What is the mean shoe size

(d) What is the median shoe size?



Example

The mean of 5 numbers is 12.

Four of the numbers are 9, 7, 15, 21.

What is the fifth number?

call it x

$$9 + 7 + 15 + 21 + x = 12 \times 5$$

5

$\times 5$

$\times 5$

$$9 + 7 + 15 + 21 + x = 60$$

$$52 + x = 60$$

$$x = 8$$

Questions

The mean of four numbers is 10. Three of the numbers are 9, 11 and 7.
Work out the fourth number.

The mean of six numbers is 5. Five of the numbers are 6, 6, 5, 3 and 1.
Work out the sixth number.

The mean of five numbers is 8.2. Four of the numbers are 8, 10, 12 and 10.
Work out the fifth number.

Example

The table shows the ages of different students at a summer club.
Using the table work out the mean, median, mode and range.

Age	Frequency
5	2
6	2
7	5
8	1

For this type of table, because there isn't a lot of data we can take of the data and list it

5, 5, 6, 6, 7, 7, 7, 7, 7, 8

$$\text{Mean} = \frac{5+5+6+6+7+7+7+7+7+8}{10} = \frac{65}{10} = 6.5 \text{ years old}$$

$$\text{Median} = 5, 5, 6, 6, 7, 7, 7, 7, 7, 8 = 7 \text{ years old}$$

$$\text{Mode} = 7 \text{ years old}$$

$$\text{Range} = 8 - 5 = 3 \text{ years old}$$

Example

The table shows the number of bedrooms in houses for sale.
Work out the mean.

Number of bedrooms	Frequency	F(x)
1	X 34	34
2	X 275	550
3	X 512	1536
4	X 179	716
Total	1000 2 2	2836 1 1 1

Too many numbers to list

Add another column called F(x)

$$\text{Mean} = \frac{\text{Total F(x)}}{\text{Total frequency}}$$

$$= \frac{2836}{1000} = 2.836$$

Questions

For each table find the mean, median, mode and range:

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

Level	Frequency
3	1
4	9
5	7
6	2
7	1

For each table find the mean:

Money Withdrawn	Frequency
£10	16
£20	19
£30	4
£40	3
£50	6
£60	2

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

Age	Frequency
16	28
17	7
18	3
19	2

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

Times visited	Frequency
0	131
1	873
2	599
3	205

Siblings	Frequency
0	71
1	25
2	14

Number of bedrooms	Frequency
1	34
2	275
3	512
4	179

Star rating	Frequency
0	9
1	12
2	17
3	19
4	21
5	8

Topic 16 - Probability

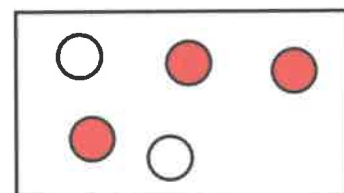
- Probability is out of 1
- Relative frequency is written as fraction, decimal or percentage
- **AND** means multiply
- **OR** means add

Example

Theo has 3 red sweets and 2 white sweets. $\rightarrow$ Total = 5
He picks a sweet at random.

(a) Write down the probability that Theo picks a red sweet

$$\frac{3}{5}$$



(b) Write down the probability that Theo picks a white sweet

$$\frac{2}{5}$$

Example

The probability of it raining tomorrow is 0.3.
What is the probability of it not raining?

$$1 - 0.3 = 0.7$$

Probability adds to 1

Example

Here is a spinner with some numbers on it.
The spinner is spun.

What is the probability of it landing on:

a) a 2? $\frac{1}{5}$

b) an even number?
(2 or 4) $\frac{2}{5}$

c) a prime number?
(2, 3 or 5) $\frac{3}{5}$

d) a square number?
(1 or 4) $\frac{2}{5}$

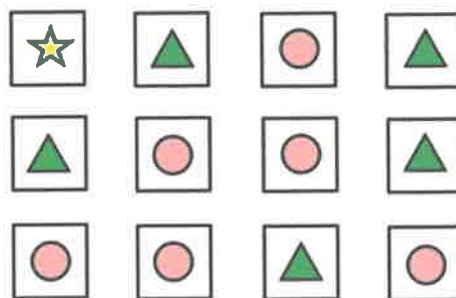
5 numbers.



Questions

Question 2: Leah has 12 cards, each with a shape on it.
She takes a card at random.

- (a) What is the probability that Leah takes a card with a star on it?
- (b) What is the probability that Leah takes a card with a triangle on it?
- (c) What is the probability that Leah takes a card with a circle on it?



Question 3: Ralph has 9 cards, each with a number on it.



He picks a card at random.

Write down the probability that the chosen card is

- (a) the number 8 (b) an even number (c) a number less than 7
- (d) a multiple of 4 (e) a square number (f) a prime number

Question 4: There are 12 red roses, 5 yellow roses and 3 white roses in a vase.
Felix takes a rose, at random, from the vase.

- (a) Write down the probability that he takes a white rose.
- (b) Write down the probability that he takes a red **or** a white rose.
- (c) Write down the probability that Felix takes a rose that is **not** red.

Leon throws a biased coin.
The probability of getting tails is 0.4
Work out the probability of getting heads.



Edith plants a daffodil bulb.
The probability that the bulb will grow is 0.8
What is the probability that the bulb will **not** grow?

Wycombe Wanderers play a match of football.
The probability that they win the match is 0.28
The probability that they draw the match is 0.55
Work out the probability that they lose the match.

Evelyn has 80 pens in a drawer.
15 pens are black and the other pens are blue.

Evelyn picks a pen at random from the drawer.

- (a) What is the probability that Evelyn picks a black pen?
- (b) What is the probability that Evelyn picks a blue pen?



Probability tables sum to 1

Examples

Complete the probability tables.

1. Shown is the probability of each student being chosen to answer a question

Name	Josh	Brian	Matt	Luke	David
Probability	0.32	0.1	0.21	0.06	0.31

= 1

① Add: $0.32 + 0.1 + 0.21 + 0.31 = 0.94$

② Subtract: $1 - 0.94 = 0.06$

2. The probability of choosing Biology is twice the probability of choosing History.

Favourite subject	Maths	Biology	History	Art
probability	0.72	$2x$ 0.12	x 0.06	0.1

= 1

$$0.72 + 2x + x + 0.1 = 1$$

$$3x + 0.82 = 1$$

$$\frac{3x}{3} = \frac{0.18}{3}$$

$$x = 0.06$$

3. Shown is the probability of selecting a coloured pen from a pot containing blue, red, pink and green pens.

Colour of pens	blue	red	Pink	green
probability	$\frac{1}{8}$	$\frac{1}{2}$ $\frac{4}{8}$	$\frac{1}{4}$ $\frac{2}{8}$	? $\frac{1}{8}$

= 1

Add:

① $\frac{1}{8} + \frac{4}{8} + \frac{2}{8} = \frac{7}{8}$

② Subtract: $1 - \frac{7}{8} = \frac{8}{8} - \frac{7}{8} = \frac{1}{8}$

Questions

Colour of pens	blue	red	pink	green
probability	0.5	0.3	0.1	?

Colour of pens	blue	red	pink	green
probability	0.12	0.43	0.22	x

Probability of History and Art are the same

Favourite subject	Maths	Biology	History	Art
probability	0.36	0.24		

The probability of History is twice the probability of Biology

Favourite subject	Maths	Biology	History	Art
probability	0.63			0.1

Colour of pens	blue	red	pink	green
probability	$\frac{2}{7}$	$\frac{3}{14}$	$\frac{5}{14}$	?

Colour of pens	blue	red	Pink	green
probability	$\frac{1}{3}$	$\frac{1}{18}$	$\frac{1}{9}$	?

A bag contains some sweets.

The flavours of the sweets are either strawberry or chocolate or mint or orange.
Sarah is going to take one sweet at random from the bag.

The table shows the probability that Sarah will take a strawberry sweet or a mint sweet or an orange sweet.

Flavour	Strawberry	Chocolate	Mint	Orange
Probability	0.32		0.17	0.2

Work out the probability that Sarah will take a chocolate sweet.

Mia spins a spinner.

The spinner can land on red or green or blue or pink.

The table shows each of the probabilities that the spinner will land on red or green or blue.

Colour	Red	Green	Blue	Pink
Probability	0.4	0.1	0.2	

Work out the probability that the spinner will land on pink.

Four teams, City, Rovers, Town and United play a competition to win a cup. Only one team can win the cup.

The table below shows the probabilities of City or Rovers or Town winning the cup.

City	Rovers	Town	United
0.38	0.27	0.15	x

Work out the value of x .

Expected outcome = probability $\times$ number of trials

Example

A biased coin is tossed 300 times.

The probability of it landing on heads is 0.2.

a) How many times would you **expect** to get a head?

$$0.2 \times 300 = 60$$

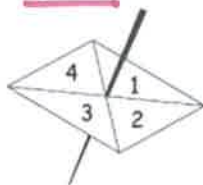
b) How many times would you **expect** to get a tail?

$$\rightarrow P(T) = 1 - 0.2 = 0.8$$

$$0.8 \times 300 = 240$$

Example

Here is a four-sided spinner. The spinner is **biased**.



This means that there is not an equal chance of landing on each number.

The table shows the probabilities that the spinner will land on 1 or on 3

Number	1	2	3	4
Probability	0.2	x	0.1	x

$$0.35$$

$$0.35$$

$$= 1$$

The probability that the spinner will land on 2 is the same as the probability that the spinner will land on 4

$$0.2 + x + 0.1 + x = 1$$

$$2x + 0.3 = 1$$

$$2x = 0.7$$

$$x = 0.35$$

(a) Work out the probability that the spinner will land on 4

$$0.35$$

Shunya is going to spin the spinner 200 times.

(b) Work out an estimate for the number of times the spinner will land on 3

$$0.1 \times 200 = 20$$

Questions

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

(a) Work out the probability that the counter taken is yellow

There are 40 counters in the bag.

(b) Work out the number of blue counters in the bag.

The probability that a biased dice will land on a five is 0.3

Megan is going to roll the dice 400 times.

Work out an estimate for the number of times the dice will land on a five.

Jack sows 300 wildflower seeds.

The probability of a seed flowering is 0.7

Work out an estimate for the number of these seeds that will flower.

A bag contains only red, green and blue counters.

The table shows the probability that a counter chosen at random from the bag will be red or will be green.

Colour	Red	Green	Blue
Probability	0.5	0.3	

Mary takes a counter at random from the bag.

(a) Work out the probability that Mary takes a blue counter.

.....

The bag contains 50 counters.

(b) Work out how many green counters there are in the bag.

.....

Marco has a 4-sided spinner.
 The sides of the spinner are numbered 1, 2, 3 and 4
 The spinner is biased.



The table shows the probability that the spinner will land on each of the numbers 1, 2 and 3

Number	1	2	3	4
Probability	0.20	0.35	0.20	

(a) Work out the probability that the spinner will land on the number 4

.....

Marco spins the spinner 100 times.

(b) Work out an estimate for the number of times the spinner will land on the number 2

There are only red counters, blue counters, white counters and black counters in a bag.
 The table shows the probability that a counter taken at random from the bag will be red or blue.

Colour	red	blue	white	black
Probability	0.2	0.5		

The number of white counters in the bag is the same as the number of black counters in the bag.
 Tania takes at random a counter from the bag.

(a) Work out the probability that Tania takes a white counter.

.....

There are 240 counters in the bag.

(b) Work out the number of red counters in the bag.

.....

Relative frequency

Example

Here are the results after 250 spins of a coin.

Heads	128
Tails	122

The coin is spun an extra 50 times.

After all 300 spins, the relative frequency of Heads is 0.49

For the **extra 50 spins**, work out number of Heads : number of Tails

Heads

New Relative frequency = 0.49

$300 \times 0.49 = 147$ ← after 300 spins

128 ← after 250 spins

$147 - 128 = 19$ ← extra 50 spins

Head = 19
Tails = $50 - 19 = 31$

Example

Today, she made 120 calls.

The table shows the results.

Result of call	Frequency
Not answered	33
Answered but sale not made	81
Answered and sale made	6

- (a) Write down the relative frequency that a call was **not answered**.

Answer _____

(1)

- (b) During the **rest of the week**, Rosle will make 500 calls.

Using the results in the table, how many sales does she expect to make during the **rest of the week**?

a) Relative frequency = $\frac{F}{D}$

$\frac{33}{33+81+6} = \frac{33}{120}$

b) Relative of sales made = $\frac{6}{120}$

"Probability X Number of trials"

$\frac{6}{120} \times 500 = 25$

Example

Adam and Bianca each throw the same biased coin.
Here is some information about their throws.

	Number of throws	Number of Heads
Adam	40	14
Bianca	60	20

Bianca says,

"My results give a better estimate of the probability of Heads than Adam's results."

Is she correct?

Tick a box.

Yes

☒

No

☐

Give a reason for your answer.

More trials = More reliable result

$$\text{Adam} = \frac{14}{40}$$

$$\text{Bianca} = \frac{20}{60} \leftarrow \text{more trials} = \text{better estimate.}$$

Yes, she is correct.

Questions

A coin is thrown 50 times.
The coin lands on heads 30 times.

Write down the relative frequency of landing on heads.

Total number of trains	50	100	150	200	250
Total number of late trains	16	21	36	38	55
Relative frequency of late trains	0.32	0.21			

(a) Complete the relative frequency graph.

A spinner can land on red, blue or green.

After 350 spins

relative frequency of red = 0.18

relative frequency of blue = 0.62

Work out the number of times the spinner landed on green.

A biased coin is thrown 250 times.

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

0.3

0.32

0.342

0.4

Circle the relative frequency that represents 13 successes out of 50 trials.

0.13

26

13 : 50

0.26

On three days, Ali throws darts at a target.

Here are his results.

	Number of throws	Number of hits	Number of misses
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Total	90	54	36

- (a) Work out **two** different estimates for the probability of Ali hitting the target.

Two way tables

Example

Pupils in a school can either choose to have a hot dinner or packed lunch. The table below shows some information about the choices of the Year 4 pupils.

	Hot Dinner	Packed Lunch	Total
Boys	12	a) 14	26
Girls	b) 15	9	24
Total	27	23	50

- a) One of the pupils is chosen at random. What is the probability that they have a packed lunch and are a boy? Give your answer as a decimal

↓
14 boys
have
packed
lunch

$$\frac{14}{50} = 0.28.$$

- b) One of the pupils that chose a hot dinner is chosen at random. What is the probability that they are a girl?

↓
15 girls

↓
only selecting from
hot dinners which is 27
pupils

$$\frac{15}{27}$$

Example

The table below shows some information about the number of employed and unemployed people in a village.

	Employed	Unemployed	Total
Male	156	40	196
Female	98	32	130
Total	254	72	326

a) Complete the two way table.

b) Find the probability that a person chosen at random is unemployed.

→ 72 in total

$$\frac{72}{326}$$

Questions

The two-way table gives some information about how 100 children travelled to school one day.

	Walk	Car	Other	Total
Boy	15		14	54
Girl		8	16	
Total	37			100

(a) Complete the two-way table.

One of the children is picked at random.

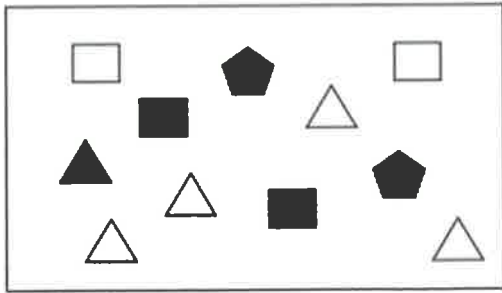
(b) Write down the probability that this child walked to school that day.

.....

One of the girls is picked at random.

(c) Work out the probability that this girl did **not** walk to school that day.

.....



The diagram shows some 3-sided, 4-sided and 5-sided shapes.

The shapes are black or white.

(a) Complete the two-way table. (3)

	Black	White	Total
3-sided shape		4	5
4-sided shape	2		
5-sided shape		0	
Total			11

Ed takes a shape at random.

(b) Write down the probability the shape is white and 3-sided.

.....

The two way table gives information about how 100 students travelled to school.

	Walk	Car	Other	Total
Boys	15			52
Girls		22	8	
Total			19	100

(a) Complete the two way table.

One of the students is picked at random.

(b) Write down the probability they walk to school.

The two way table gives information about the favourite subject of 200 students.

	Maths	English	Other	Total
Year 10			26	98
Year 11	47			
Total	88	41		

(a) Complete the two way table.

One of the students is picked at random.

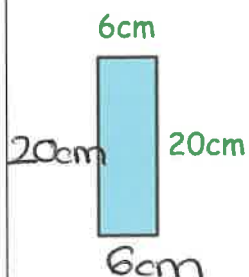
(b) Write down the probability they are a year 10 student whose favourite subject is maths.

Topic 17 – Perimeter and area of shapes

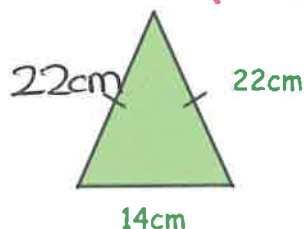
Perimeter is the total length around the shape

Examples

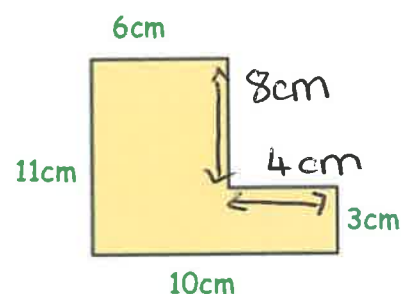
Find the perimeter of each shape



$$P = 6 + 6 + 20 + 20 = 52\text{cm}$$



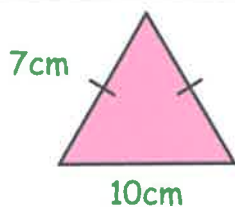
$$P = 22 + 22 + 14 = 58\text{cm}$$



$$P = 6 + 8 + 4 + 3 + 10 + 11 = 42\text{cm}$$

** Fill in all missing lengths first **

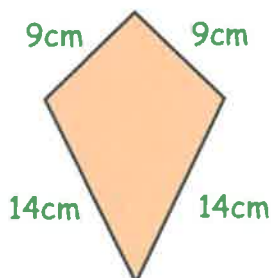
Questions – Find the perimeter of each shape

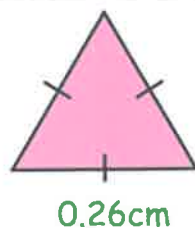




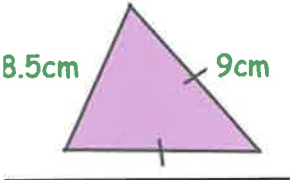
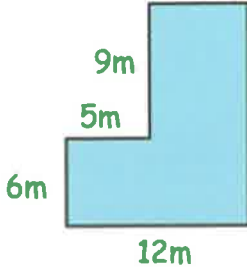
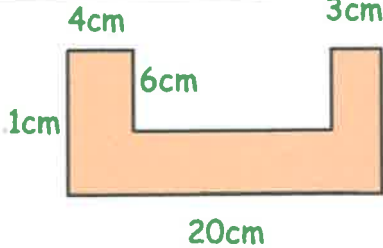
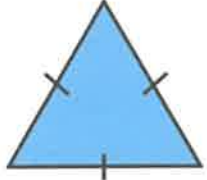
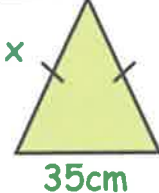


Square







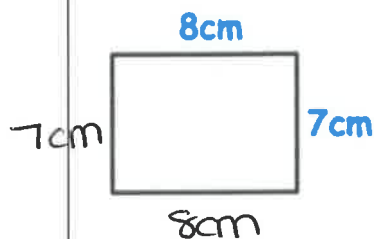
		
 Perimeter = 36cm	 Perimeter = 79cm	 Perimeter = 80cm

Area is the space inside a shape

Examples

Square/Rectangle

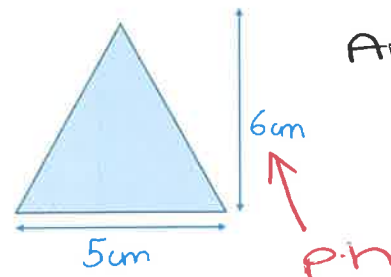
$$\text{Area} = \text{base} \times \text{height}$$



$$\text{Area} = 8 \times 7 = 56\text{cm}^2$$

Triangle

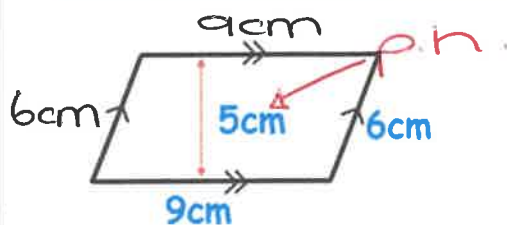
$$\text{Area} = \frac{\text{base} \times \text{perpendicular height}}{2}$$



$$\text{Area} = \frac{5 \times 6}{2} = 15\text{cm}^2$$

Parallelogram

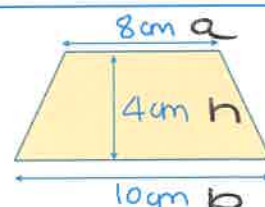
$$\text{Area} = \text{base} \times \text{perpendicular height}$$



$$\text{Area} = 9 \times 5 = 45\text{cm}^2$$

Trapezium

$$\text{Area} = \frac{(a+b) \times \text{height}}{2}$$

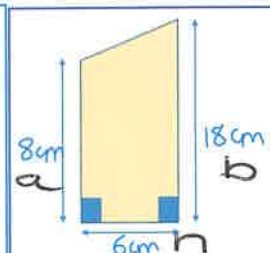


$$\text{Area} = \frac{a+b}{2} \times h$$

$$\text{Area} = \frac{10+8}{2} \times 4$$

$$\frac{18}{2} \times 4$$

$$9 \times 4 = 36\text{cm}^2$$



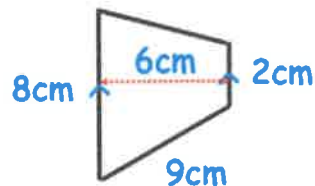
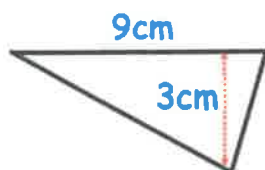
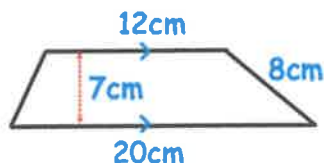
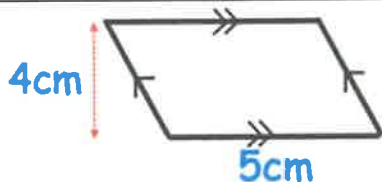
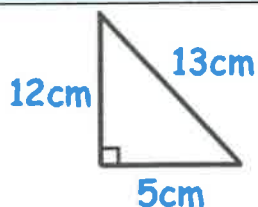
$$\text{Area} = \frac{a+b}{2} \times h$$

$$\text{Area} = \frac{18+8}{2} \times 6$$

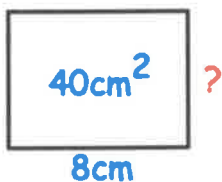
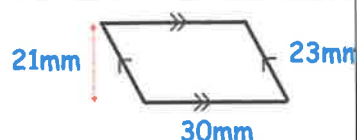
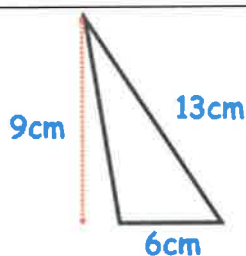
$$\frac{26}{2} \times 6$$

$$13 \times 6 = 78\text{cm}^2$$

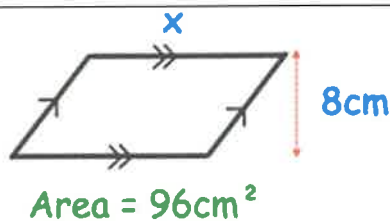
Questions – Find the area of each shape



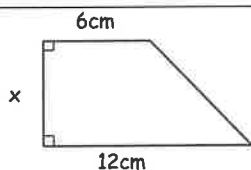
Square



A rectangle has an area of 80cm^2 and a perimeter of 4. Find the length and width of the rectangle.



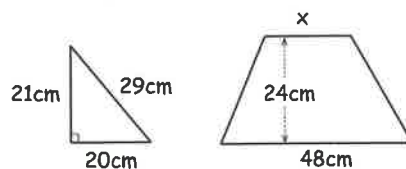
The area of the triangle is 20cm^2 , find x .



The area of the trapezium is 63cm^2 .

Work out the value of x .

Shown below is a triangle and a trapezium.



The area of the trapezium is four times the area of the triangle.

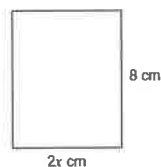
Find x .

The length of a rectangle is five times the width.
The area of the rectangle is 1620cm^2

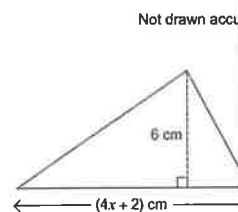


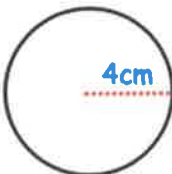
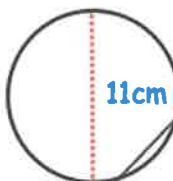
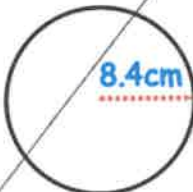
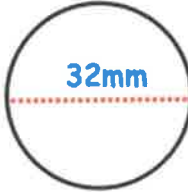
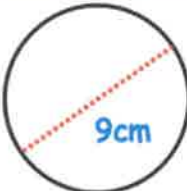
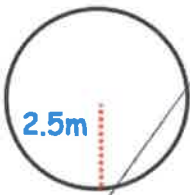
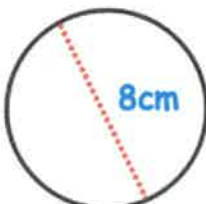
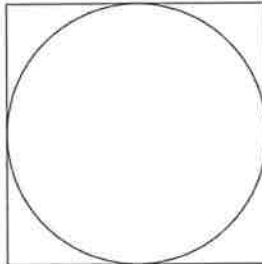
Work out the width of the rectangle.

The area of the rectangle and the area of the triangle are equal.



Work out the value of x .



Circles – Area and circumference			
r = radius		d = diameter	
The circumference of a circle is the distance around the outside (perimeter)			
Area = $\pi \times r^2$		Circumference = $\pi \times d$	
Examples  Every time you see a circle you work out these 4 things!!! $r = 4\text{ cm}$ $d = 8\text{ cm}$		Examples  $r = 11\text{ cm}$ $d =$	
Questions			
			
		A circle with radius 7cm	
A circle has radius 4.2 cm Work out the length of the circumference. Give your answer to 1 decimal place.		Here is a circle touching a square.  The area of the square is 64 cm^2 Work out the area of the circle. Give your answer in terms of π .	

Circles – Area and circumference

r = radius

d = diameter

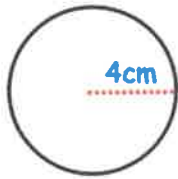
The circumference of a circle is the distance around the outside (perimeter)

Area = $\pi \times r^2$

Circumference = $\pi \times d$

Examples

Every time you see a circle
you work out these 4 things!!!



$r = 4\text{cm}$

$d = 8\text{cm}$

Area = $\pi \times r^2$
 $= \pi \times 4^2$
 $= 16\pi \text{ cm}^2$

Circumference =
 $\pi \times d$
 $= \pi \times 8$
 $= 8\pi \text{ cm}$

left in
terms
of π

Examples

$r = 5.5\text{cm}$

$d = 11\text{cm}$

Area = $\pi \times r^2$
 $= \pi \times 5.5^2$
 $= 95.0\text{cm}^2$

circumference = $\pi \times d$
 $= \pi \times 11$
 $= 11\pi \text{ cm}$
 $= 34.5\text{cm}$

Questions

10m

8.4cm

32mm

9cm

2.5m

8cm

A circle with radius 7cm

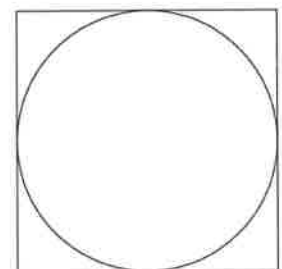
A circle with diameter 10cm

A circle has radius 4.2 cm

Work out the length of the circumference.

Give your answer to 1 decimal place.

Here is a circle touching a square.



The area of the square is 64 cm^2

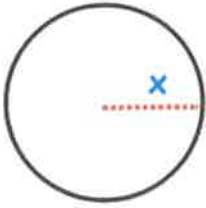
Work out the area of the circle.

Give your answer in terms of π .

Example

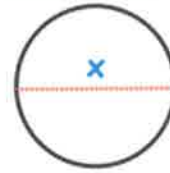
Find the radius

$$\text{Area} = 20\text{cm}^2$$

**Example**

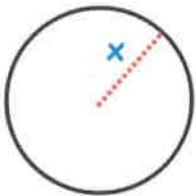
Find the diameter

$$\text{Circumference} = 12\text{cm}$$

**Questions**

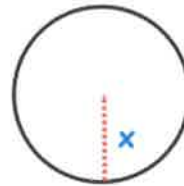
Find the radius

$$\text{Area} = 65\text{cm}^2$$



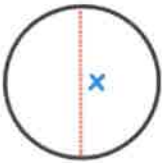
Find the radius

$$\text{Area} = 100\text{cm}^2$$



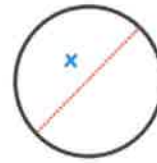
Find the diameter

$$\text{Circumference} = 50\text{cm}$$

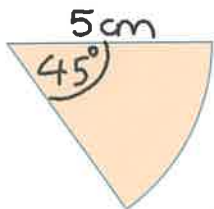


Find the diameter

$$\text{Circumference} = 3\text{m}$$



$$\text{Area of part circles} = \pi \times r^2 \times \frac{\text{angle}}{360}$$

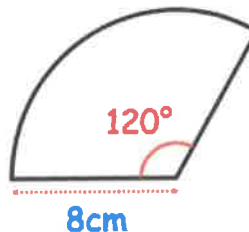
Example - Area

$$\frac{45}{360} = \frac{1}{8}$$

$$\begin{aligned} \text{Area} &= \pi \times 5^2 \times \frac{45}{360} \\ &= \pi \times 5^2 \times \frac{1}{8} \\ &= \pi \times 25 \times \frac{1}{8} \end{aligned}$$

$$\text{Area} = \frac{25\pi}{8} \text{ cm}^2$$

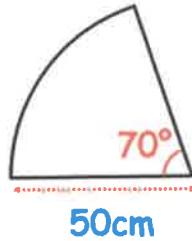
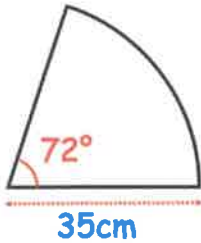
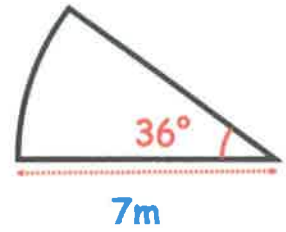
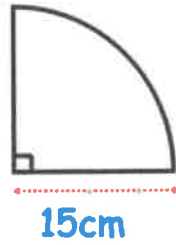
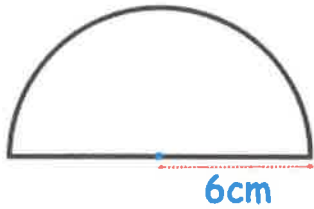
in term of π

Example - Area

$$\frac{120}{360} = \frac{1}{3}$$

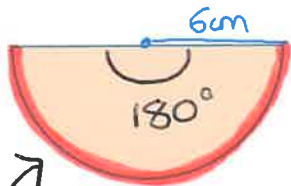
$$\begin{aligned} \text{Area} &= \pi \times 8^2 \times \frac{120}{360} \\ &= \pi \times 8^2 \times \frac{1}{3} \\ &= \pi \times 64 \times \frac{1}{3} \\ &= 67\text{cm}^2. \end{aligned}$$

Questions – Find the area



$$\text{Arc length} = \pi \times d \times \frac{\text{angle}}{360}$$

Example – Arc length

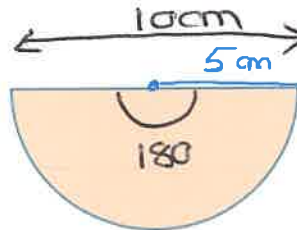


$$r = 6\text{cm}$$

$$d = 12\text{cm}$$

$$\begin{aligned} \text{Arc length} &= \pi \times 12 \times \frac{180}{360} \\ &= 6\pi \text{ cm} \\ &= 18.8\text{cm} \end{aligned}$$

Example – Perimeter



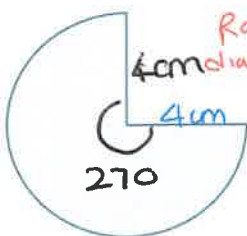
$$r = 5\text{cm}$$

$$d = 10\text{cm}$$

$$\begin{aligned} \text{Arc length} &= \pi \times 10 \times \frac{180}{360} \\ &= 5\pi \end{aligned}$$

$$\text{Perimeter} = 5\pi + 10 \text{ cm}$$

Example



$$\text{Radius} = 4\text{cm}$$

$$\text{Diameter} = 8\text{cm}$$

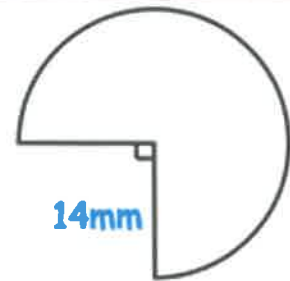
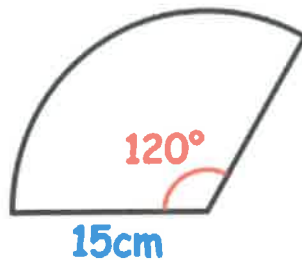
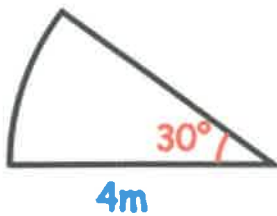
$$\frac{270}{360} = \frac{3}{4}$$

$$\text{Perimeter} = \text{Arc length} + \text{radius} + \text{radius}$$

$$\begin{aligned} \text{arc length} &= 6\pi \\ &= \frac{24\pi}{4} + 4 + 4 \end{aligned}$$

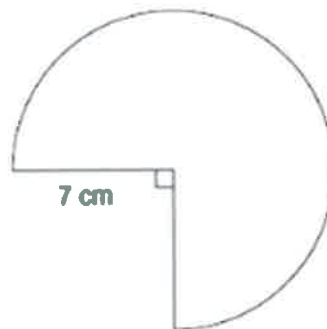
$$\underline{\underline{6\pi + 8 = \text{Perimeter}}}$$

Questions



The diagram shows a sector of a circle.
The radius is 7 cm

Not drawn accurately



Work out the perimeter of the shape.

Work out the arc length, in metres, of a semicircle of radius 6 metres.

Circle your answer.

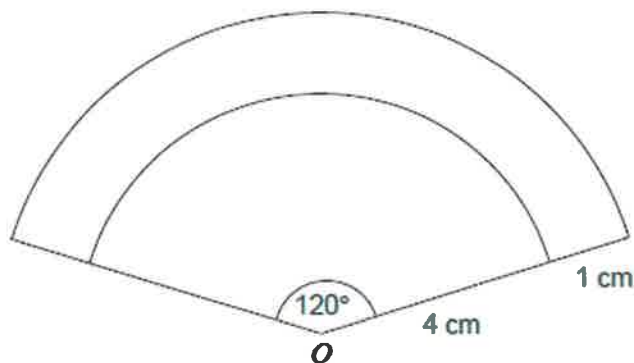
3π

6π

12π

18π

The diagram shows two circular arcs with centre O .



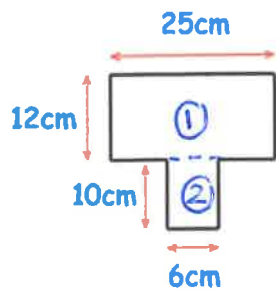
How much longer is the big arc than the small arc?
Give your answer to 1 decimal place.

Compound shapes are two or more shapes that have been joined together.

To find the area we need to 'split' them into shapes that we know.

Examples

Find the area



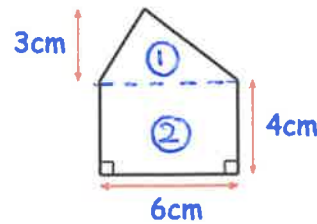
① 12×25
 $A = 12 \times 25$
 $= 300$

② 6×10
 $A = 6 \times 10$
 $= 60$

Total area = $300 + 60$
 $= 360 \text{ cm}^2$

Examples

Find the area

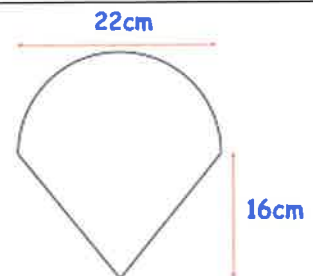
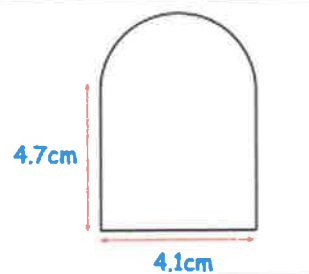
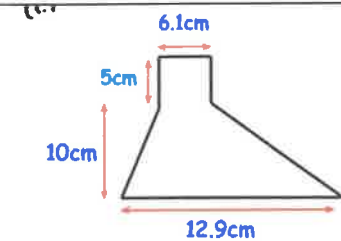
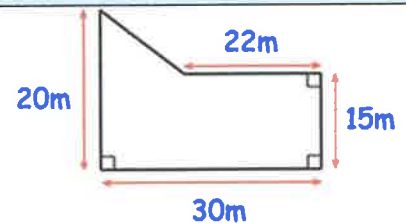
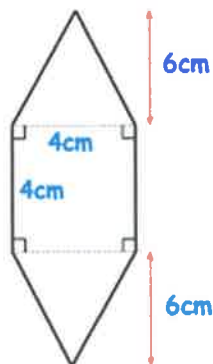
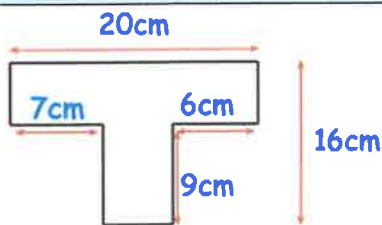


① $\frac{6 \times 3}{2}$
 $A = \frac{6 \times 3}{2} = 9$

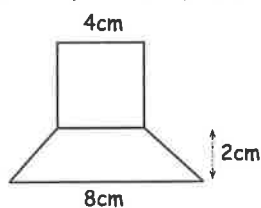
② 6×4
 $A = 6 \times 4 = 24$

Total area = $9 + 24$
 $= 33 \text{ cm}^2$

Questions – Find the area



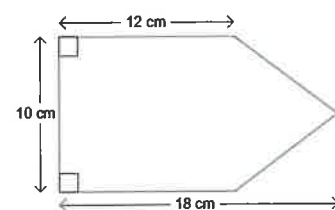
A club logo is made from a square and a trapezium.



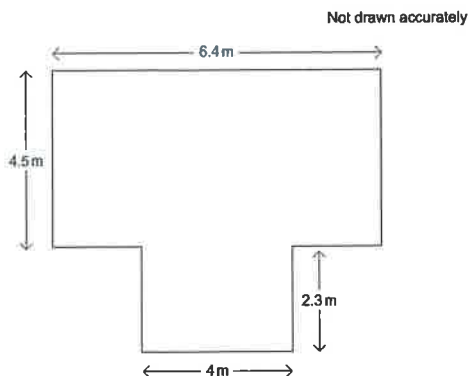
Calculate the area of the logo

Work out the area of this pentagon.

Not drawn accurately



This diagram shows Adam's garden.
It is in the shape of two rectangles joined together.



(a) Work out the area of the garden.

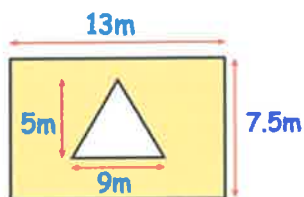
(b) Adam makes a flower bed.
It is a circle of radius 1.7 m.

Work out the area of the flower bed.

Shaded area

Example

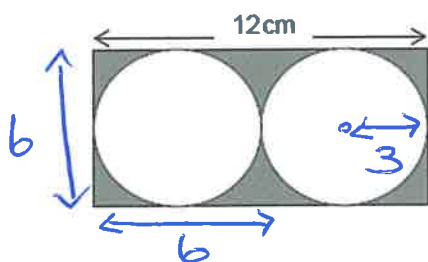
Work out the shaded area



$$\begin{aligned}
 & 13 \times 7.5 - \frac{9 \times 5}{2} = \text{shaded area} \\
 & (13 \times 7.5) - \left(\frac{9 \times 5}{2} \right) \\
 & 97.5 - 22.5 = 75 \text{ cm}^2
 \end{aligned}$$

Example

Two identical circles just fit inside a rectangle as shown.



Not drawn accurately

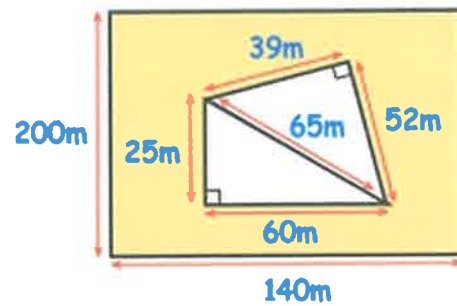
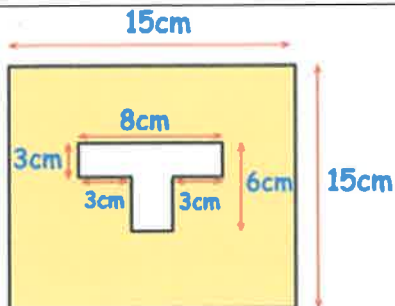
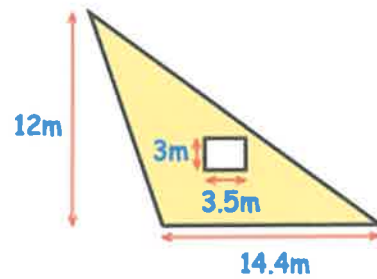
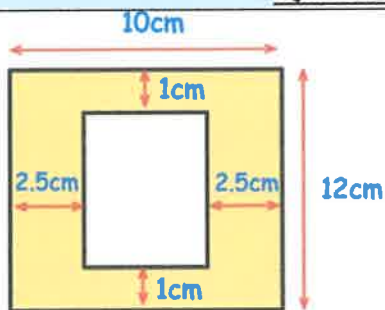
Work out the area of the shaded section.

Give your answer in terms of π

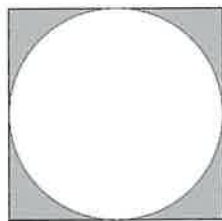
$$\begin{aligned}
 \text{shaded area} &= 6 \times 12 - 2 \times \pi \times 3^2 \\
 &= (12 \times 6) - (2 \times 9\pi) \\
 &= 72 - 18\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= \pi \times r^2 \\
 &= \pi \times 3^2 \\
 &= 9\pi
 \end{aligned}$$

Questions – find the shaded area

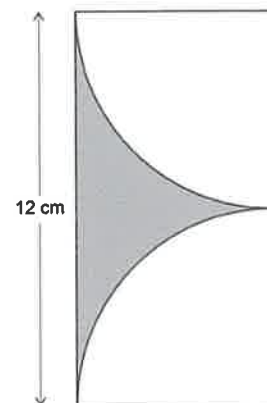


The diagram shows a circle of radius 5 cm inside a square.



Work out the shaded area.

Two identical quarter circles are cut from a rectangle as shown.



Work out the shaded area.

Topic 18 - Distance Time Graphs

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

A distance-time graph can be used to show the distance travelled over a period of time.

The **gradient** of the line shows the **speed**.

Stationary is shown by a **horizontal line**. (ie the speed is 0)

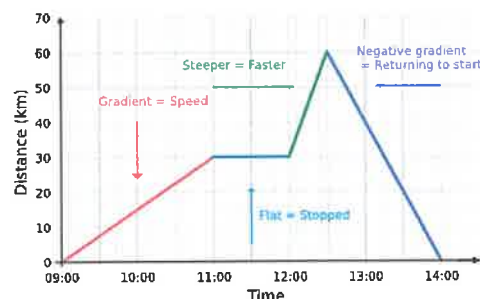
$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

Units of speed:

- mph = miles per hour
- km/h = kilometres per hour
- m/s = meters per second

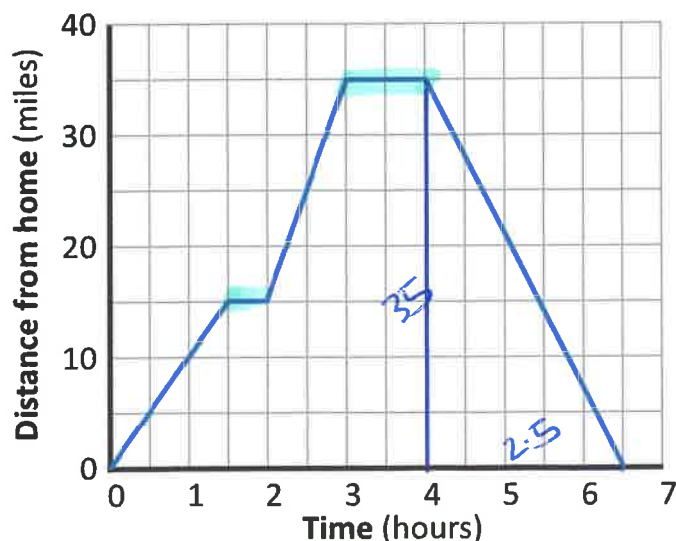
Distance time graphs – Key things to remember:

- 1) The gradient of the line = speed
- 2) A flat section means no speed (stopped)
- 3) The steeper the graph the greater the speed
- 4) Negative gradient = returning to start point (coming back)



Example

The graph shows a cyclist's journey, from home into the city and back.



a) How long in total was the cyclist stationary during his journey?

$$0.5 \text{ h} + 1 \text{ h} = 1.5 \text{ hours}$$

b) How far did the cyclist travel in total?

$$35 + 35 = 70 \text{ miles}$$

c) What was the cyclist's speed on the way home?

$$S = \frac{D}{T} = \frac{35}{2.5} = 14 \text{ mph}$$

Example

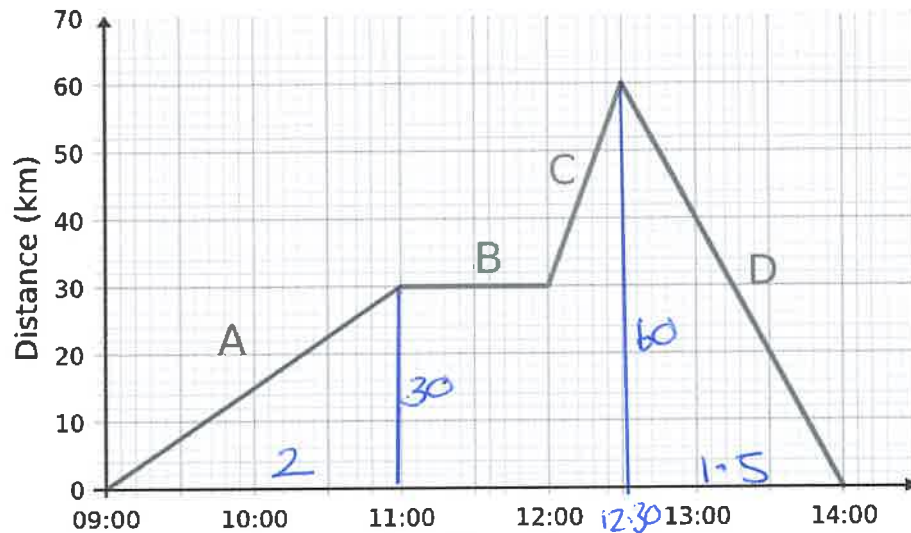
The graph below describes a journey that has several parts to it, each represented by a different straight line.

Part A: 09 : 00 – 11 : 00, the person travelled 30 km away from their starting point and that took them 2 hours.

Part B: 11 : 00 – 12 : 00, we can see that the line is flat, so the distance from their starting point did not change – they were stationary.

Part C: 12 : 00 – 12 : 30, they moved a further 30 km away from their starting point.

Part D: 12 : 30 – 14 : 00, they travelled the full 60 km back to where they began.



a) What was the total distance travelled?

$$60 + 60 = 120 \text{ km}$$

b) What was the speed between 9:00 and 11:00? (A)

$$s = \frac{d}{t} = \frac{30}{2} = 15 \text{ km/h}$$

c) What was the speed between 11:00 and 12:00? (B)

$$\text{Stationary} = 0 \text{ km/h}$$

d) What was the speed between 12:30 and 14:00? (D)

$$s = \frac{d}{t} = \frac{60}{1.5} = 40 \text{ km/h}$$

e) Between which times was the person travelling the fastest? Justify your answer.

12:00 - 12:30 as the line is steeper.

Valentina is going for a bike ride. Below is a distance-time graph that describes her full journey.

a) How long was she stationary for?

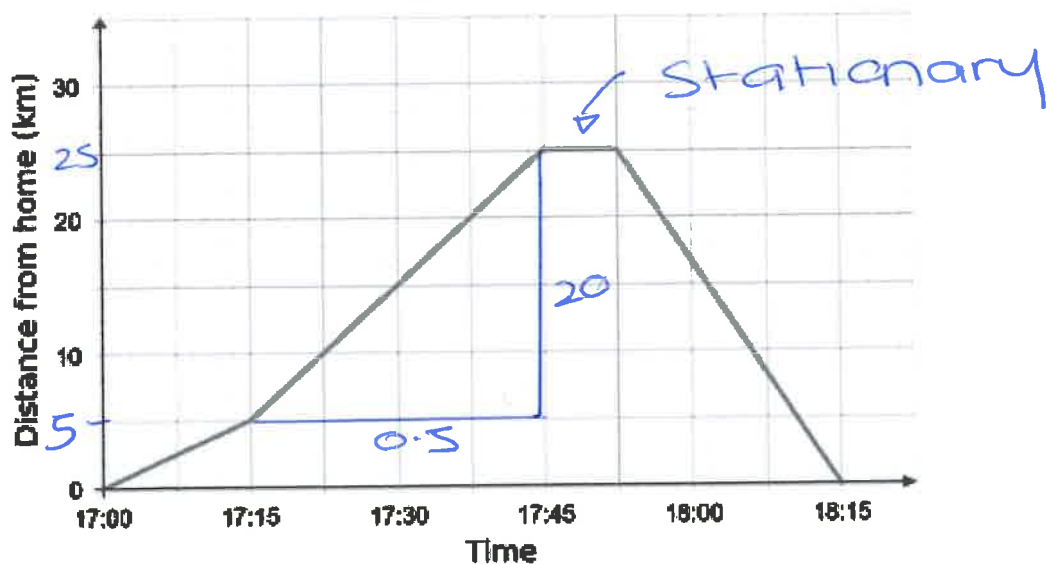
7.5 minutes

b) What was the total distance travelled during her journey?

$25 + 25 = 50$ km

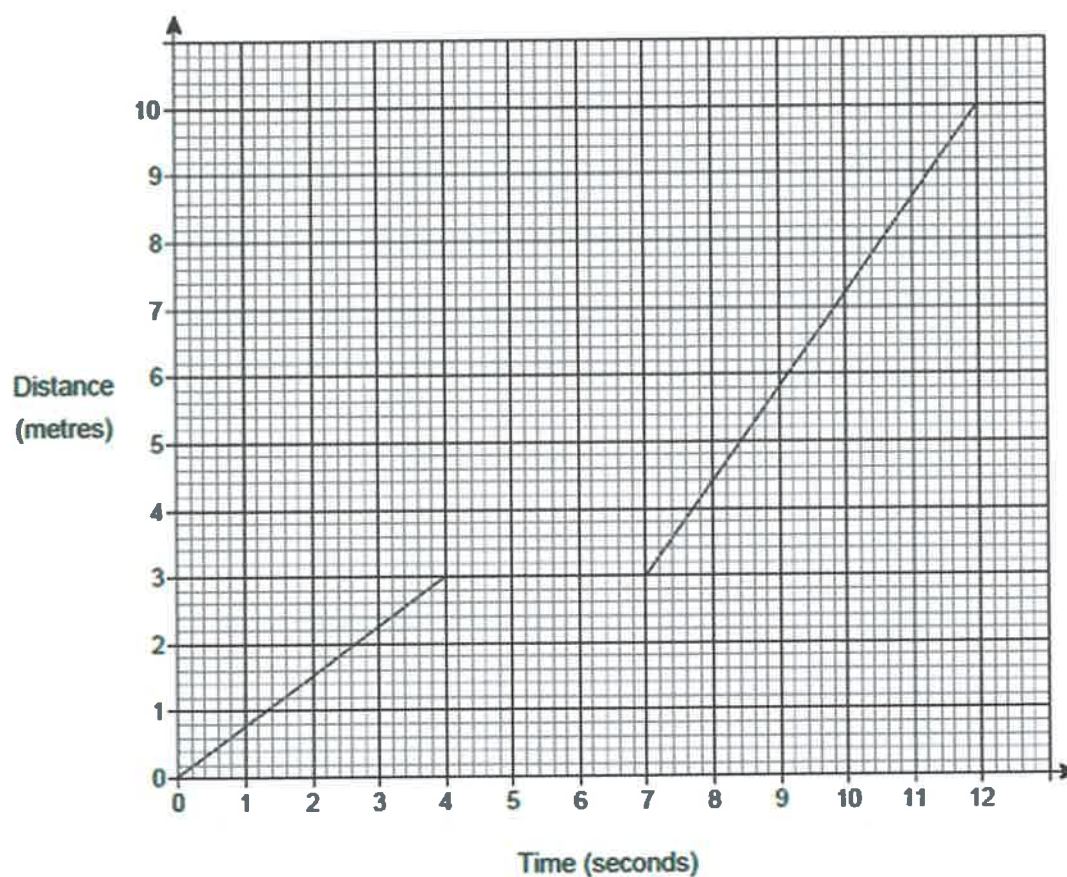
c) What was her average speed in kilometres per hour between 17 : 15 and 17 : 45?

$$S = \frac{d}{t} = \frac{20}{0.5} = 40 \text{ km/h}$$



Questions

The distance-time graph shows the journey of a toy car.



(a) For how long is the toy car stationary?

(1)

(b) Work out the average speed for the last five seconds of the journey.

(2)

a) Here are some descriptions of the first **two** parts of the journey.

Circle the letter of the correct description.

A Accelerates then stationary

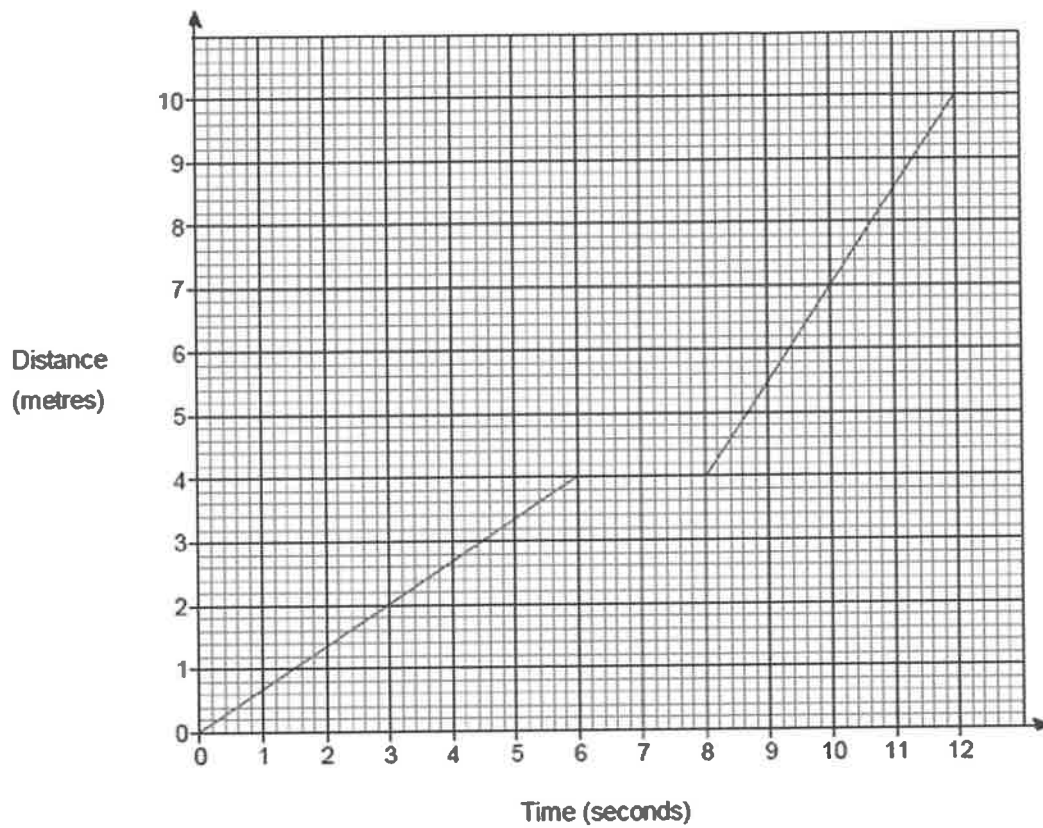
B Moves at constant speed then stationary

C Accelerates then moves at constant speed

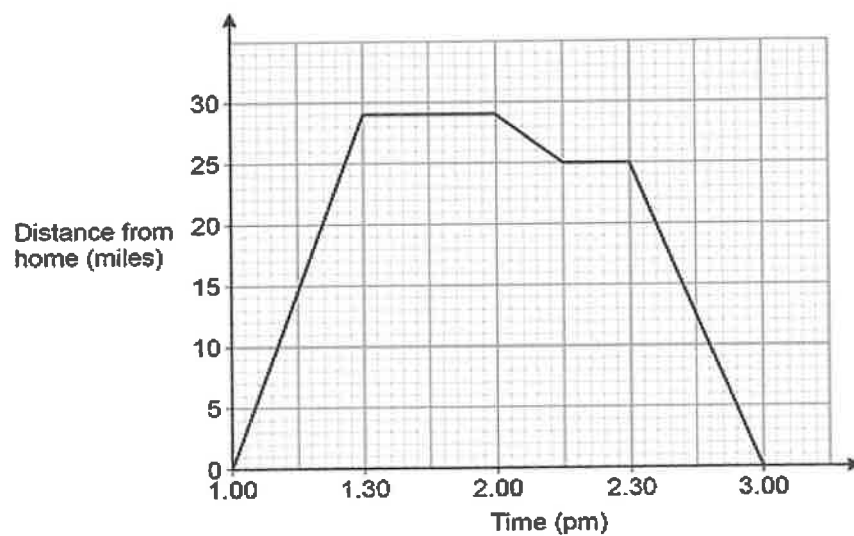
D Stationary then moves at constant speed

b) Work out the average speed for the last four seconds of the

The distance-time graph shows the journey of a toy train.



Here is the distance-time graph for a car between 1 pm and 3 pm



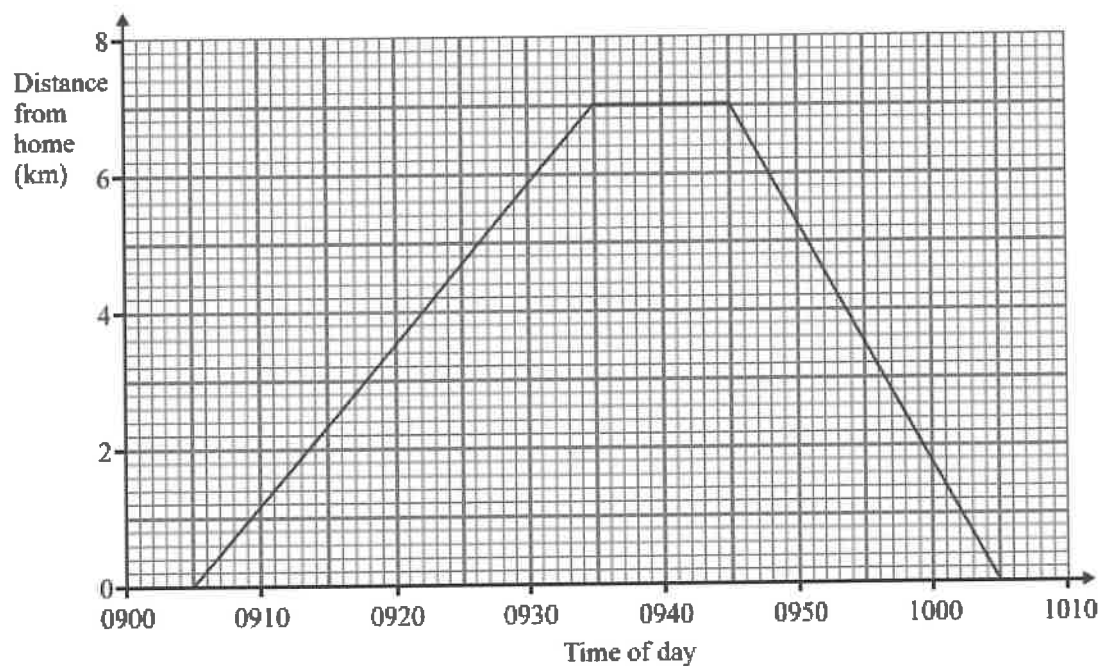
- (a) Work out the **total** time that the car is **not** moving between 1 pm and 3 pm
State the units of your answer.

(2)

- (b) Work out the **total** distance the car travels between 1 pm and 3 pm

(2)

2. Anil cycled from his home to the park.
 Anil waited in the park.
 Then he cycled back home.
 Here is a distance-time graph for Anil's complete journey.



- (a) At what time did Anil leave home?

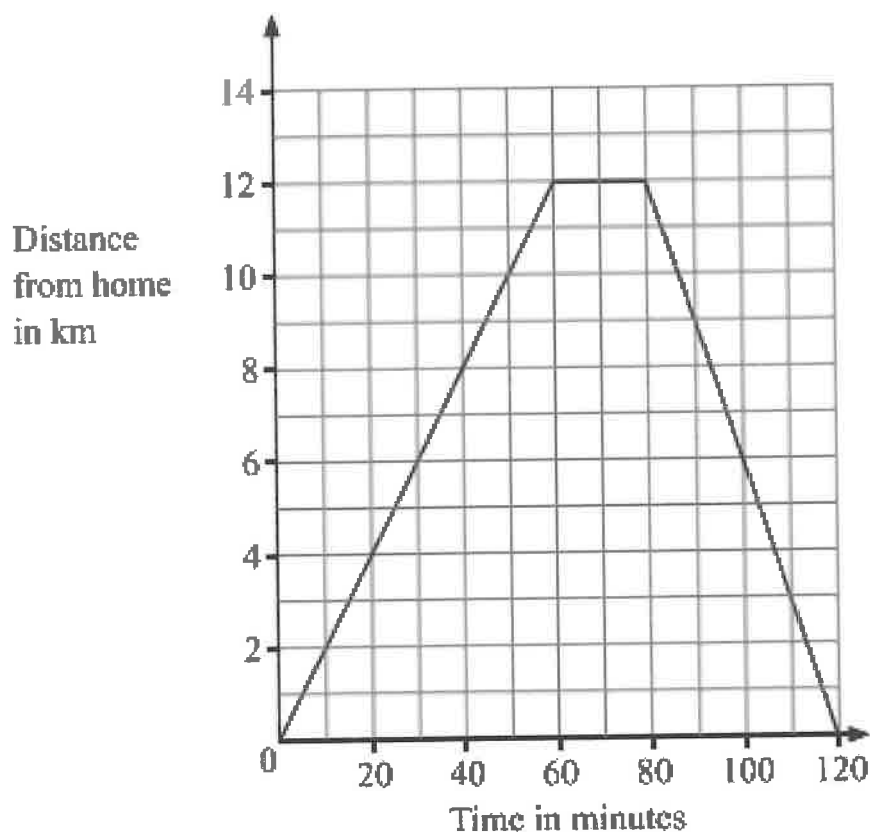
.....

- (b) What is the distance from Anil's home to the park?

..... km

- (c) How many minutes did Anil wait in the park?

4. Margaret went on a cycle ride.
The travel graph shows Margaret's distance from home on this cycle ride.



- (a) How far had Margaret cycled after 30 minutes?

..... km (1)

After 60 minutes, Margaret stopped for a rest.

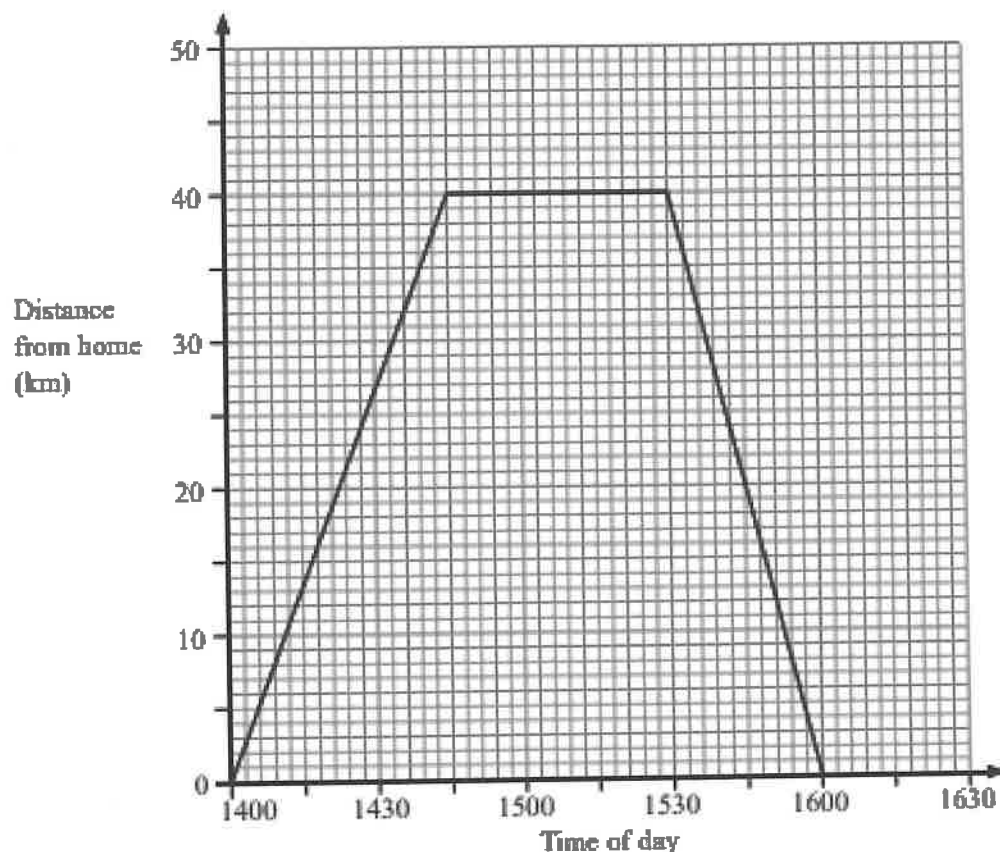
- (b) For how many minutes did she rest?

..... minutes (1)

- (c) How far did Margaret cycle in total on her ride?

..... km (1)
(Total 3 marks)

5. Judy drove from her home to the airport.
She waited at the airport.
Then she drove home.
Here is the distance-time graph for Judy's complete journey.



- (a) What is the distance from Judy's home to the airport?

..... km

(1)

- (b) For how many minutes did Judy wait at the airport?

..... minutes

(1)

- (c) Work out Judy's average speed on her journey home from the airport.
Give your answer in kilometres per hour.

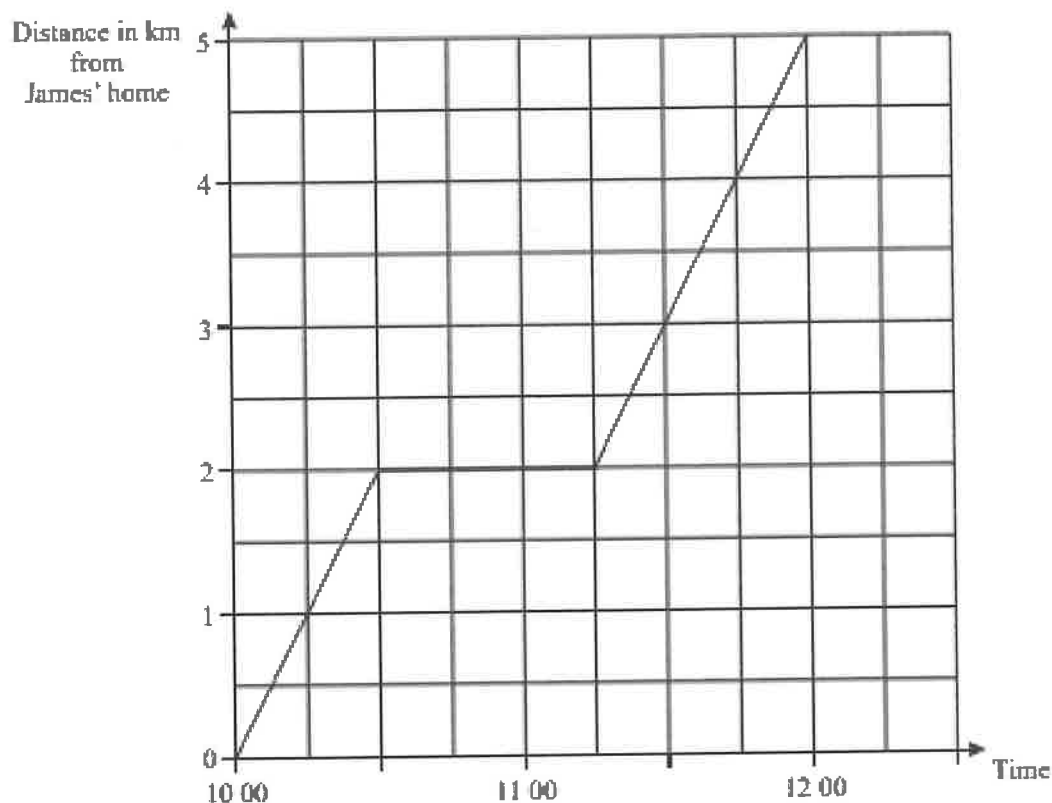
..... kilometres per hour

(2)

(Total 4 marks)

7. James left home at 10 00 am.
He walked to the swimming pool.
On the way to the swimming pool he stopped to talk to a friend.

Here is the distance-time graph for his complete journey.



- (a) For how many minutes did James stop and talk to his friend?

..... minutes

(1)

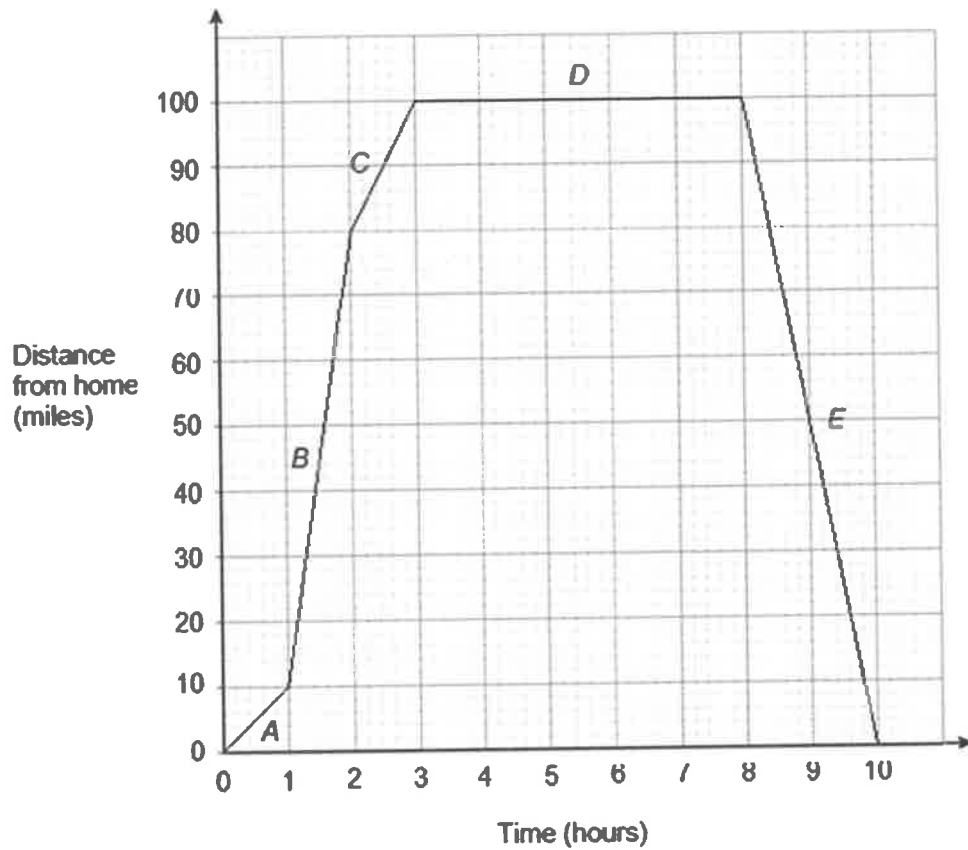
- (b) What is the distance from James' home to the swimming pool?

..... km

(1)

(Total 2 marks)

The distance time graph shows Josh's journey from his house to Canterbury and back.



(a) Match each statement to one section of the journey.

Average speed = 20 miles per hour _____

Average speed = 50 miles per hour _____

Stationary _____

Fastest part of the journey _____

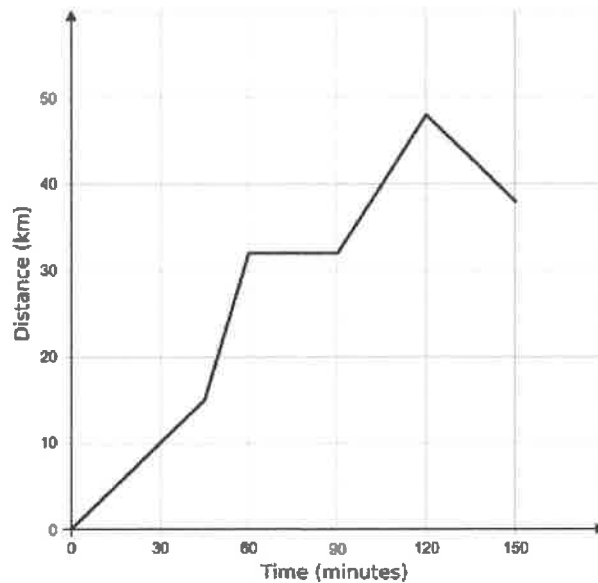
(b) How far did Josh travel altogether?

Question 3: A cyclist on a training ride records the distance she travels away from home. The data only shows the first 150 minutes of the ride before her cycling computer ran out of battery.

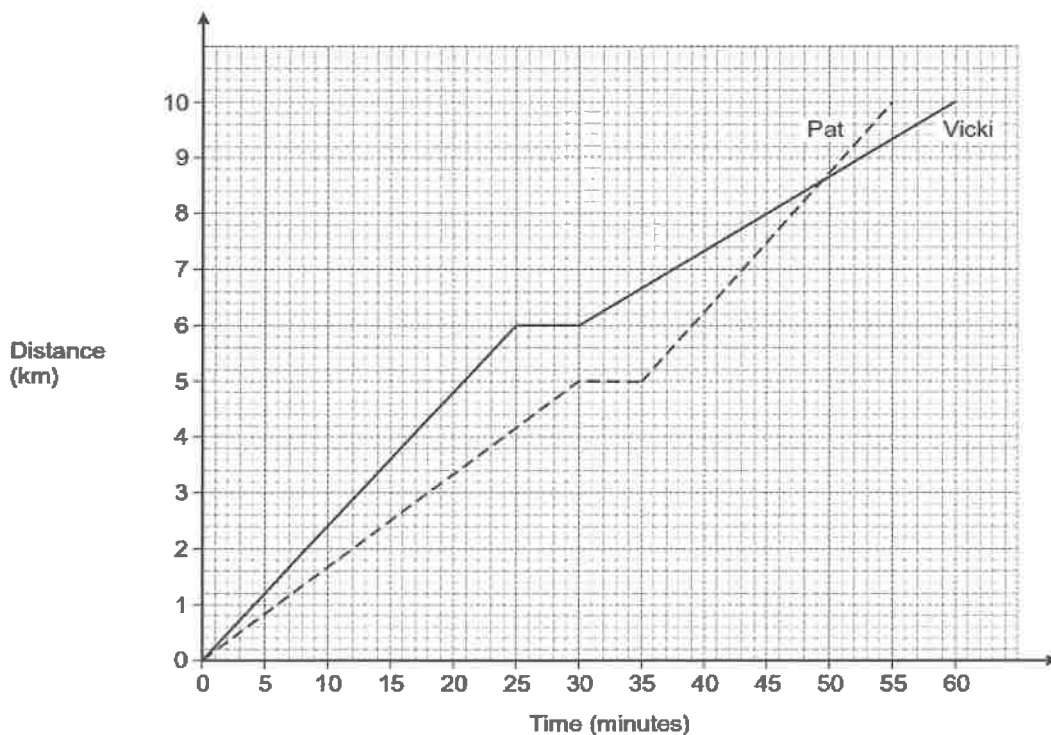
[3 marks]

(a) How far did she ride in the first 150 minutes?

(b) For how long was she stationary?



The graph shows two training runs by Pat and Vicki.



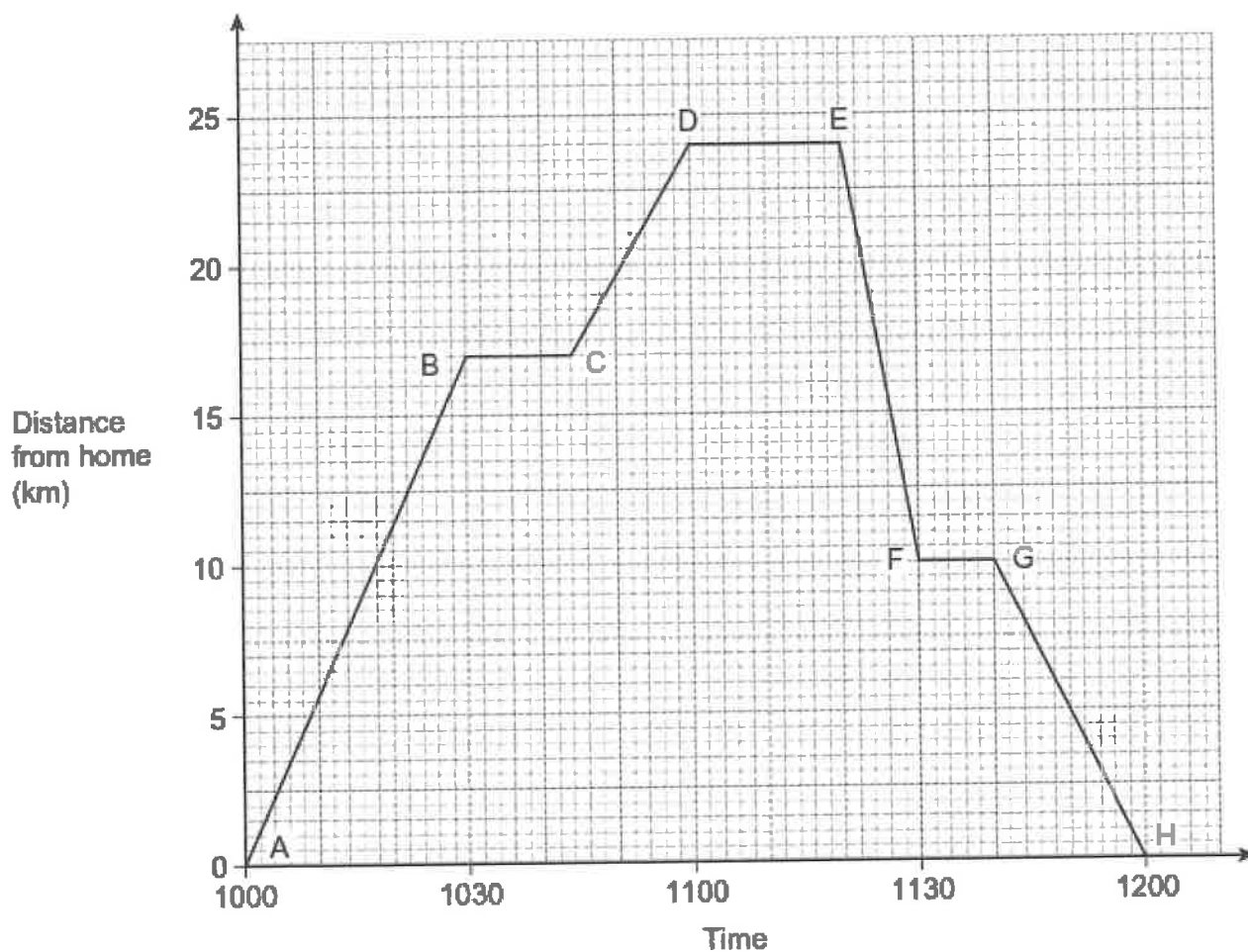
(a) After how many minutes does Pat overtake Vicki?

(1)

(b) How far ahead is Vicki when Pat starts again after her rest?

(2)

Amy leaves home in her car at 1000 and returns at 1200.
The graph shows her journey.

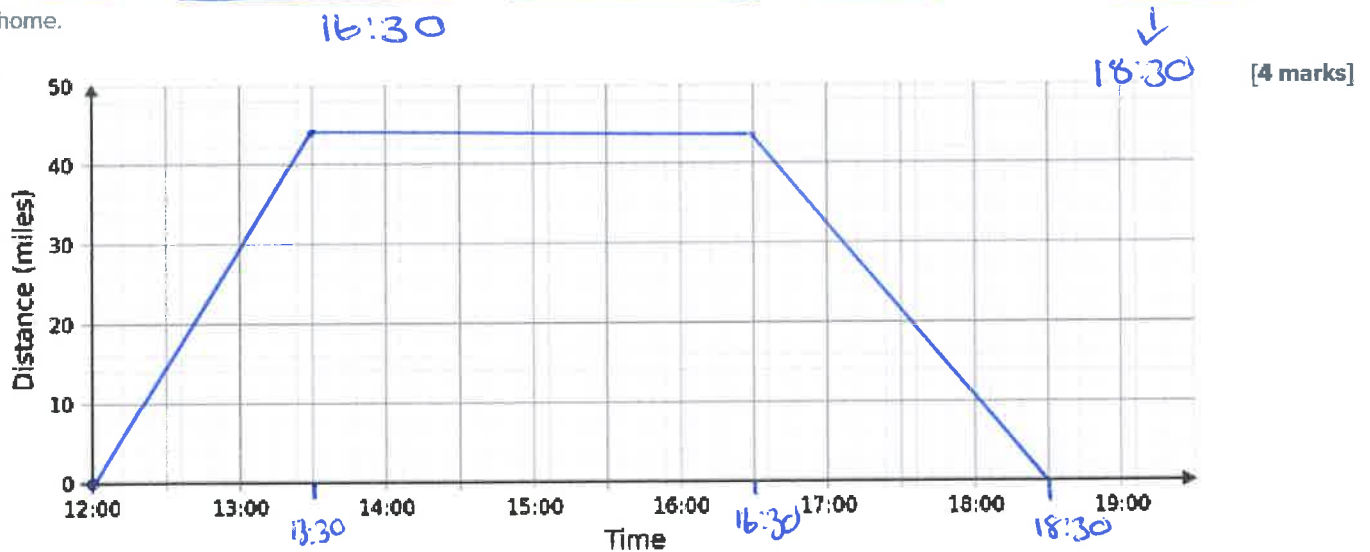


- (a) How far does she travel in her car altogether? (1)
- (b) For how long does the car stop altogether? (2)
- (c) On which part of her journey is she travelling at the fastest speed?
Give a reason for your answer. (2)

Example

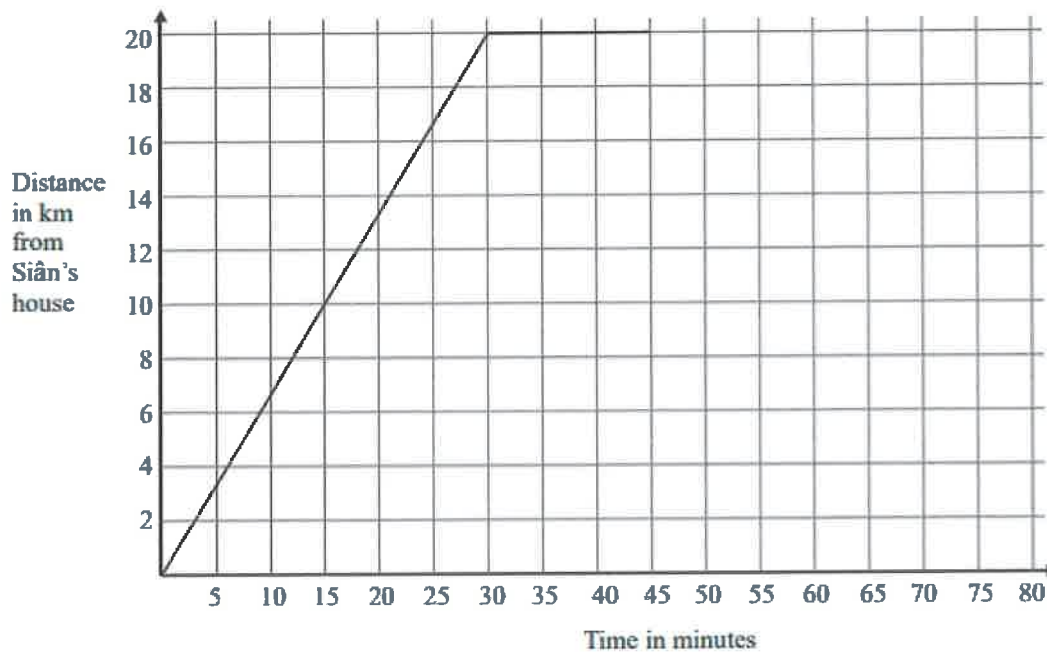
Question 1: From the description of Neil's journey below, construct a distance-time graph.

Neil left home at 12 : 00 and after an hour and a half of moving at a constant speed, he had travelled 44 miles, at which point he stopped. After 3 hours of being stopped, he drove towards home at a constant speed, and it took him 2 hours in total to get home.



Questions

Here is part of a travel graph of Siân's journey from her house to the shops and back.

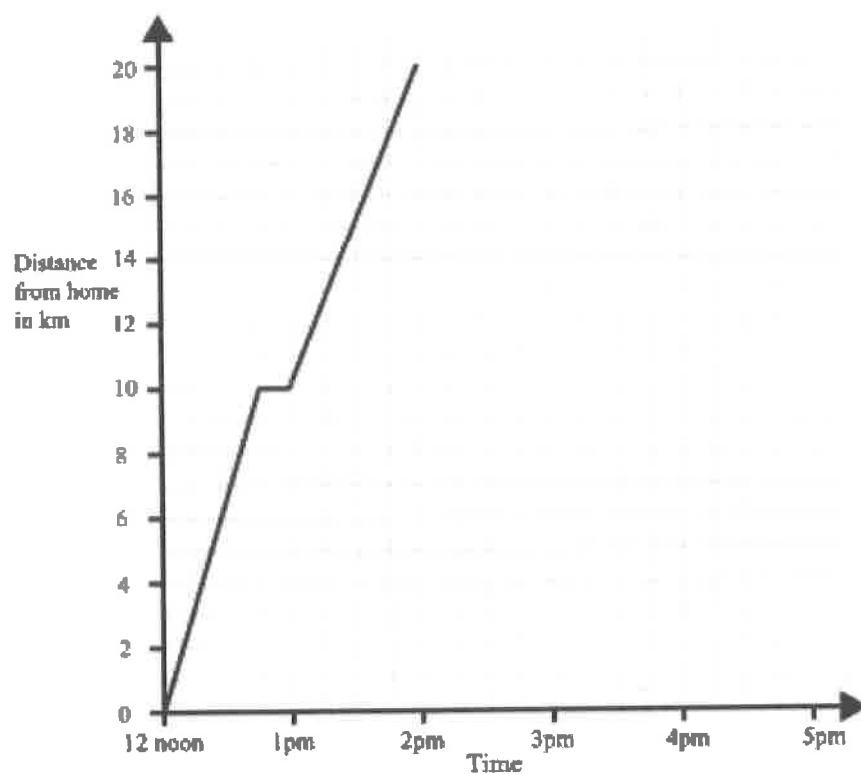


Siân spends 15 minutes at the shops.
She then travels back to her house at 60 km/h.

(b) Complete the travel graph.

(2)

A man left home at 12 noon to go for a cycle ride.
The travel graph represents part of the man's journey.



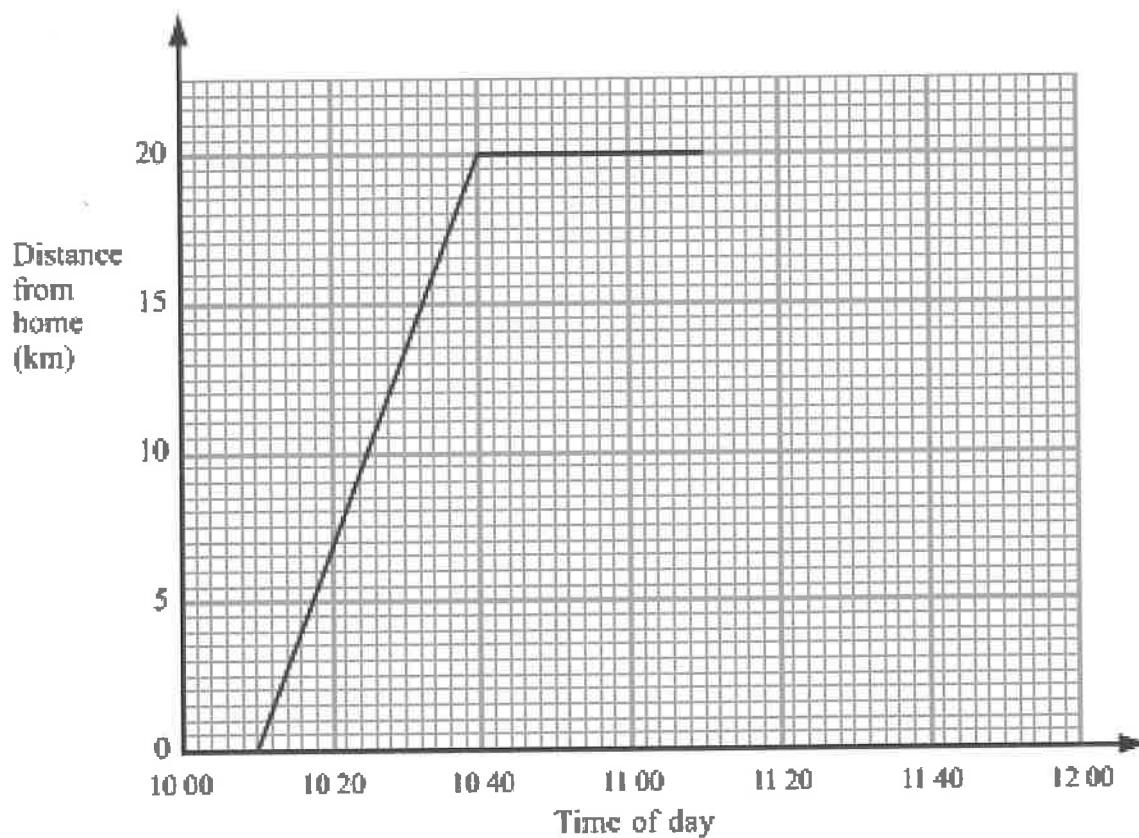
The man stopped for another rest at 2pm.

He rested for one hour.

Then he cycled home at a steady speed. It took him 2 hours.

(c) Complete the travel graph.

Jamie travelled 20 km from his home to his friend's house.
Jamie then spent some time at his friend's house before returning home.
Here is the travel graph for part of Jamie's journey.



Jamie returned home at a steady speed.
He arrived home at 11:50

(d) Complete the travel graph.

(1)

Topic 19 - Compound Measures

60 seconds = 1 minute

60 minutes = 1 hour

Example

Convert 2 hours and 45 minutes into hours

$$\text{Hours} + \frac{\text{Minutes}}{60}$$

$$2 + \frac{45}{60} = 2 + 0.75 = 2.75 \text{ hours}$$

Example

Convert 186 minutes into hours

$$\frac{\text{Minutes}}{60}$$

$$\frac{186}{60} = 3.1 \text{ hours}$$

Questions

Question 4: Convert the times from hours into hours/minutes, without a calculator.

(a) 0.75 hours

(b) 1.25 hours

(c) 5.5 hours

(d) 1.3333... hours

(e) 2.6666... hours

(f) 10.75 hours

Question 2: Convert the times from hours/minutes into hours.
You may use a calculator if needed.

(a) 18 minutes

(b) 54 minutes

(c) 1 hour 3 minutes

(d) 1 hour 36 minutes

(e) 2 hours 48 minutes

(f) 2 hours 33 minutes

(g) 8 hours 51 minutes

(h) 3 hours 21 minutes

(i) 27 minutes

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Units of speed:

- mph = miles per hour
- km/h = kilometres per hour
- m/s = meters per second

Example

A car is travelling at 40mph.

How far will the car travel in 3 hours?

D	T
40 miles	1 hour
x3	x3
120 miles	3 hours
120 miles.	

Example

A car is travelling at 60mph.

How far will the car travel in 1 hour 45 minutes?

D	T
60 miles	1 hour
x1.75	x1.75
105 miles	1.75 hours
105 miles	

1.75 hours

Example

A plane is travelling at 250km per hour for 4 hours. How far does it travel?

D	T
250 km	1 hour
x4	x4
1000 km	4 hours
1000 km	

Example

A person runs at a speed of 9mph for 20 minutes. How far do they run?

D	T
9 miles	60 mins
÷3	÷3
3 miles	20 mins
3 miles	

9 miles every 60 mins

Questions – Calculate the distance travelled

- (a) A car travels at a speed of 50mph for 3 hours.
- (b) A plane flies at a speed of 230 kilometres per hour for 2 hours.
- (c) A lorry drives for 4 hours at a speed of 45 miles per hour.
- (d) A man runs at a speed of 8 metres per second for 15 seconds.

- (a) A car drives at a speed of 60mph for 30 minutes.
- (b) A taxi travels for 30 minutes at a speed of 28 mph.
- (c) A car travels at a speed of 44mph for 15 minutes.
- (d) A lorry drives at a speed of 51mph for 20 minutes.
- (e) An airplane travels at a speed of 441mph for 20 minutes.
- (f) A car drives at a speed of 48mph for 45 minutes.

- (a) A car drives at a speed of 40mph for 1 hour 30 minutes
- (b) A bird flies at a speed of 32 kilometres per hour for 1 hour 30 minutes
- (c) A lorry travels for 2 hours 30 minutes at a speed of 52 mph
- (d) A F1 race car drives for 1 hour 15 minutes at a speed of 124 mph
- (e) A helicopter flies at a speed of 104 mph for 1 hour 45 minutes

- (a) A runner runs at a speed of 8m/s for 2 minutes
- (b) A jog runs at a speed of 4m/s for 10 minutes.
- (c) A car drives at 60mph for 90 seconds.
- (d) A lorry drives at 30 mph for 150 seconds.

Example

A car travels 40 miles in 2 hours.

Work out the average speed.

D	T
40 miles	2 hours
$\div 2$	$\div 2$
20 miles	1 hour
20 mph	

Example

A dog runs 1.2km in 10 minutes.

Work out his average speed.

D	T
1.2 km	10 mins
$\times 6$	$\times 6$
7.2 km	60 mins
7.2 km/h	

Example

A car travels 105 miles in 2 hours 30 minutes.

Work out the average speed.

D	T
105 miles	2.5 hours
$\div 2.5$	$\div 2.5$
42 miles	1 hour
42 mph	

Example

A runner runs 26 miles in 1 hour 18 minutes.

Work out his average speed.

D	T
26 miles	1.3 hours
$\div 1.3$	$\div 1.3$
20 miles	1 hour
20 mph	

Questions – Calculate the average speed

- (a) A car travels 60 miles in 2 hours
- (b) A lorry travels 120 miles in 3 hours
- (c) A cyclist travels 45 miles in 5 hours
- (d) A jogger travels 30km in 4 hours
- (e) A runner runs 100 metres in 10 seconds
- (f) A car travels 195 miles in 3 hours
- (g) A helicopter travels 425 miles in 5 hours
- (h) A helicopter flies 840 miles in 7 hours

- | | |
|---|---|
| (a) A car travels 20 miles in 30 minutes | (b) A lorry travels 32 miles in 30 minutes |
| (c) A bird flies 17 kilometres in 30 minutes | (d) A man jogs 2 kilometres in 15 minutes. |
| (e) A helicopter flies 18 miles in 15 minutes | (f) An F1 car travels 32 miles in 15 minutes. |

- (a) A car travels 63 miles in 1 hour 30 minutes
 (b) A man runs 15 miles in 2 hours 30 minutes
 (c) A helicopter flies 238 miles in 3 hours 30 minutes
 (d) A car travels 85.5 miles 2 hours 15 minutes
 (e) An airplane flies 315 kilometres in 1 hour 45 minutes

- (a) A man jogs 6 miles in 1 hour 12 minutes
 (b) A motorcycle drives 130 miles in 2 hours 36 minutes
 (c) A helicopter flies 152 miles in 1 hour 54 minutes
 (d) A plane travels 1272 kilometres in 5 hours 18 minutes
 (e) A car travels 98 miles in 2 hours 27 minutes

Example

A car travels 150 miles at a speed of 50mph.
 How long does the journey take?

D	T
50 miles	1 hour
x3	x3
150 miles	3 hours
3 hours	

Example

A car travels 140km at a speed of 40km per hour.
 How long does the journey take?

D	T
40 km	1 hour
x3.5	x3.5
140 km	3.5 hours
3.5 hours.	

Questions – Work out how long each journey takes

- (a) A car drives 120 miles at a speed of 40 mph.
 (b) A lorry drives 250 miles at a speed of 50 mph.
 (c) A bird flies 330 kilometres at a speed of 55 kilometres per hour.
 (d) An object travels 48 miles at speed of 16 mph.
 (e) A man runs 240 metres at a speed of 6m/s
 (f) A dog runs 168 metres at a speed of 12m/s
- (a) A car travels 100 miles at a speed of 40mph.
 (b) A lorry travels 90 miles at a speed of 60 mph.
 (c) A bus drives at a speed of 48mph and covers a distance of 60 miles.
 (d) A helicopter flies 105 kilometres at a speed of 140 km/h
 (e) A bird covers a distance of 95 miles at a speed of 20 miles per hour.
 (f) A car travels at 50 mph and covers a distance of 110 miles.
 (g) A lorry drives a distance of 452.4 kilometres at a speed of 52 km/h.

Density is the amount of mass in a volume. It tells us how tightly matter is packed together.  	
$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$	Units of density: • g/cm^3 = grams per cm^3 • kg/m^3 = kilograms per m^3
Example A small statue has a mass is 1200g and a volume is 30cm^3 . Work out the density of the statue. $\text{Density} = \frac{1200\text{g}}{30\text{cm}^3} = 40\text{g/cm}^3$	Example A metal rod has a mass of 1.2kg and a volume of 300cm^3 . Work out the density. ↗ 1200g $\text{Density} = \frac{1200\text{g}}{300\text{cm}^3} = 4\text{g/cm}^3$
Questions - Work out the density of each of the following.	
State the units of each answer. (a) A piece of wood has a mass of 7g and a volume of 10cm^3 (b) A rod of aluminium has a mass of 575.4g and a volume of 210cm^3 (c) A piece of nickel has a mass of 3.48kg and a volume of 400cm^3 (d) An iron statue with volume of 0.05m^3 and a mass of 394kg (e) 2.1m^3 of oil with a mass of 1775g	
Example A vase has a volume of 200cm^3 and is made from a material which has a density of 4g/cm^3 . Work out the mass of the vase.	
Questions - Work out the mass of each of the following.	
State the units of each answer. (a) A statue with a volume of 120cm^3 made from ceramic which has a density of 2g/cm^3 . (b) A rod with a volume of 50cm^3 made from copper which has a density of 8.9g/cm^3 . (c) A block with a volume of 1.8m^3 made from silver which has a density of 10490kg/m^3 (d) A statue with a volume of 3m^3 made from zinc which as a density of 7.14g/cm^3 (e) 2800cm^3 of butter which has a density of 911kg/m^3	
Example A piece of wood has a mass of 200kg and a density of 3.5kg/m^3 . Work out the volume.  $\text{Volume} = \frac{m}{D} = \frac{200}{3.5} = 57.1\text{m}^3$	Example A sheet of metal has a mass of 2.3kg and a density of 2.4g/cm^3 . ↗ 2300g Work out the volume. $\text{Volume} = \frac{m}{D} = \frac{2300}{2.4} = 958.3\text{cm}^3$

Questions - Work out the volume of each of the following.

State the units of each answer.

- (a) A 50g piece of wood which has a density of 0.4g/cm^3
- (b) A 770g block made of brass which has a density of 8.67g/cm^3
- (c) A 4kg sheet of glass which has a density of 2.42g/cm^3
- (d) 80kg of rye which has a density of 720kg/m^3
- (e) 5 tonnes of gold which has a density of 19300kg/m^3

Calculate the density in kg/m^3 .



A wooden cube has lengths 8cm. It has a mass of 307.2g. Find the density of the cube.

Lead has a density of 11.5g/cm^3 .
A rectangular block of lead measures $4\text{cm} \times 5\text{cm} \times 2\text{cm}$.
Find the mass.

Olympic medals have a diameter of 6cm and a thickness of 1cm.
Gold has a density of 19g/cm^3 . Work out mass of a gold medal (to 2 dp).

Pressure is the amount of force applied in an given area.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Units of pressure:

- N/m^2 = newtons per m^2
- N/cm^2 = newtons per cm^2

Example

An elephant's weight pushes down on the ground with a force of 1,800N.
The total area of its feet is 565cm^2 .
Calculate the pressure.

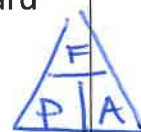
$$\begin{aligned} \text{Pressure} &= \frac{1800\text{N}}{565\text{cm}^2} \\ &= 3.2\text{N/cm}^2 \end{aligned}$$

(1dp)

Example

A rectangular laptop exerts a downward pressure of 330N/m^2 .
The weight of the laptop is 23.1N.
Work out the area of the base of the laptop.

$$\text{Area} = \frac{F}{P} = \frac{23.1}{300} = 0.077\text{m}^2$$



Questions

Question 1: Work out the pressure for each of the following.
Give suitable units for each answer.

- (a) A box is placed on a table and exerts a force of 250N on an area of 20cm^2
- (b) An object is placed on the ground and exerts a force of 3000N on an area of 4m^2
- (c) An object is placed on the ground and exerts a force of 54N on an area of 0.5cm^2
- (d) A box is placed on a table and exerts a force of 124 newtons on an area of 10.5cm^2
- (e) An object is placed on the ground and exerts a force of 25958N on an area of 1.4m^2

Question 2: Work out the force for each of the following.
In each case a box has been placed on the floor.

- (a) The area of contact is 16cm^2 and the pressure exerted is 10N/cm^2
- (b) The area of contact is 1.5m^2 and the pressure exerted is 5000N/m^2
- (c) The area of contact is 660cm^2 and the pressure exerted is 8.2N/cm^2
- (d) The area of contact is 0.2m^2 and the pressure exerted is 1.2N/cm^2
- (e) The area of contact is 500cm^2 and the pressure exerted is 450000N/m^2

Question 3: Work out the area of contact for each of the following.
In each case an object has been placed on the floor.
Give suitable units for each answer.

- (a) The object exerts a force of 420N on the floor and the pressure on the floor is 20N/cm^2
- (b) The object exerts a force of 8590N on the floor and the pressure on the floor is 900N/m^2
- (c) The object exerts a force of 30N on the floor and the pressure on the floor is 600N/m^2
- (d) The object exerts a force of 3945N on the floor and the pressure on the floor is 200N/cm^2

Exam Questions

A solid statue has volume 512 cm^3
The statue has mass 3.6 kilograms.

density of iron = 7.87 grams per cubic centimetre

Is the statue made of iron?
You **must** show your working.

A solid has volume 300 cm^3 and density 2 g/cm^3
Circle the mass of the solid.

150g

298g

302g

600g

A ball contains 5000 cm^3 of air.

More air is pumped into the ball at a rate of 160 cm^3 per second.

The ball is full of air when it becomes a sphere with radius 15 cm



$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3 \text{ where } r \text{ is the radius}$$

Does it take **less than** 1 minute to fill the ball?

You **must** show your working.

A machine takes 4 seconds to fill a packet of crisps.

(a) In total, how many packets can 35 of these machines fill in 8 hours?

(b) Each packet of crisps contains 32.5 grams of crisps.

At what rate does a machine put the crisps into the packets?

Give your answer in grams per second.

A solid piece of silver has
mass 2.625 kilograms
volume 250 cm^3

Work out the density of the piece of silver.

Give your answer in grams per cubic centimetre.

A car travels 3.5 miles in 5 minutes.

Work out the average speed in miles per hour.

A car journey is in two stages.

Stage 1 The car travels 110 miles in 2 hours.

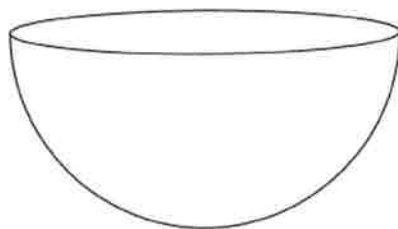
Stage 2 The car travels 44 miles at the same average speed as Stage 1

Work out the time for Stage 2

Give your answer in minutes.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3 \text{ where } r \text{ is the radius}$$

A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

The distance by road from Newport to London is 140 miles.

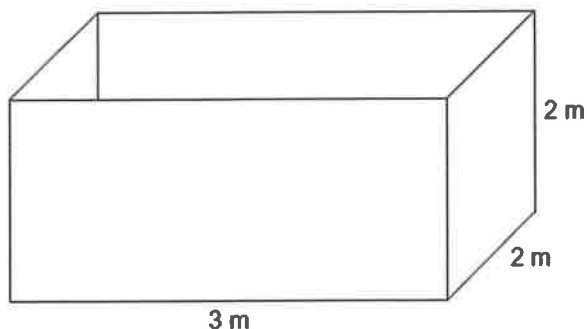
Tom travels by coach from Newport to London.

The coach leaves Newport at 1.30 pm

(a) He assumes the coach will travel at an average speed of 50 mph

Use his assumption to work out the arrival time in London.

An empty tank is in the shape of a cuboid as shown.



The tank is to be filled with water at 1.25 litres per second.

$$1 \text{ m}^3 = 1000 \text{ litres}$$

Work out the time taken to fill the tank.

Give your answer in hours and minutes.

Convert 11.2 kilometres into miles.

Use $8 \text{ km} = 5 \text{ miles}$

The heel of a shoe exerts a pressure of 198 pounds per square inch.

Convert this pressure into kilograms per square centimetre.

Use

$$1 \text{ pound} = 0.45 \text{ kilograms}$$

$$1 \text{ square inch} = 6.25 \text{ square centimetres}$$

In this question use 1 cubic foot = 6.23 gallons 1 cubic foot = 0.028 cubic metres Convert 3115 gallons into cubic metres.	In this question, use 1 foot = 12 inches 1 inch = 2.5 centimetres Change 5 feet 8 inches to centimetres.
In this question, use 1 kilogram = 2.2 pounds 1 stone = 14 pounds Change 70 kilograms into stones.	

Topic 20 – Solving and Representing Inequalities

Definition

= means equal to

< means less than

$\leq$ means less than or equal to

> means greater than

$\geq$ means greater than or equal to

Examples

$x > 2$ means that x is greater than 2

$x \leq 5$ means that x is less than or equal to 5

$-2 < x \leq 3$ means that x is greater than -2 but less than or equal to 3

Questions – Write down the meaning of each inequality

(a) $x > 6$

(b) $x < 2$

(c) $x \geq 1$

(d) $x \leq 4$

(e) $x \geq 0$

(f) $x \leq -4$

(g) $x < -2$

(h) $x > 20$

(i) $x < y$

(j) $a \geq b$

(k) $c > 5$

(l) $y \leq 100$

(a) $3 < x < 5$

(b) $2 < x < 9$

(c) $19 \leq x < 20$

(d) $5 \leq x \leq 10$

(e) $0 < x \leq 4$

(f) $-4 \leq x < 1$

(g) $-8 \leq x \leq -6$

(h) $100 < x < 200$

Examples – write an inequality for each of the following

x is greater than or equal to 10 : $x \geq 10$

x is less than -3 : $x < -3$

x is greater than -4 but less than or equal to 7 : $-4 < x \leq 7$

Questions – Write an inequality for each of the following

(a) x is greater than 8

(b) x is less than 3

(c) x is less than or equal to 1

(d) x is greater than or equal to 0

(e) x is less than 7

(f) x is greater than or equal to -2

(g) x is less than or equal to -10

(h) x is greater than 5

(a) x is greater than 2, but less than 5

(b) x is greater than 0, but less than 4

(c) x is greater than 1, but less than or equal to 7

(d) x is greater than -5, but less than or equal to 2

(e) x is greater than or equal to -8, but less than 3

(f) x is greater than or equal to 10, but less than 20

(g) x is greater than or equal to 3, but less than or equal to 6

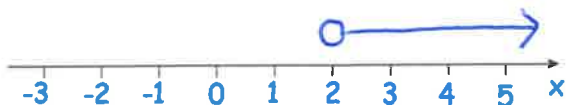
(h) x is greater than or equal to 8, but less than or equal to 11

Inequalities can be represented on a number line.

- means that the number is equal to
- means that the number is **not** equal to

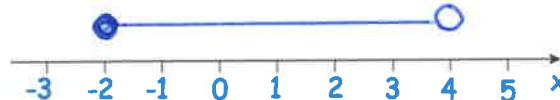
Example

Represent the inequality $x > 2$ on this number line



Example

Represent the inequality $-2 \leq x < 4$ on this number line



Example

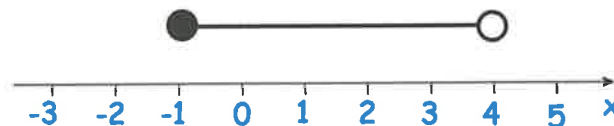
Write down the inequality shown on the number line



$$x > -2$$

Example

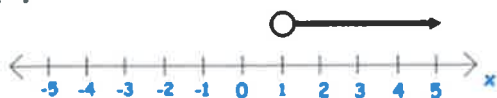
Write down the inequality shown on the number line



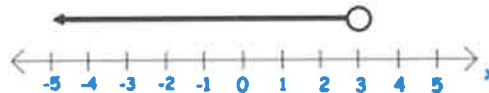
$$-1 \leq x < 4$$

Questions – Write down the inequality shown on each number line

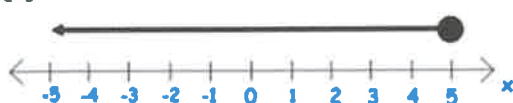
(a)



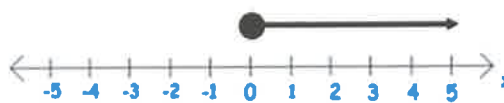
(b)

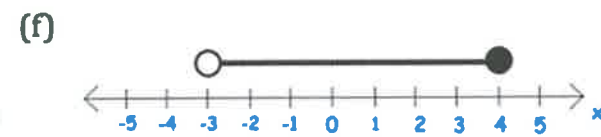
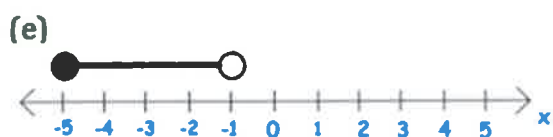
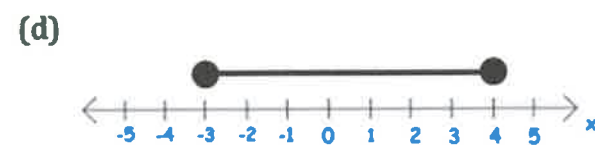
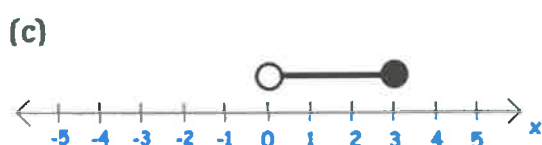
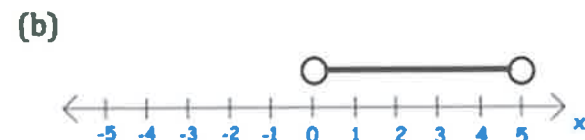
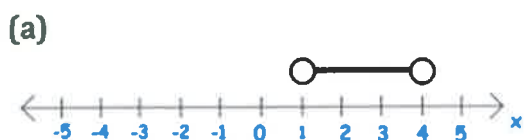
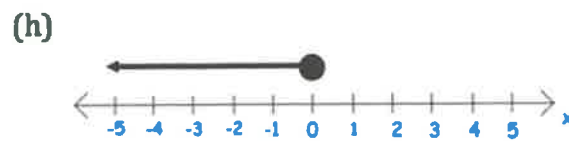
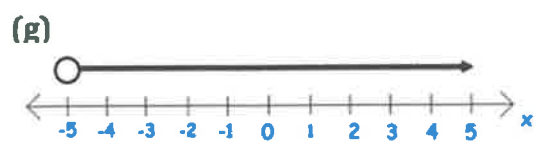
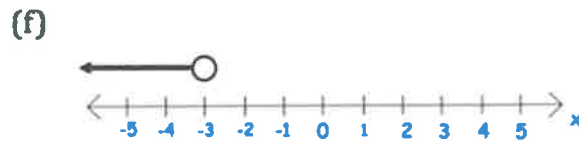
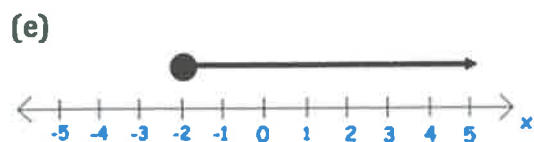


(c)



(d)





Example

List all of the integers that satisfy the inequality $-4 < x \leq 3$

$-3, -2, -1, 0, 1, 2, 3$

Questions – List all of the integers that satisfy the inequalities shown

(a) $2 < x < 6$

(b) $5 < x < 10$

(c) $4 \leq x < 8$

(d) $12 \leq x \leq 15$

(e) $-2 < x \leq 3$

(f) $-5 \leq x < 1$

(g) $-10 \leq x \leq -5$

(h) $-4 < x < 4$

We can solve inequalities in the same way that we solve equations

Example

Solve

$$\begin{array}{l} x - 2 < 5 \\ +2 \quad +2 \\ \hline x < 7 \end{array}$$

Example

Solve

$$\begin{array}{l} 3x \geq 15 \\ \div 3 \quad \div 3 \\ \hline x \geq 5 \end{array}$$

Example Solve $3x + 2 \leq 14$ $\begin{array}{rcl} 3x + 2 & \leq & 14 \\ -2 & -2 & \\ \hline 3x & \leq & 12 \\ \hline x & \leq & 4 \end{array}$	Example Solve $\frac{x-3}{4} > 2$ $\begin{array}{rcl} \frac{x-3}{4} & > & 2 \\ \times 4 & \times 4 & \\ \hline x-3 & > & 8 \\ +3 & +3 & \\ \hline x & > & 11 \end{array}$
Example Solve $3(2x + 1) \geq 15$ $\begin{array}{rcl} 3(2x + 1) & \geq & 15 \\ 6x + 3 & \geq & 15 \\ -3 & -3 & \\ \hline 6x & \geq & 12 \\ \hline x & \geq & 2 \end{array}$	Example Solve $4x + 1 < 2x + 10$ $\begin{array}{rcl} 4x + 1 & < & 2x + 10 \\ -2x & -2x & \\ \hline 2x + 1 & < & 10 \\ -1 & -1 & \\ \hline 2x & < & 9 \\ \hline x & < & 4.5 \end{array}$
Questions – Solve the following inequalities	
(a) $x + 4 > 9$ (b) $x - 3 < 2$ (c) $2x \leq 14$ (d) $8x < 16$ (e) $5x \geq 15$ (f) $\frac{x}{3} > 4$ (g) $\frac{x}{5} \leq 2$ (h) $x + 6 \geq 4$	
(a) $2x + 1 \leq 9$ (b) $3x - 5 > 16$ (c) $4x + 8 < 32$ (d) $5x - 2 \geq 68$ (e) $\frac{x}{2} + 1 \leq 5$ (f) $\frac{x}{9} - 6 > 4$ (g) $\frac{x+3}{2} \geq 5$ (h) $\frac{x-5}{4} > 2$	
(a) $5(x - 3) \geq 40$ (b) $6(x + 2) < 42$ (c) $2(5x + 1) \leq 36$	
(a) $4x + 3 > 2x + 11$ (b) $x + 1 \geq 3x - 18$ (c) $13x - 12 < 3x + 13$ (d) $7x - 5 \geq 3x + 11$	

Question 6: Find the largest integer that satisfies each inequality below.

(a) $x + 3 < 9$

(b) $2x + 5 < 12$

(c) $7x + 10 \leq 31$

(d) $3x - 5 \leq 9$

(e) $\frac{x}{4} + 3 \leq 8$

(f) $4x + 14 \leq 8$

Question 7: Find the smallest integer that satisfies each inequality below.

(a) $2x - 5 \geq 12$

(b) $4x > 9$

(c) $\frac{x+9}{3} \geq 7$

(d) $7x + 1 > 60$

(e) $10x - 16 \geq 76$

(f) $9x + 4 > 7x + 15$

Example

Solve

$$\begin{array}{r} -6 < 3x \leq 12 \\ \hline 3 \quad 3 \quad 3 \\ -2 < x \leq 4 \end{array}$$

Example

Solve

$$\begin{array}{r} 5 \leq 2x + 1 < 13 \\ \hline -1 \quad -1 \quad -1 \\ 4 \leq 2x < 12 \\ \hline 2 \leq x < 6 \end{array}$$

Questions – Solve each inequality

(a) $6 < x + 3 < 10$

(b) $4 \leq 2x \leq 7$

(c) $1 \leq 3x < 9$

(d) $4 < \frac{x}{5} < 6$

(e) $9 \leq 2x + 3 \leq 25$

(f) $-3 \leq \frac{x}{4} - 1 < 0$

(d) $16 \leq 5x + 1 < 31$

(e) $0 \leq \frac{x-6}{2} < 2$

(f) $-9 < \frac{x}{4} - 1 < -8$

When we are solving inequalities, if we have to multiply or divide by a negative number then the inequality sign 'flips'

Example

Solve

$$\begin{array}{r} -4x < 24 \\ \hline -4 \quad -4 \\ x > -6 \end{array}$$

Example

Solve

$$\begin{array}{r} 12 - 2x \geq 20 \\ \hline -12 \quad -12 \\ -2x \geq 8 \\ \hline -2 \quad -2 \\ x \leq -4 \end{array}$$

Questions – Solve the inequalities

h) $6 - x > 2$

i) $15 - 4x < 3$

j) $13 - 2x \leq 7$

k) $5 - 3x > -4$

l) $9 - 5x \geq -16$

m) $2(4 - 3x) \geq 32$

Solve $8 > 3 - \frac{1}{2}x$

Topic 21 - Ratio

Definition - A ratio compares values. It tells us how much of one thing there is compared to another.

A ratio can be scaled up to make an equivalent ratio.

Examples – Scale up to find the equivalent ratio

$$\begin{array}{ccccccc}
 4:5 & & 2:7 & & 3:15 & & 3:15 \\
 \swarrow \times 8 & \searrow \times 8 & \swarrow \times 5 & \searrow \times 5 & \swarrow \times 5 & \searrow \times 5 & \swarrow \times 3 & \searrow \times 3 \\
 32:40 & & 10:35 & & 45:225 & & 9:45 & &
 \end{array}$$

Questions

Q1) Use multiplication to complete these equivalent ratios

a	b	c	d	e
2 : 3	2 : 3	2 : 3	2 : 6	1 : 6
4 :	6 :	: 6	: 18	: 18
f	g	h	i	j
1 : 3	1 : 3	2 : 3	2 : 9	2 : 9
: 18	18 :	18 :	18 :	: 18
k	l	m	n	o
9 : 2	3 : 2	3 : 1	2 : 1	5 : 1
27 :	27 :	27 :	: 27	: 27

A ratio can be simplified to make an equivalent ratio.

Length	Mass
$\begin{array}{ccccccc} & \times 1000 & & \times 100 & & \times 10 & \\ & \curvearrowright & & \curvearrowright & & \curvearrowright & \\ \text{Km} & & \text{m} & & \text{cm} & & \text{mm} \\ & \curvearrowleft & & \curvearrowleft & & \curvearrowleft & \\ & \div 1000 & & \div 100 & & \div 10 & \end{array}$	$\begin{array}{ccccccc} & \times 1000 & & \times 1000 & & \times 1000 & \\ & \curvearrowright & & \curvearrowright & & \curvearrowright & \\ \text{tonne} & & \text{kg} & & \text{g} & & \text{mg} \\ & \curvearrowleft & & \curvearrowleft & & \curvearrowleft & \\ & \div 1000 & & \div 1000 & & \div 1000 & \end{array}$

Examples – Fully simplify each ratio

$$\begin{array}{cccc}
 35:49 & 12:48:30 & 88\text{ab}:40\text{a} & 300\text{m}:1.8\text{Km} \\
 \swarrow \div 7 & \swarrow \div 6 \quad \downarrow \div 6 \quad \searrow \div 6 & \swarrow \div 8 & \swarrow \div 3 \quad \uparrow 1800\text{m} \\
 5:7 & 2:8:5 & 11\text{b}:5 & 300\text{m}:1800\text{m} \\
 & & & \downarrow \div 3 \\
 & & & 1\text{m}:6\text{m}
 \end{array}$$

Questions

(a) 4 : 6

(b) 14 : 8

(c) 15 : 10

(d) 6 : 15

(e) 30 : 10

(f) 12 : 16

(g) 6 : 18

(h) 45 : 10

(i) 12 : 28

(j) 24 : 36

(k) 25 : 60

(l) 27 : 63

Simplify:
14:77:21

Simplify:
78:12:66

Simplify:
21:39:9

Simplify:
8:28:20

Simplify:
18:30:78

Simplify:
22:14:26

Simplify:
24ab:88a

Simplify:
20ab:52a

Simplify:
30ab:42a

Simplify:
42ab:78a

Simplify:
24ab:56a

Simplify:
78ab:66a

Simplify:
 $20a^4b^9:8ba^8$

Simplify:
 $30a^2b^4:66ba^5$

Simplify:
 $66a^8b^6:30ba^5$

(a) 8 days to 2 weeks

(b) 1 hour to 15 minutes

(c) 2 hours to 1 day

(d) 95p to £3.00

(e) 400m to 1.5km

(f) 15kg to 900g

(g) 4500ml to 2 litres

(h) 8km to 50mm

(i) 90 minutes to 2 days

A ratio can be expressed in the form 1:n or n:1

Examples

Express the following ratios in the form 1:n

$$\begin{array}{l} 2:5 \\ \div 2 \quad \downarrow \quad \div 2 \\ 1:2.5 \end{array}$$

$$\begin{array}{l} 8:3 \\ \div 8 \quad \downarrow \quad \div 8 \\ 1:0.375 \end{array}$$

Examples

Express the following ratios in the form n:1

$$\begin{array}{l} 3:4 \\ \div 4 \quad \downarrow \quad \div 4 \\ 0.75:1 \end{array}$$

$$\begin{array}{l} 5:2 \\ \div 2 \quad \downarrow \quad \div 2 \\ 2.5:1 \end{array}$$

Questions

Question 5: Express each of the following ratios in the form $1 : n$

- | | | | |
|--------------|--------------|-----------------|----------------|
| (a) $2 : 3$ | (b) $5 : 4$ | (c) $4 : 10$ | (d) $10 : 7$ |
| (e) $8 : 13$ | (f) $5 : 81$ | (g) $100 : 131$ | (h) $200 : 77$ |

Question 6: Express each of the following ratios in the form $n : 1$

- | | | | |
|--------------|---------------|----------------|---------------|
| (a) $7 : 2$ | (b) $9 : 5$ | (c) $11 : 3$ | (d) $5 : 8$ |
| (e) $3 : 10$ | (f) $19 : 20$ | (g) $207 : 50$ | (h) $38 : 55$ |

We can represent a ratio in a table and use this to work out totals and differences.

Example

In a bag there are 3 red counters for every 4 yellow counters.

Given that there are 12 red counters in the bag, work out:

$$\begin{array}{rclcl}
 R & : & Y & = & T & D \\
 3 & : & 4 & = & 7 & 1 \\
 \times 4 & & \times 4 & & \times 4 & \times 4 \\
 12 & : & 16 & = & 28 & 4
 \end{array}$$

a) How many yellow counters are there?

16 yellow.

b) How many counters are there in total?

28 in total

Example

Ali and Beth share some sweets in the ratio $3:2$.

They share 30 sweets in total.

How many sweets do they each get?

$$\begin{array}{rclcl}
 A & : & B & = & T & D \\
 3 & : & 2 & = & 5 & 1 \\
 \times 6 & & \times 6 & & \times 6 & \times 6 \\
 18 & : & 12 & = & 30 & 6
 \end{array}$$

Ali = 18 sweets

Beth = 12 sweets.

Questions

1. The ratio of yellow counters to red counters is $3:4$. There are 9 yellow counters in the bag.

a) How many red counters were in the bag?

b) How many counters were there in total?

2. In a class there are 5 boys for every 3 girls.

There are 12 girls in the class.

a) How many boys are in the class?

b) How many children are in the class in total?

3. The ratio of circles to squares is 2:5. There are 28 shapes in total.

a) How many squares are there?

b) How many circles are there?

4. In a pot there are 2 blue pens for every 7 black pens. There are 45 pens in the pot in total.

a) How many blue pens are there?

b) How many black pens are there?

5. At a party there is 1 adult for every 3 children. There are 21 children at the party. How many adults are there?

6. On a farm cows : sheep = 5:8. There are 78 animals on the farm in total. How many sheep are there on the farm?

7. A length of wood is cut into two smaller lengths in the ratio 5:3. There are 48 pieces of wood in total. How many pieces of each length are there?

8. The ratio of men to women in a sports club is 7:5

There are 63 women. How many men are there?

Example

Ali and Beth share some money in the ratio 5:3.

Ali gets £12 more than Beth. (D)

How much money do they each get?

$$\begin{array}{cccc} A & : & B & = & T & D \\ 5 & : & 3 & & 8 & 2 \\ \times 6 & & \times 6 & & \times 6 & \times 6 \\ 30 & : & 18 & & 48 & 12 \end{array}$$

$$Ali = £30$$

$$Beth = £18$$

Example

In a school

the number of boys : number of girls is 2 : 5

There are 99 less boys than girls. (D)

Work out the total number of students at the school.

$$\begin{array}{cccc} B & : & G & = & T & D \\ 2 & : & 5 & & 7 & 3 \\ \times 33 & & \times 33 & & \times 33 & \times 33 \\ 66 & : & 165 & & 231 & 99 \end{array}$$

231 students.

Questions

A bag contains yellow and blue blocks in the ratio 1:3

There are 8 more blue blocks than yellow blocks.

(a) How many yellow blocks are there?

(b) How many blue blocks are there?

The ratio of boys to girls in a class is 2:3

There are 6 more girls than boys in the class.

How many girls are in the class?

Thomas and Emma share some money in the ratio 3:5
Emma receives £30 more than Thomas.

- (a) How much money does Emma receive?
(b) How much money does Thomas receive?

In a survey, the ratio of the number of people who preferred tea to those who preferred coffee was 9:5
36 more people preferred tea to coffee.
How many people were in the survey?

The ratio of Mollie's age to Heather's age is 4:9
Heather is 40 years older than Mollie
How old is Mollie?

A box contains red, purple and green beads in the ratio 4:6:7
There are 1428 more green beads than red beads.
How many green beads are in the box?

Mixed Questions

- a) Charlie and Dave share some money in the ratio 2:7.
Dave gets £30 more than Charlie.
How much money did they share?
- b) Adil and Arman share some money in the ratio 2:5. Arman gets £63 more than Adil. How much money does Adil get?
- c) A metal alloy block of mass 56kg consists of copper, zinc and tin in the ratio 4:3:1. Find the mass of each metal.
- d) Two numbers are in the ratio 4:3. Their sum is 70. Find the numbers.
- e) Two numbers, A and B, have a ratio 5:3. A is 50 more than B. Work out A+B.
- f) Amy and Arman share £140 in the ratio 2:5
How much more does Arman get?
- g) A recipe for a cake says to mix 40g of flour for every 12g of sugar. Altogether, Alex mixed 208g. How many grams of each ingredient did he use?
- h) Alan Becky and Charlie share 72 sweets in the ratio 2:3:4. How many sweets do they each receive?
- i) Two lengths are in the ratio 2:7. The longest length is 35cm more than the shortest length.
Work out the size of each length.

Sometimes we can have two ratios that are overlapping. When this happens, we have to find the common part and make it the same in both ratios.

Example

The ratio of $a:b = 2:3$

The ratio of $b:c = 4:5$

Write the ratio of $a:b:c$

$$\begin{array}{cc} a:b & b:c \\ 2:3 & 4:5 \\ \times 4 & \times 3 \\ 8:12 & 12:15 \end{array}$$

$$a:b:c \\ 8:12:15$$

b is the overlap

Example

In a bag there are red, green and blue counters. The ratio of red counters to green counters is $3:4$. The ratio of green counters to blue counters is $5:2$.

Write down the ratio of red to green to blue counters.

$$\begin{array}{cc} R:G & G:B \\ 3:4 & 5:2 \\ \times 5 & \times 4 \\ 15:20 & 20:8 \end{array}$$

$$R:G:B \\ 15:20:8$$

Questions

a	The ratio of $a:b$ is $2:3$ The ratio of $b:c$ is $5:4$ Write the ratio of $a:b:c$	b	The ratio of $a:b$ is $2:5$ The ratio of $b:c$ is $3:4$ Write the ratio of $a:b:c$
c	The ratio of $a:c$ is $2:3$ The ratio of $b:c$ is $4:5$ Write the ratio of $a:b:c$	d	The ratio of $a:b$ is $2:3$ The ratio of $a:c$ is $4:5$ i) Write the ratio of $a:b:c$ ii) What fraction is b ?
e	The ratio of $a:b$ is $3:5$ The ratio of $b:c$ is $4:5$ i) Write the ratio of $a:b:c$ ii) What fraction is c ?	f	The ratio of $a:b = 1:2$ The ratio of $b:c = 1:1$ i) Write the ratio of $a:b:c$ ii) What percentage of the total is c ?

In a bag there are blue, green and yellow counters.

The ratio of blue counters to green counters is $3:2$

The ratio of green counters to yellow counters is $2:5$

(a) Write down the ratio of blue to green to yellow counters in the bag.

(b) What percentage of the beads are green?

At a safari park, the ratio of lions to tigers is 7:4.

The ratio of elephants to tigers is 1:2

Write down the ratio of lions to tigers to elephants in the safari park.

A bag contains three different shaped pieces of card.

The ratio of circles to triangles is 2:3

The ratio of triangles to rectangles is 2:5

Find the ratio of circles to triangles to rectangles.

In a drawer, there are white, black and grey socks.

The ratio of white socks to black socks is 3:2

The ratio of white socks to grey socks is 9:4

(a) Write down the ratio of white socks to black socks to grey socks.

Elsie says there is an odd white sock.

(b) Explain why Elsie might be wrong.

A quadrilateral, ABCD, is drawn.

The ratio of the size of angle A to angle B is 1:3

The ratio of the size of angle B to angle D is 5:3

The ratio of the size of angle C to angle A is 7:5

Find the difference in size between the largest and smallest angles in quadrilateral ABCD.

The ratio of red pens to black pens is 2:9

The ratio of black pens to blue pens is 5:4

Show less than 50% of the pens are black.

We can represent a ratio as a fraction.

Example

In a bag, the ratio of yellow counters to red counters is 2:5.

$$\begin{array}{l} 4 : 2 = 2 \\ 2 : 5 = \frac{2}{5} \end{array}$$

a) What fraction of the counters are red? $\frac{5}{7}$

b) What fraction of the counters are yellow? $\frac{2}{7}$

Example

The ratio of men to women is 4:3.

$$\begin{array}{l} m : w = 4 : 3 \\ 4 : 3 = \frac{4}{3} \end{array}$$

a) What proportion of people are men? $\frac{4}{7}$

b) What proportion of people are women? $\frac{3}{7}$

Questions

1. At a concert, the ratio of men to women is 2:3

- What fraction of the people are men?
- What fraction of people are women?

2. At a party there are 30 children and 20 adults. 8 more adults arrive.

- Write the number of children to adults as a ratio in its simplest form.
- What fractions of the people are children?

3. At a school there are 44 teachers. 24 teachers wear glasses and 20 do not wear glasses. 4 more teacher start school who wear glasses.

- Write the number of teachers who wear glasses and who do not wear glasses as a ratio in its simplest form.
- What proportion of teachers wear glasses?

4. There are triple the number of boys compared to girls.

- Write the number of girls to boys as a ratio.
- What fractions are girls?

5. In a classroom half of the pupils are boys. The rest are girls.

- Write the ratio of boys: girls.
- There are 27 pupils in total, how many boys and how many girls are there?

6. There are double the number of male teachers than female teachers.

- Write this as a ratio of male to female teachers
- Write the ratio of female to male teachers.
- What fraction of the teachers are male?

5) Chris makes a drink by mixing lemonade and orange juice in the ratio 13:7

- What fraction of the drink is orange juice?
- What fraction of the drink is lemonade?

6) Olivia has blue, pink and orange counters in a bag. The ratio of blue to pink to orange counters is 4:5:2

- What fraction of the counters are blue?
- What fraction of the counters are orange?

We can represent a fraction as a ratio.

Example

There are blue and green counters in a bag.

The fraction of green counters is $\frac{1}{4}$.

$$\begin{array}{rcl} B : G & = & T \\ 3 : 1 & & 4 \end{array}$$

$$4 - 1 = 3 \text{ Blue}$$

- What fractions are blue counters? $\frac{3}{4}$

- What is the ratio of green counters to blue counters?

$$\begin{array}{l} 1 : B \\ 1 : 3 \end{array}$$

- There are 15 blue counters in the bag. How many green counters are there?

$$B : G$$

$$5 \text{ Green}$$

$$\times 5 \left(\begin{array}{l} 3 : 1 \\ 15 : 5 \end{array} \right) \times 5$$

Questions

1. In a bag there red and yellow counters. The fractions of red counters is $\frac{2}{5}$. a) What is the ratio of red to yellow counters? b) If there are 12 red counters, how many yellow counters are there?	2. On a farm there are cows and sheep. The fractions of sheep is $\frac{4}{7}$. a) What is the ratio of cows to sheep. b) There were 40 sheep on the farm, but 4 went missing. Write the number of cows to sheep as a ratio in its simplest form.
3. In a bag there are red and yellow counters. The fraction of red counters is $\frac{4}{9}$. a) Write the ratio of red counters to yellow counters. b) There are 81 counters in total. How many counters are there of each colour?	4. A bag of sweets contains cola bottle and jelly babies. $\frac{1}{3}$ of the sweets are cola bottles. What is the ratio of cola bottles to jelly babies?
5. A bag of sweets contains cola bottle and jelly babies. $\frac{2}{5}$ of the sweets are cola bottles. What is the ratio of jelly babies to cola bottles?	6. A bag of sweets contains cola bottle and jelly babies. $\frac{3}{15}$ of the sweets are cola bottles. Write the ratio of jelly babies to cola bottles in its simplest form.

We can represent a ratio as an equation.

Example

Write $x:y = 4:5$ as an equation

$$\begin{array}{c} x : y \\ 4 : 5 \\ \hline 5x = 4y \end{array}$$

Example

Write $x:y = 3:2$ as an equation

$$\begin{array}{c} x : y \\ 3 : 2 \\ \hline 2x = 3y \end{array}$$

Questions

- Write $x:y = 1:5$ as an equation
- Write $x:y = 2:7$ as an equation
- Write $y:x = 3:4$ as an equation
- Write $a:b = 5:2$ as an equation
- Write $b:a = 9:4$ as an equation
- $X:Y = 4:3$ write as an equation
- $A:B = 6:1$ write as an equation
- If $x:y = 4:5$, form an equation

We can represent an equation as a ratio.

Examples

Given that $3a = 2b$
Write the ratio of $a:b$

$$\begin{array}{l} 3a = 2b \\ \times \\ a : b \\ 2 : 3 \end{array}$$

Given that $4x = 5y$
Write the ratio of $y:x$

$$\begin{array}{l} 4x = 5y \\ \times \\ x : y \\ 5 : 4 \\ \text{so} \\ y : x \\ 4 : 5 \end{array}$$

Example

Given that $a = \frac{9b}{7}$
Write the ratio of $a : b$

$$\begin{array}{l} a = \frac{9b}{7} \\ \times 7 \quad \quad \times 7 \\ 7a = 9b \\ \times \\ a : b \\ 9 : 7 \end{array}$$

Given that $x = \frac{2y}{5}$
Write the ratio of $y:x$

$$\begin{array}{l} x = \frac{2y}{5} \\ \times 5 \quad \quad \times 5 \\ 5x = 2y \\ \times \\ x : y \\ 2 : 5 \\ \text{so} \\ y : x \\ 5 : 2 \end{array}$$

Questions

a	$2a = 3b$	Write the ratio <u>$a:b$</u>	b	$5a = 7b$	Write the ratio <u>$b:a$</u>
c	$4x = 9y$	Write the ratio <u>$x:y$</u>	d	$3x = 8y$	Write the ratio <u>$y:x$</u>
e	$3a = 7b$	Write the ratio <u>$a:b$</u>	f	$11x : 5y$	Write the ratio of <u>$x:y$</u>
g) Given that $x = \frac{3y}{4}$ Write the ratio of $x:y$			h) Given that $a = \frac{4b}{3}$ Write the ratio of $a:b$		
i) Given that $x = \frac{7y}{2}$ Write the ratio of $y:x$			j) Given that $b = \frac{5a}{7}$ Write the ratio of $a:b$		

Problem Solving and Exam Questions

Andrew and Bruce share some money in the ratio 5 : 6

Bruce gets £96

Andrew gives $\frac{1}{4}$ of his share to Carl.

Bruce gives $\frac{2}{3}$ of his share to Carl.

How much money does Carl receive?

Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as 10p coins
- £17.70 in total.

Work out total **value** of 2p coins : total **value** of 5p coins

Give your answer in its simplest form.

Jing has £2450

She saves some and gives the rest to her four brothers.

money saved : money given to brothers = 2 : 5

She gives each of her **four** brothers the **same** amount.

Does each brother receive more than £430 ?

You **must** show your working.

Divide 62 in the ratio 3 : 7

Write $(3^6 \times 3^5) : 3^7$ in the form $n : 1$ where n is an integer.

a is 10% more than b .

Circle the ratio $a : b$

10 : 11

10 : 1

11 : 10

1 : 10

Joe, Kim and Lisa each have an amount of money.

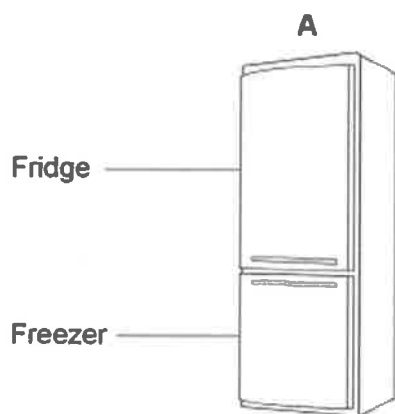
Joe has £72

Joe's amount : Kim's amount = 6 : 5

Lisa's amount is $1\frac{1}{2}$ times Joe's amount.

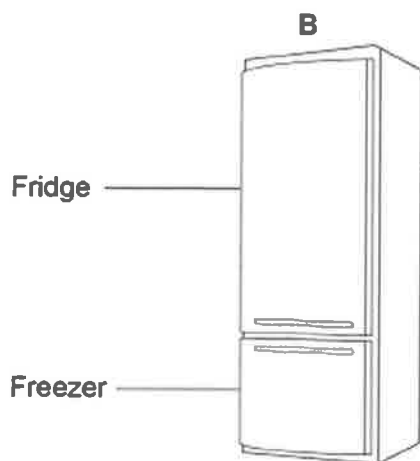
Show that, in total, they have **less** than £250

Information about two fridge-freezers, A and B, is shown.



Total capacity is 330 litres

fridge capacity : freezer capacity = 3 : 2



Fridge capacity is 294 litres

fridge capacity : freezer capacity = 7 : 3

Grace buys one of these fridge-freezers.

She buys the one with the greater **freezer** capacity.

Which one does she buy?

You **must** show your working.

There are between 20 and 30 students in a class.

The ratio of left-handed students to right-handed students is 3 : 8

How many students are in the class?

Here are two piles of the same type of paper.

Each sheet of paper weighs 5 g.

The taller pile weighs 7.5 kg.



weight of taller pile : weight of shorter pile = 5 : 3

Work out the number of sheets of paper in the shorter pile.

Jeff and Kaz share £270 in the ratio Jeff : Kaz = 2.6 : 1

How much **more** than Kaz does Jeff get?

The cost of a holiday is £2400

Rana pays a deposit followed by monthly payments, in the ratio

deposit : total of the monthly payments = 3 : 5

She makes 6 equal monthly payments.

Work out her monthly payment.

At a party there are 90 people.

48 are women and 42 are men.

Some women leave.

Some men arrive.

The ratio of women to men is now 10 : 11

Are there now more than 90 people at the party?

Tick **one** box.

☐

Yes

☐

No

☐

Cannot tell

Show working to support your answer.

Here are two sets of numbers, A and B.

Set A

200	160
104	100

Set B

270	400	483
300	x	

mean of Set A : mean of Set B = 3 : 8

Work out the value of x .

Saj makes Rose Pink paint and Cherry Pink paint.

He mixes red paint with white paint as shown.

Rose Pink

red : white = 1 : 2

Cherry Pink

red : white = 4 : 3

He makes 60 litres of Rose Pink paint.

To this Rose Pink paint he adds

80 litres of red paint and 28 litres of white paint.

Has he now made Cherry Pink paint?

You **must** show your working.

Topic 22 - Expanding and Factorising Quadratics

$$ax^2 + bx + c = 0$$

A Quadratic Equation in Standard Form

(a, b, and c can have any value, except that a can't be 0.)

Definition – Expanding means removing brackets. We do this by multiplying.

Examples – Expand and simplify

$$(x + 3)(x + 5)$$

$$= x^2 + 5x + 3x + 15 \checkmark$$

$$= x^2 + 8x + 15 \checkmark$$

Examples – Expand and simplify

$$(2x + 3)(x - 5)$$

$$= 2x^2 - 10x + 3x - 15 \checkmark$$

$$= 2x^2 - 7x - 15 \checkmark$$

Examples – Expand and simplify

$$(x - 4)(3x - 1)$$

$$= 3x^2 - 1x - 12x + 4 \checkmark$$

$$= 3x^2 - 13x + 4 \checkmark$$

Examples – Expand and simplify

$$(2x + 4)(3x + 5)$$

$$= 6x^2 + 10x + 12x + 20 \checkmark$$

$$= 6x^2 + 22x + 20 \checkmark$$

Questions – Expand and simplify

Expand
 $(x + 3)(x + 6)$

Expand
 $(x - 6)(x - 7)$

Expand
 $(x + 5)(x - 5)$

Expand
 $(x + 5)(x + 6)$

Expand
 $(x - 1)(x - 5)$

Expand
 $(x + 4)(x - 4)$

Expand
 $(2x + 3)(x + 4)$

Expand
 $(x - 4)(3x + 2)$

Expand
 $(4x - 1)(x + 3)$

Expand
 $(2x + 6)(8x - 7)$

Expand
 $(5x + 4)(6x + 7)$

Expand
 $(7x + 8)(8x + 3)$

Expand
 $(4x + 6)(4x - 5)$

Expand
 $(3x + 2)(4x - 7)$

Expand
 $(8x - 6)(5x + 5)$

Expand
 $(7x + 6)(6x + 2)$

Expand
 $(8x + 8)(4x + 3)$

Expand
 $(6x - 5)(4x + 8)$

Remember...squared means to multiply by itself

Examples – Expand and simplify

$$\begin{aligned}
 & (x+4)^2 \\
 &= (x+4)(x+4) \\
 &= x^2 + 4x + 4x + 16 \\
 &= x^2 + 8x + 16
 \end{aligned}$$

Examples – Expand and simplify

$$\begin{aligned}
 & (2x-1)^2 \\
 &= (2x-1)(2x-1) \\
 &= 4x^2 - 2x - 2x + 1 \\
 &= 4x^2 - 4x + 1
 \end{aligned}$$

Questions – Expand and simplify

(a) $(a+2)^2$

(b) $(x+7)^2$

(c) $(z-9)^2$

(d) $(p+1)^2$

(i) $(3g+2)^2$

(j) $(2b-1)^2$

(k) $(3m-5)^2$

(l) $(2v+9)^2$

(m) $(7-a)^2$

(n) $(4-3s)^2$

(o) $(8+5h)^2$

(p) $(7-2p)^2$

(a) $(a+2)(a+3) + (a+4)(a+1)$

(b) $(2w+3)(w-1) + (w-3)(w-2)$

(c) $(x+9)(3x+4) - (x+3)(x-1)$

(d) $2(x+1)(x+4) - (x+1)(x+2)$

(e) $(x+4)^2 + (x+1)^2$

(f) $(2x+1)^2 - (x-5)^2$

Factorising a quadratic is to put it back into brackets

Example

Factorise $x^2 + 7x + 12$

$a=1$

$b=7$

$c=12$

$(x+3)(x+4)$

x	+12	c
+3	+4	
+	+7	b

Example

Factorise $x^2 - 10x + 21$

$a=1$

$b=-10$

$c=21$

$(x-3)(x-7)$

x	+21	c
-3	-7	
+	-10	b

ExampleFactorise $x^2 + 2x - 15$

$a = 1$

$b = 2$

$c = -15$

$(x+5)(x-3)$

x	-15	c
+5	-3	
+	2	b

Questions

Factorise

$x^2 + 12x + 36$

Factorise:

$x^2 - 6x - 16$

Factorise:

$x^2 - 12x + 36$

Factorise

$x^2 + 4x + 4$

Factorise:

$x^2 - x - 12$

Factorise:

$x^2 - 8x + 12$

Factorise

$x^2 + 11x + 28$

Factorise:

$x^2 + 2x - 24$

Factorise:

$x^2 - 10x + 21$

Factorise

$x^2 + 9x + 14$

Factorise:

$x^2 + x - 20$

Factorise:

$x^2 - 11x + 24$

ExampleFactorise $x^2 - 100$

DOTS

$(x+10)(x-10)$

ExampleFactorise $x^2 - 36$

DOTS

$(x+6)(x-6)$

Questions

Qu 1A Factorise: $x^2 - 169$	Qu 1B Factorise: $x^2 - 4$	Qu 1C Factorise: $x^2 - 121$	Qu 1D Factorise: $x^2 - 49$
Qu 2A Factorise: $x^2 - 64$	Qu 2B Factorise: $x^2 - 16$	Qu 2C Factorise: $x^2 - 81$	Qu 2D Factorise: $x^2 - 25$
Qu 3A Factorise: $x^2 - 144$	Qu 3B Factorise: $x^2 - 9$	Qu 3C Factorise: $x^2 - 196$	Qu 3D Factorise: $x^2 - 100$

To solve a quadratic it must be equal to zero.

Example

Solve $(x + 3)(x - 2) = 0$

$$\begin{array}{l|l} x+3=0 & x-2=0 \\ \hline -3 & +2 \\ \hline x=-3 & x=2 \\ \checkmark & \checkmark \end{array}$$

Example

Solve $(2x + 3)(4x - 5) = 0$

$$\begin{array}{l|l} 2x+3=0 & 4x-5=0 \\ \hline -3 & +5 \\ \hline \frac{2x}{2} = -\frac{3}{2} & \frac{4x}{4} = \frac{5}{4} \\ \hline x = -1.5 & x = 1.25 \\ \checkmark & \checkmark \end{array}$$

Questions - Solve

(a) $(x - 1)(x - 3) = 0$

(b) $(y - 4)(y - 9) = 0$

(c) $(m + 1)(m + 6) = 0$

(d) $(x - 3)(x + 2) = 0$

(e) $(t + 7)(t - 3) = 0$

(f) $(k - 10)(k + 9) = 0$

(g) $(w + 5)(w + 11) = 0$

(h) $(y - 8)(y - 2) = 0$

(i) $(x + 3)(x - 9) = 0$

Example

Factorise and solve $x^2 + 6x + 8 = 0$

$a=1$
 $b=6$
 $c=8$

x	8	c
+2	+4	
+6	b	

$$(x+2)(x+4)=0$$

$$\begin{array}{l|l} x+2=0 & x+4=0 \\ \hline -2 & -4 \\ \hline x=-2 & x=-4 \\ \checkmark & \checkmark \end{array}$$

Example

Factorise and solve $x^2 - 11x + 18 = 0$

$a=1$
 $b=-11$
 $c=18$

x	18	c
-2	-9	
+[-11]	b	

$$(x-2)(x-9)=0$$

$$\begin{array}{l|l} x-2=0 & x-9=0 \\ \hline +2 & +9 \\ \hline x=2 & x=9 \\ \checkmark & \checkmark \end{array}$$

Questions

(d) $y^2 + 3y - 4 = 0$

(e) $x^2 - 2x - 8 = 0$

(f) $m^2 - 7m + 12 = 0$

(g) $y^2 - 10y + 25 = 0$

(h) $y^2 - 4y - 45 = 0$

(i) $x^2 - x - 56 = 0$

(j) $y^2 + 10y + 24 = 0$

(k) $x^2 + 9x + 18 = 0$

(l) $x^2 + 23x + 22 = 0$

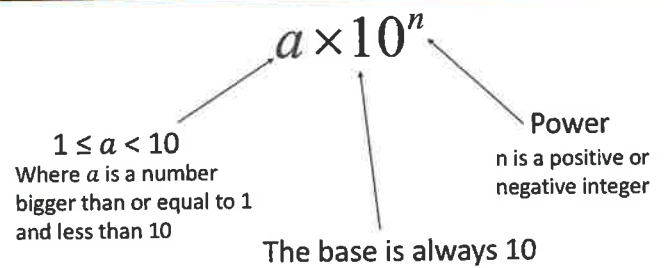
(m) $y^2 - 13y + 22 = 0$

(n) $x^2 + x - 12 = 0$

(o) $m^2 - 6m - 27 = 0$

Topic 23 - Standard Form

Definition – standard form is used as a convenient way to write very big or very small numbers



Circle the number written in standard form.

0.5×10^4

5×10^{-4}

50×10^4

$5 \times 10^{0.4}$

Which number is **not** in standard form?
Circle your answer.

1.01×10^9

0.99×10^{-2}

$9.8 - 10^6$

4.632×10^{-5}

Circle the number that is in standard form.

0.25×10^4

6×10^7

38×10^{-3}

$4 \times 10^{\frac{1}{2}}$

Examples

Write these ordinary numbers in standard form:

$$\begin{array}{l} 400 \\ \text{uu} \\ \text{2 jumps} \\ \rightarrow \end{array} = 4 \times 10^2$$

$$\begin{array}{l} 40500 \\ \text{uuu} \\ \text{4 jumps} \\ \rightarrow \end{array} = 4.05 \times 10^4$$

$$\begin{array}{l} 4243.5 \\ \text{uu} \\ \text{3 jumps} \\ \rightarrow \end{array} = 4.2435 \times 10^3$$

Examples

Write these ordinary numbers in standard form:

$$\begin{array}{l} 0.35 \\ \text{u} \\ \text{1 jump} \\ \leftarrow \end{array} = 3.5 \times 10^{-1}$$

$$\begin{array}{l} 0.00305 \\ \text{uuu} \\ \text{3 jumps} \\ \leftarrow \end{array} = 3.05 \times 10^{-3}$$

$$\begin{array}{l} 0.002 \\ \text{uu} \\ \text{3 jumps} \\ \leftarrow \end{array} = 2 \times 10^{-3}$$

Example

Write the number six million five thousand two hundred in standard form.

Questions – write in standard form

Standard

60000

900

0.09

0.0000007

80000000

0.00002

Challenge

5800

690000

0.085

0.00074

47000

0.000000062

Super Challenge

4 million

5.1 million

7.2 billion

0.0302

0.004521

9

Example

Work out 3000×50000

Give your answer in standard form

150000000
8 jumps
 $\rightarrow$
 $= 1.5 \times 10^8$

Example

Work out $20\,000\,000 \div 40\,000$

Give your answer in standard form

$2000 \div 4 = 500$
 5×10^2
2 jumps
 $\rightarrow$

Questions

Work out $80\,000\,000 \div 200$

Give your answer in standard form.

Work out $2000 \times 70\,000$

Give your answer in standard form.

Work out $\frac{2 \times 10^{14}}{8 \times 10^9}$ Give your answer in standard form.	Work out $32000000 \div 8000$ Give your answer in standard form
2.4×0.002 Give your answer in standard form	Work out 14000000×2000 Give your answer in standard form
Examples Write as an ordinary number 5.6×10^3 3 jumps $\rightarrow$  4.234×10^2 2 jumps $\rightarrow$  2×10^4 4 jumps $\rightarrow$ 	Examples Write as an ordinary number 5.6×10^{-3} 3 jumps $\leftarrow$  4.234×10^{-2} 2 jumps $\leftarrow$  2×10^{-4} 4 jumps $\leftarrow$ 
Questions – write as an ordinary number	
(a) 3×10^4 (e) 5×10^7 (i) 7.7×10^4 (m) 4.05×10^6 (q) 1.0051×10^6	(a) 2×10^{-3} (e) 4.8×10^{-4} (i) 3.16×10^{-5} (m) 2.05×10^{-8}

a) 4×10^2

b) 3×10^{-4}

c) 7.7×10^1

d) 3.51×10^{-2}

e) 2.9×10^{-3}

f) 4.05×10^4

g) 8.1958×10^{-2}

h) 7.8209×10^3

i) 2.143578×10^8

j) 2.4×10^{-4}

k) 1.0008×10^5

l) 2.518×10^3

Work out

$$\frac{4.2 \times 10^3}{2 \times 10^2}$$

Work out

$$\frac{3 \times 10^5}{4 \times 10^3}$$

Work out

$$\frac{2 \times 10^2}{5 \times 10^{-1}}$$

Work out

$$\frac{1.8 \times 10^2}{3 \times 10^{-1}}$$

A teacher asks Amy and Jack to convert 101 376 into standard form.

(a) Amy writes 10.1376×10^4

Criticise Amy's answer.

(b) Jack writes 1.01376×10^{-5}

Criticise Jack's answer.

Example

Write these numbers in order of size, smallest to largest.

2.04×10^3

250

0.3×10^3

4×10^{-2}

2040

$0300 = 300$

0.04

$4 \times 10^{-2}, 250, 0.3 \times 10^3, 2.04 \times 10^3$

Questions

Here is a list of numbers.

Put them into descending order

1 000 000 4.6×10^4 63 000 5×10^3 1.7×10^5

Write these numbers in ascending order:

5.12×10^4 , 512999, 5.3×10^3 , 53.1×10^3

Write these numbers in ascending order.

9812 9.82×10^2 9.81×10^3

Put these numbers in order from smallest to largest.

8×10^{-4} 4×10^{-2} 6×10^{-4} 0.07

Write these numbers in **descending** order.

9563 9.56×10^3 9.56×3^{10}

Write these numbers in **descending** order.

9563 9.56×10^3 9.56×3^{10}

Sometimes a number looks like it is in standard form but it isn't. When this happens, we need to 'fix' it so that it is standard form. We can do this by compensating.

Examples

Write these numbers in standard form:

51×10^3 61×10^3
 $\div 10$ $\times 10$ $\div 10$ $\times 10$
 5.1×10^4 6.1×10^4

Examples

Write these numbers in standard form:

0.5×10^3 0.423×10^3
 $\times 10$ $\div 10$ $\times 10$ $\div 10$
 5×10^2 4.23×10^2

Questions – Fix these numbers so that they are in standard form

(a) 72×10^3

(b) 84×10^6

(c) 500×10^2

(d) 210×10^4

(e) 0.8×10^7

(f) 0.46×10^5

(g) 0.06×10^8

(h) 0.007×10^{12}

Example

Work out. Give your answer in standard form.

$$(2 \times 10^5) \times (3 \times 10^3)$$

$$6 \times 10^8$$

Example

Work out. Give your answer in standard form.

$$(4.3 \times 10^5) \times (2 \times 10^3)$$

$$8.6 \times 10^8$$

Example

Work out. Give your answer in standard form.

$$(3 \times 10^2) \times (5 \times 10^3)$$

$$15 \times 10^5$$

$$\div 10 \quad \times 10$$

$$1.5 \times 10^6$$

Example

Work out. Give your answer in standard form.

$$(4.3 \times 10^5) \times (2 \times 10^3)$$

$$8.6 \times 10^8$$

Example

Work out. Give your answer in standard form.

$$(2 \times 10^5) \times (4 \times 10^{-3})$$

$$8 \times 10^2$$

Example

Work out. Give your answer in standard form.

$$(4.5 \times 10^{-2}) \times (6 \times 10^{-4})$$

$$27 \times 10^{-6}$$

$$\div 10 \quad \times 10$$

$$2.7 \times 10^{-5}$$

Questions – work out and give your answers in standard form

(a) $2 \times 10^3 \times 3 \times 10^4$

(b) $(4 \times 10^8) \times (2 \times 10^5)$

(c) $1.2 \times 10^4 \times 6 \times 10^4$

(d) $(3 \times 10^{-9}) \times (3 \times 10^3)$

(e) $4 \times 10^{-7} \times 3 \times 10^{-2}$

(f) $6 \times 10^{10} \times 4 \times 10^8$

(g) $7 \times 10^{12} \times 8 \times 10^{-9}$

(h) $3.7 \times 10^5 \times 5 \times 10^6$

(i) $(8 \times 10^3) \times (5 \times 10^5)$

(j) $5 \times 10^{-14} \times 4 \times 10^{-7}$

(k) $1.8 \times 10^2 \times 2 \times 10^{-8}$

(l) $5.8 \times 10^6 \times 4 \times 10^7$

Topic 24 - Rearranging

Changing the subject is the same as 'solving'. We rearrange to solve for a particular letter.
For example, 'make x the subject' means that we solve for x (ie $x = \dots$)

Example

Make x the subject

$$x + 3a = m$$

$$\quad \quad \quad -3a \quad -3a$$

$$x = m - 3a$$

Example

Make a the subject

$$2ax = B$$

$$\quad \quad \quad \cancel{2x} \quad \cancel{2x}$$

$$a = \frac{B}{2x}$$

Example

Make y the subject

$$y^2 - x = m$$

$$\quad \quad \quad +x \quad +x$$

$$y^2 = m + x$$

$$\sqrt{\quad} \quad \quad \sqrt{\quad}$$

$$y = \sqrt{m + x}$$

Example

Make x the subject

$$\frac{5}{x} = A$$

$$\quad \quad \quad \times x \quad \times x$$

$$\frac{5}{A} = \frac{Ax}{A}$$

$$\frac{5}{A} = x$$

Questions

Question 1: Make y the subject of each of the following

(a) $y + w = c$

(b) $y - p = m$

(c) $m + y = s$

(d) $y - 2g = n$

(e) $3y = c$

(f) $ay = w$

(g) $\frac{y}{c} = w$

(h) $\frac{y}{a} = 2c$

(i) $a = y + p$

(j) $c = y - k$

(k) $y^2 = s$

(l) $y^3 = x$

(m) $\sqrt{y} = g$

(n) $\pi y = c$

(o) $n - y = t$

(p) $ry = c$

(q) $4\pi y = b$

(r) $y + 7t = c + r$

(s) $\frac{r}{y} = w$

(t) $y^2 = k + x$

(u) $A = xy$

ExampleMake x the subject

$$\begin{aligned}
 2x + a &= m \\
 -a &-a \\
 \frac{2x}{2} &= \frac{m-a}{2} \\
 x &= \frac{m-a}{2}
 \end{aligned}$$

ExampleMake y the subject

$$\begin{aligned}
 \frac{y-a}{c} &= x \\
 \times c &\times c \\
 y-a &= cx \\
 +a &+a \\
 y &= cx+a
 \end{aligned}$$

ExampleMake x the subject

$$\begin{aligned}
 4t + ax^2 &= y \\
 -4t &-4t \\
 \frac{ax^2}{a} &= \frac{y-4t}{a} \\
 x^2 &= \frac{y-4t}{a} \\
 \sqrt{} &
 \end{aligned}$$

$$x = \sqrt{\frac{y-4t}{a}}$$

QuestionsQuestion 2: Make x the subject of the following formulae

(a) $4x + c = w$

(b) $dx - t = 8$

(c) $x^2 + 3 = h$

(d) $2x + 2y = p$

(e) $s = x^2 - 3$

(f) $y = xz + s$

(g) $\frac{x}{n} + 2 = w$

(h) $\frac{x}{6} - 5 = w$

(i) $\frac{x+3}{c} = h$

(j) $3y = 4x + 1$

(k) $x^2 + a = v$

(l) $x^3 - 4 = 5y$

(m) $\frac{x+t}{m} = 2c$

(n) $\frac{w+x}{u} = 3z$

(o) $\Lambda = \pi x^2$

(p) $\Lambda = \frac{1}{2}bx$

(q) $V = abx$

(r) $v^2 = u^2 + 2ax$

(s) $\frac{a+b}{x} = r$

(t) $\frac{5cx}{b} = a$

(u) $\sqrt[3]{\frac{x}{k}} = w$

The circumference of a circle is given as $c = 2\pi r$
 Make the radius, r , the subject of the formula.

The formula to convert degrees Fahrenheit to degree Celsius is $\frac{5}{9}(F - 32) = C$

Find the formula to convert from degrees Celsius to degrees Fahrenheit by making F the subject.

Spot the mistake:

Make y the subject of the formula:

$$k = y^2 + a$$

$$\sqrt{k} = y + a$$

$$\sqrt{k} - a = y$$

$$y = \sqrt{k} - a$$

Spot the mistake:

Express v in terms of t

$$t = \frac{v}{4} + 1$$

$$t - 1 = \frac{v}{4}$$

$$\frac{t-1}{4} = v$$

Topic 25 - Linear Simultaneous Equations

To solve simultaneous equations we use the elimination method. This means that we need to make one of the letters the same coefficient. Sign doesn't matter! We can choose any letter to make the same.

Type 1 – no scaling

Example

Solve the simultaneous equations

$$x + 7y = 64 \quad (1)$$

$$- \quad x + 3y = 28 \quad (2)$$

$$\begin{array}{r} 4y = 36 \\ \hline 4 \quad 4 \\ y = 9 \end{array}$$

Substitute into (2):

$$x + 3(9) = 28$$

$$x + 27 = 28$$

$$\begin{array}{r} -27 \quad -27 \\ x = 1 \end{array}$$

$$x = 1, y = 9$$

Example

Solve the simultaneous equations

$$5x + y = 11 \quad (1)$$

$$+ \quad 3x - y = 9 \quad (2)$$

$$\begin{array}{r} 8x = 20 \\ \hline 8 \quad 8 \\ x = 2.5 \end{array}$$

Substitute into (1):

$$5(2.5) + y = 11$$

$$12.5 + y = 11$$

$$\begin{array}{r} -12.5 \quad -12.5 \\ y = -1.5 \end{array}$$

$$x = 2.5, y = -1.5$$

Example

Solve the simultaneous equations

$$4x - 4y = 24 \quad (1)$$

$$- \quad x - 4y = 3 \quad (2)$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

Substitute into (2):

$$4(7) - 4y = 24$$

$$28 - 4y = 24$$

$$-28 \quad -28$$

$$\frac{-4y}{-4} = \frac{-4}{-4}$$

$$y = 1$$

$$x = 7, y = 1$$

Example

Solve the simultaneous equations

$$2x + 4y = 14 \quad (1)$$

$$+ \quad 4x - 4y = 4 \quad (2)$$

$$\frac{6x}{6} = \frac{18}{6}$$

$$x = 3$$

Substitute into (1):

$$2(3) + 4y = 14$$

$$6 + 4y = 14$$

$$-6 \quad -6$$

$$\frac{4y}{4} = \frac{8}{4}$$

$$y = 2$$

$$x = 3, y = 2$$

Questions

Question 1: Solve the following simultaneous equations by using elimination.

(a) $6x + y = 18$
 $4x + y = 14$

(b) $4x + 2y = 10$
 $x + 2y = 7$

(c) $9x - 4y = 19$
 $4x + 4y = 20$

(d) $2x + y = 36$
 $x - y = 9$

(e) $6x - 3y = 12$
 $4x - 3y = 2$

(f) $3x - 6y = 6$
 $2x - 6y = 3$

(g) $8x + 7y = 39$
 $8x + 2y = 34$

(h) $x + 3y = 38$
 $x + 6y = 53$

(i) $6x + 3y = 48$
 $6x + y = 26$

(j) $2x - 4y = 10$
 $2x + 3y = 24$

(k) $5x - 2y = 120$
 $5x + y = 165$

(l) $x - 2y = 8$
 $x - 3y = 3$

(m) $3x + 2y = 54$
 $2x - 2y = 16$

(n) $7x - 4y = 80$
 $3x - 4y = -80$

(o) $5x - 2y = -23$
 $5x - 6y = -39$

(p) $6x + 2y = -26$
 $2x + 2y = -10$

(q) $x - 5y = 65$
 $2x - 5y = 85$

(r) $10x - 10y = -40$
 $10x + 4y = 16$

Type 2 – Scaling one equation

Example

Solve the simultaneous equations

$$2x + 4y = 26 \quad (1)$$

$$3x - y = 4 \quad (2) \times 4$$

$$\begin{array}{r} 2x + 4y = 26 \\ + 12x - 4y = 16 \\ \hline 14x = 42 \\ \frac{14x}{14} = \frac{42}{14} \\ x = 3 \end{array}$$

Substitute into (1):

$$\begin{array}{r} 2(3) + 4y = 26 \\ 6 + 4y = 26 \\ -6 \quad -6 \\ \hline 4y = 20 \\ \frac{4y}{4} = \frac{20}{4} \\ y = 5 \end{array}$$

$$x = 3, y = 5$$

Example

Solve the simultaneous equations

$$3x - y = 23 \quad (1) \times 3$$

$$2x + 3y = 8 \quad (2)$$

$$\begin{array}{r} 9x - 3y = 69 \\ + 2x + 3y = 8 \\ \hline 11x = 77 \\ \frac{11x}{11} = \frac{77}{11} \\ x = 7 \end{array}$$

Substitute into (2):

$$\begin{array}{r} 2(7) + 3y = 8 \\ 14 + 3y = 8 \\ -14 \quad -14 \\ \hline 3y = -6 \\ \frac{3y}{3} = \frac{-6}{3} \\ y = -2 \end{array}$$

$$x = 7, y = -2$$

Questions

Question 2: Solve the following simultaneous equations by using elimination.

(a) $\begin{array}{l} 3x + 2y = 23 \\ 2x - y = 6 \end{array}$

(b) $\begin{array}{l} 3x - 3y = 9 \\ 2x + y = 12 \end{array}$

(c) $\begin{array}{l} 4x + 2y = 34 \\ 3x + y = 21 \end{array}$

(d) $\begin{array}{l} 9x - 4y = 59 \\ 2x - y = 12 \end{array}$

(e) $\begin{array}{l} 2x + 8y = 43 \\ x + 3y = 18 \end{array}$

(f) $\begin{array}{l} 6x + 3y = 45 \\ 2x - 2y = 12 \end{array}$

(g) $\begin{array}{l} 5x + 4y = 130 \\ x + 6y = 130 \end{array}$

(h) $\begin{array}{l} 10x - 15y = 25 \\ x - 2y = 1 \end{array}$

(i) $\begin{array}{l} 3x + 8y = 97 \\ 2x + 4y = 58 \end{array}$

(j) $\begin{array}{l} 3x - y = 4 \\ 5x + 4y = 52 \end{array}$

(k) $\begin{array}{l} 4x + 9y = 10 \\ 2x + 3y = 2 \end{array}$

(l) $\begin{array}{l} 5x - 3y = 33 \\ 3x - 9y = 63 \end{array}$

(m) $\begin{array}{l} 2x + 4y = -2 \\ 4x + 2y = -10 \end{array}$

(n) $\begin{array}{l} 8x + 4y = -28 \\ 3x - 12y = 30 \end{array}$

(o) $\begin{array}{l} 15x - 4y = 82 \\ 5x - 9y = 12 \end{array}$

Type 3 – Scaling both equations

Example *Make x same*

Solve the simultaneous equations

$$3x + 5y = 1 \quad (1) \times 2$$

$$2x - 3y = 7 \quad (2) \times 3$$

$$\begin{array}{r} 6x + 10y = 2 \\ - 6x - 9y = 21 \\ \hline \end{array}$$

$$\frac{19y}{19} = \frac{-19}{19}$$

$$y = -1$$

Substitute into (1):

$$3x + 5(-1) = 1$$

$$3x - 5 = 1$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

$$x = 2, y = -1$$

Example

Make y same

Solve the simultaneous equations

$$2x + 9y = 43 \quad (1) \times 2$$

$$3x + 2y = 7 \quad (2) \times 9$$

$$\begin{array}{r} 4x + 18y = 86 \\ - 27x + 18y = 63 \\ \hline \end{array}$$

$$\frac{-23x}{-23} = \frac{28}{-23}$$

$$x = -1$$

Substitute into (1):

$$2(-1) + 9y = 43$$

$$-2 + 9y = 43$$

$$\frac{9y}{9} = \frac{45}{9}$$

$$y = 5$$

$$x = -1, y = 5$$

Questions

Question 3: Solve the following simultaneous equations by using elimination.

(a) $2x + 2y = 14$
 $5x - 3y = 19$

(b) $2x + 3y = 1$
 $7x + 2y = -22$

(c) $5x + 3y = 22$
 $2x + 4y = 20$

(d) $5x - 6y = 28$
 $4x - 4y = 24$

(e) $3x + 2y = 7$
 $2x + 9y = 43$

(f) $3x + 3y = -6$
 $4x - 4y = -24$

(g) $3x + 8y = 31$
 $5x + 3y = 31$

(h) $7x - 15y = 2.5$
 $3x - 2y = 5.5$

(i) $3x + 2y = 53$
 $2x + 5y = 72$

(j) $5x - 3y = 18$
 $2x + 4y = 54$

(k) $2x + 9y = 11$
 $9x + 3y = -63$

(l) $2x - 4y = 4$
 $5x - 3y = 24$

(m) $3x + 3y = 42$
 $2x + 4y = 38$

(n) $6x + 2y = -2$
 $4x - 3y = 29$

(o) $4x - 4y = 8$
 $5x - 3y = 18$

(p) $4x + 3y = 9$
 $5x + 2y = 13$

(q) $4x - 2y = 18$
 $2x - 3y = 15$

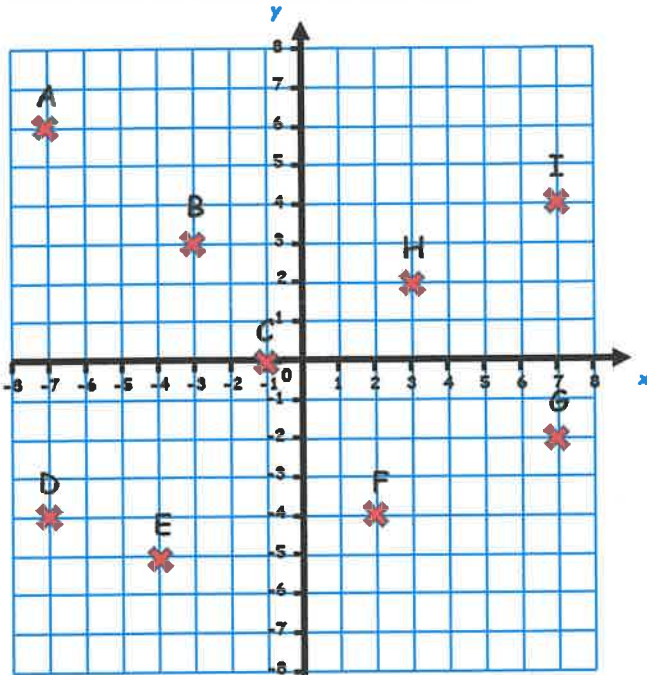
(r) $5x + 2y = 38$
 $2x - 3y = 19$

Topic 26 - Drawing Straight Line Graphs

The equation of a straight line is given by $y = mx + c$

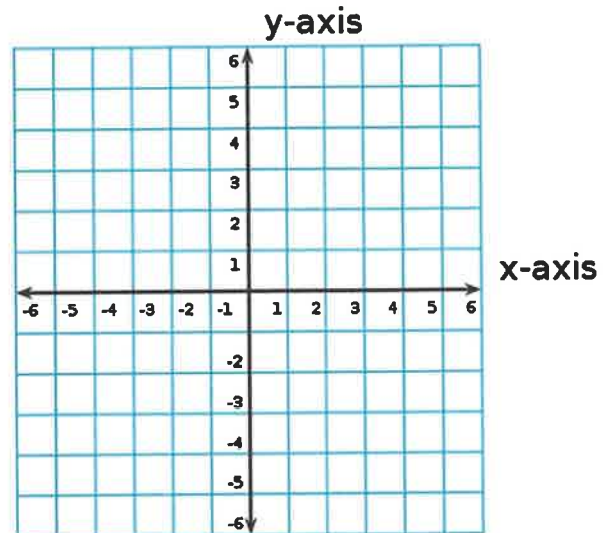
- M is the gradient. This tells us how steep the line is.
- C is the y-intercept. This tells us where the line meets the y-axis.

Write down the coordinates of each letter



Plot each coordinate on the graph:

- A = (4,4) B = (6,1) C = (3, -2)
D = (-3,4) E = (-1,5) F = (-6, -4)



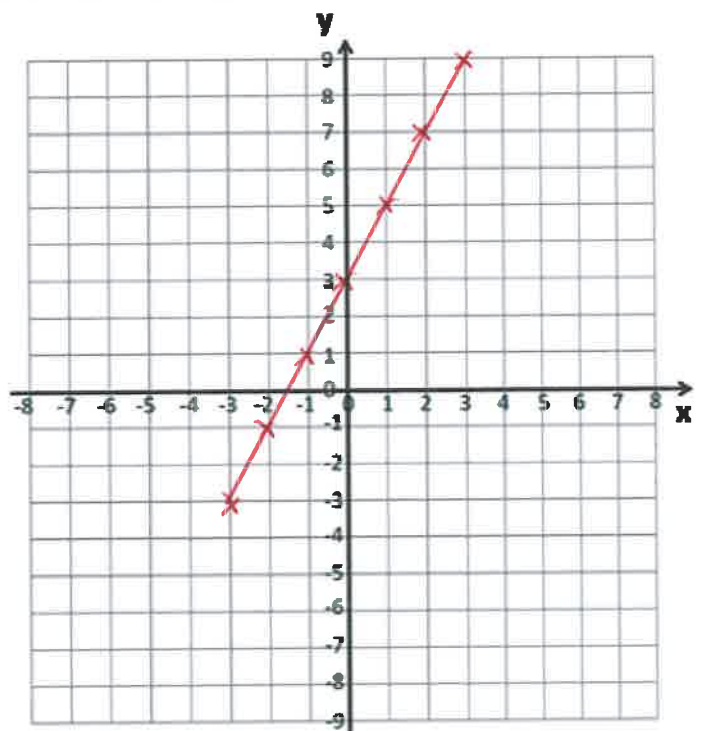
Example

Draw the graph of $y = 2x + 3$ for x values from $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

- Gradient = 2
- y-intercept = 3

- $2(3) + 3 = 6 + 3 = 9$
- $2(2) + 3 = 4 + 3 = 7$
- $2(1) + 3 = 2 + 3 = 5$
- $2(0) + 3 = 0 + 3 = 3$
- $2(-1) + 3 = -2 + 3 = 1$
- $2(-2) + 3 = -4 + 3 = -1$
- $2(-3) + 3 = -6 + 3 = -3$



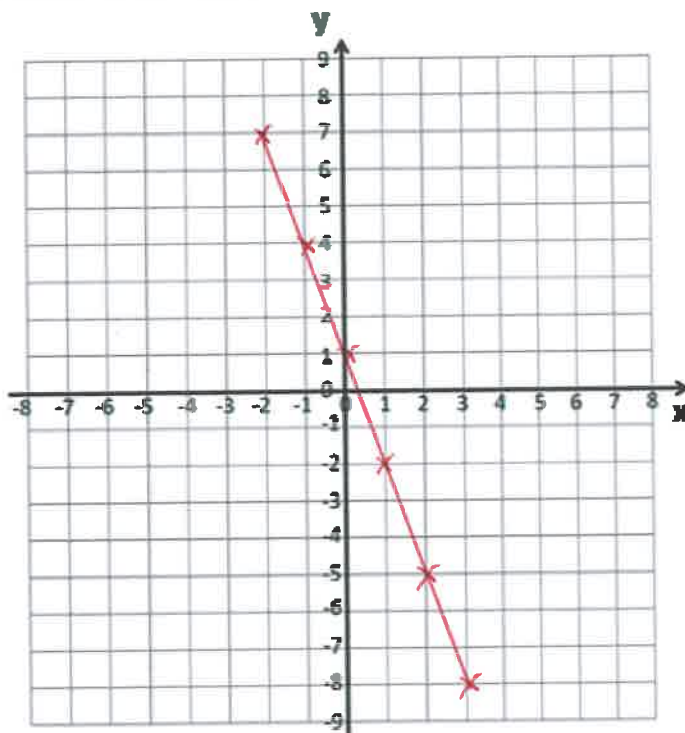
Example

Draw the graph of $y = -3x + 1$ for x values from $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	10	7	4	1	-2	-5	-8

- Gradient = ~~2~~ -3
- y-intercept = ~~3~~ 1

- $-3(3) + 1 = -9 + 1 = -8$
- $-3(2) + 1 = -6 + 1 = -5$
- $-3(1) + 1 = -3 + 1 = -2$
- $-3(0) + 1 = 0 + 1 = 1$
- $-3(-1) + 1 = 3 + 1 = 4$
- $-3(-2) + 1 = 6 + 1 = 7$
- $-3(-3) + 1 = 9 + 1 = 10$



Questions

Question 1: For each equation, complete the table of values and draw its graph for values of x from -1 to 3 .

(a) $y = 2x + 1$

x	-1	0	1	2	3
y	-1	1			7

(b) $y = 3x - 1$

x	-1	0	1	2	3
y	-4			5	

(c) $y = 2x - 3$

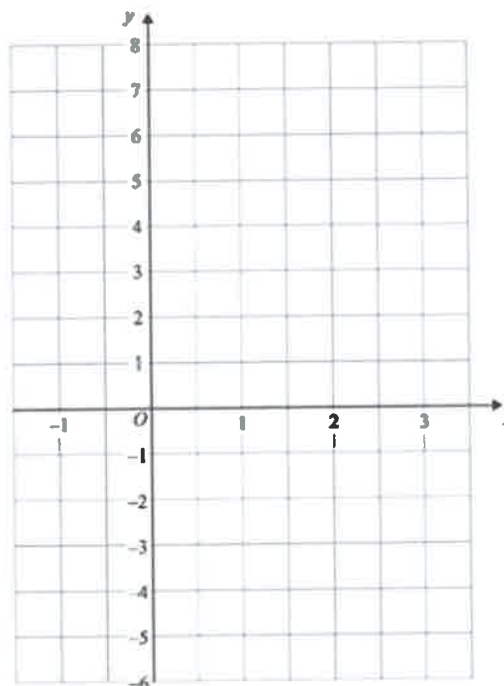
x	-1	0	1	2	3
y		-3	-1		

(d) $y = x + 4$

x	-1	0	1	2	3
y					7

(e) $y = 2x$

x	-1	0	1	2	3
y		0			6



Question 2: For each equation, complete the table of values and draw its graph for values of x from -2 to 3 .

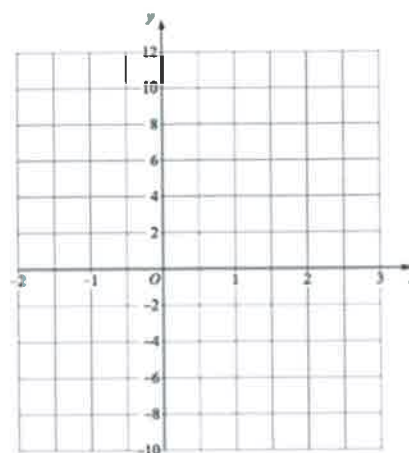
(a) $y = 2x + 4$

x	-2	-1	0	1	2	3
y						

(b) $y = 4x - 2$

x	-2	-1	0	1	2	3
y						

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Question 3: For each equation, complete the table of values and draw its graph for values of x from -2 to 2 .

(a) $y = 3x + 3$

x	-2	-1	0	1	2
y					

(b) $y = x + 9$

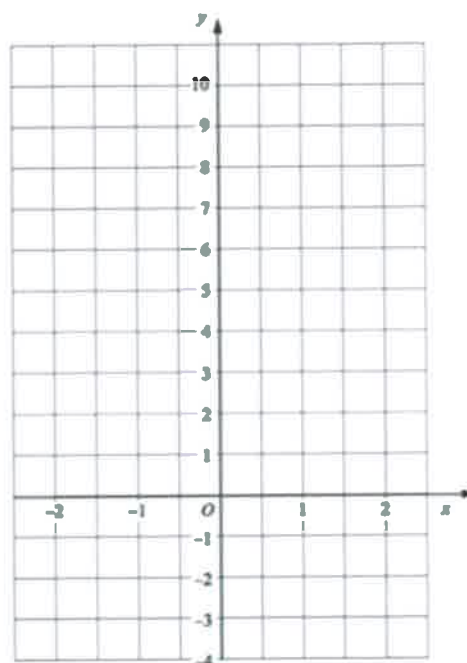
x	-2	-1	0	1	2
y					

(c) $y = x - 2$

x	-2	-1	0	1	2
y					

(d) $y = x$

x	-2	-1	0	1	2
y					



Question 5: For each equation, complete the table of values and draw its graph for values of x from -1 to 3 .

(a) $y = -2x + 5$

x	-1	0	1	2	3
y					

(b) $y = -x - 2$

x	-1	0	1	2	3
y					

(c) $y = -2x$

x	-1	0	1	2	3
y					

(d) $y = 6 - x$

x	-1	0	1	2	3
y					

